NH equations, academic tests

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NH EQUATIONS

NH equations

The original Euler Equations (EE) system :

- takes into acount compressibility of the fluid
- valid at any scale for a perfect fluid
  (perfect gaz, one phase, inviscid,…)
- sources linked to diabatism, viscosity, turbulence, etc., are added afterward

NH equations

The original Euler Equations (EE) system is first written locally in z coordinate

Then transformed to mass coordinate

Then for use in meteorology (with orography), transformed in terrain-following coordinate

NH equations (in z)

The original Euler Equations (EE) system :

$$\frac{dv}{dt} = -\frac{1}{\rho}\nabla_z p$$

$$\frac{dw}{dt} = -g - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\frac{dT}{dt} = -\frac{RT}{C_v}D_3 + \frac{Q}{C_v}$$

$$\frac{d\rho}{dt} = -\rho D_3$$

where :

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V}.\nabla_z + w\frac{\partial}{\partial z}$$

$$D_3 = \nabla_z.\mathbf{V} + \frac{\partial w}{\partial z}$$

$$p = \rho RT$$

Transformation to "mass coordinate"

Choose vertical coordinate by:

$$\frac{\partial \pi}{\partial z} = -\rho g$$

Transformation rules:

$$\frac{\partial}{\partial z} = -\frac{gp}{RT}\frac{\partial}{\partial \pi}$$

$$\nabla_z = \nabla_\pi + \frac{gp}{RT}\left(\nabla_\pi z\right)\frac{\partial}{\partial \pi}$$

NH equations (in π)

The EE system in mass coordinate:

$$\frac{dv}{dt} = -\frac{RT}{p}\nabla_{\pi} p - \frac{\partial p}{\partial \pi}\nabla_{\pi}\phi$$

$$\frac{dw}{dt} = g\left(\frac{\partial p}{\partial \pi} - 1\right)$$

$$\frac{dT}{dt} = -\frac{RT}{C_v} D_3 + \frac{Q}{C_v}$$

$$\frac{dp}{dt} = -p\left(\frac{C_p}{C_v}\right) D_3 + \frac{Qp}{C_v T}$$

where :

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V}.\nabla_{\pi} + \dot{\pi}\frac{\partial}{\partial \pi}$$

$$D_3 = \nabla_{\pi}.\mathbf{V} + \frac{p}{RT}(\nabla_{\pi}\phi).\left(\frac{\partial V}{\partial \pi}\right) - \frac{gp}{RT}\frac{\partial w}{\partial \pi}$$

$$\dot{\pi} = -\int_{\pi_T}^{\pi}(\nabla_{\pi}.\mathbf{V})\, d\pi'$$

$$\phi = \int_{\pi}^{\pi_s}\frac{RT}{p}\, d\pi'$$

This EE system in π is adapted only for domains with constant and uniform values of π at top and bottom (π_T and π_s).

➨ Not well suited for NWP

➨ Reformulate in terrain-following coordinate

Pure π–based terrain following : $\sigma = (\pi_{/}\pi_s)$

have large variations of geopotential field near

top : not well-suited for NWP as well.

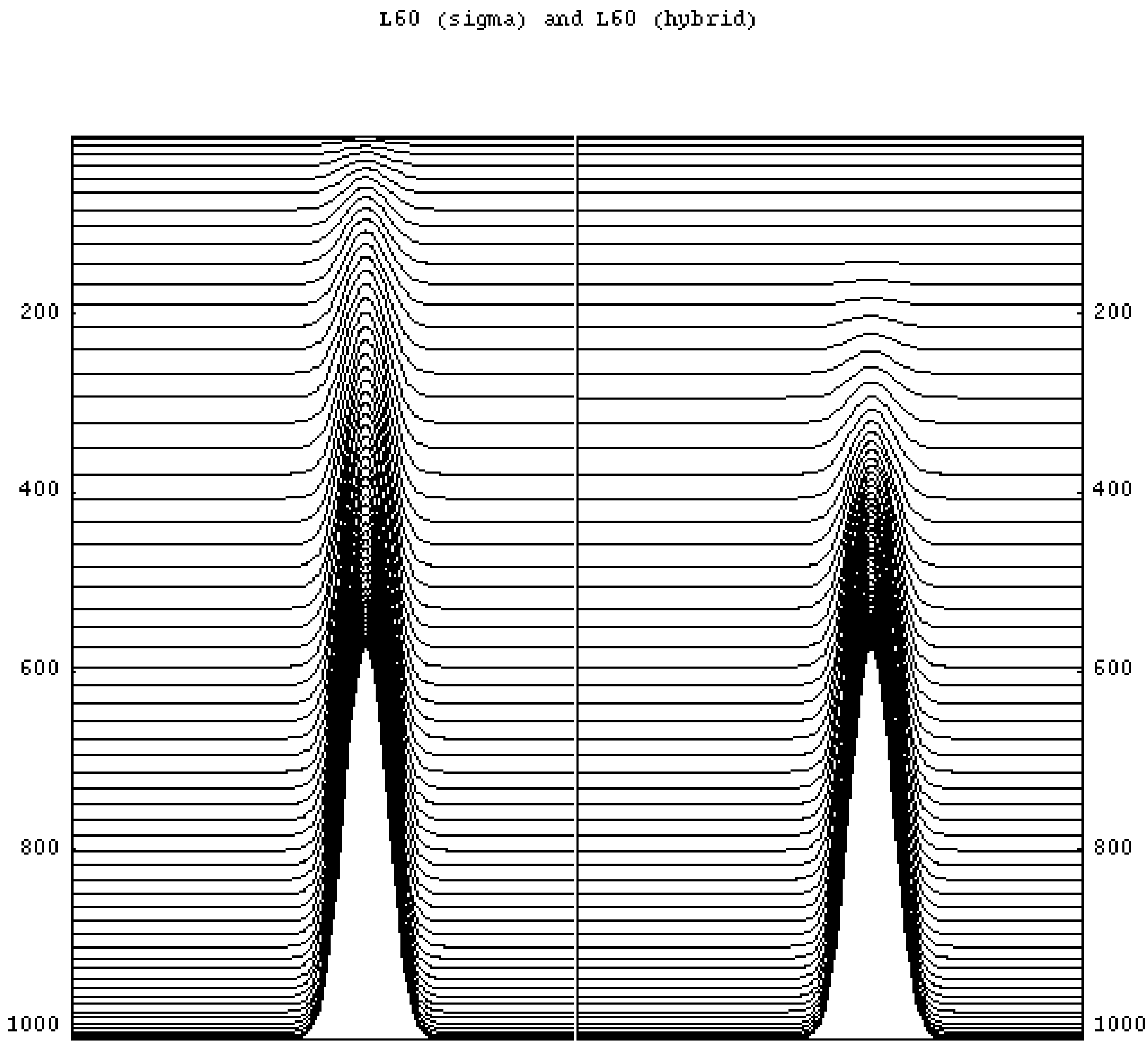
Reformulate in hybrid terrain-following mass coordinate:

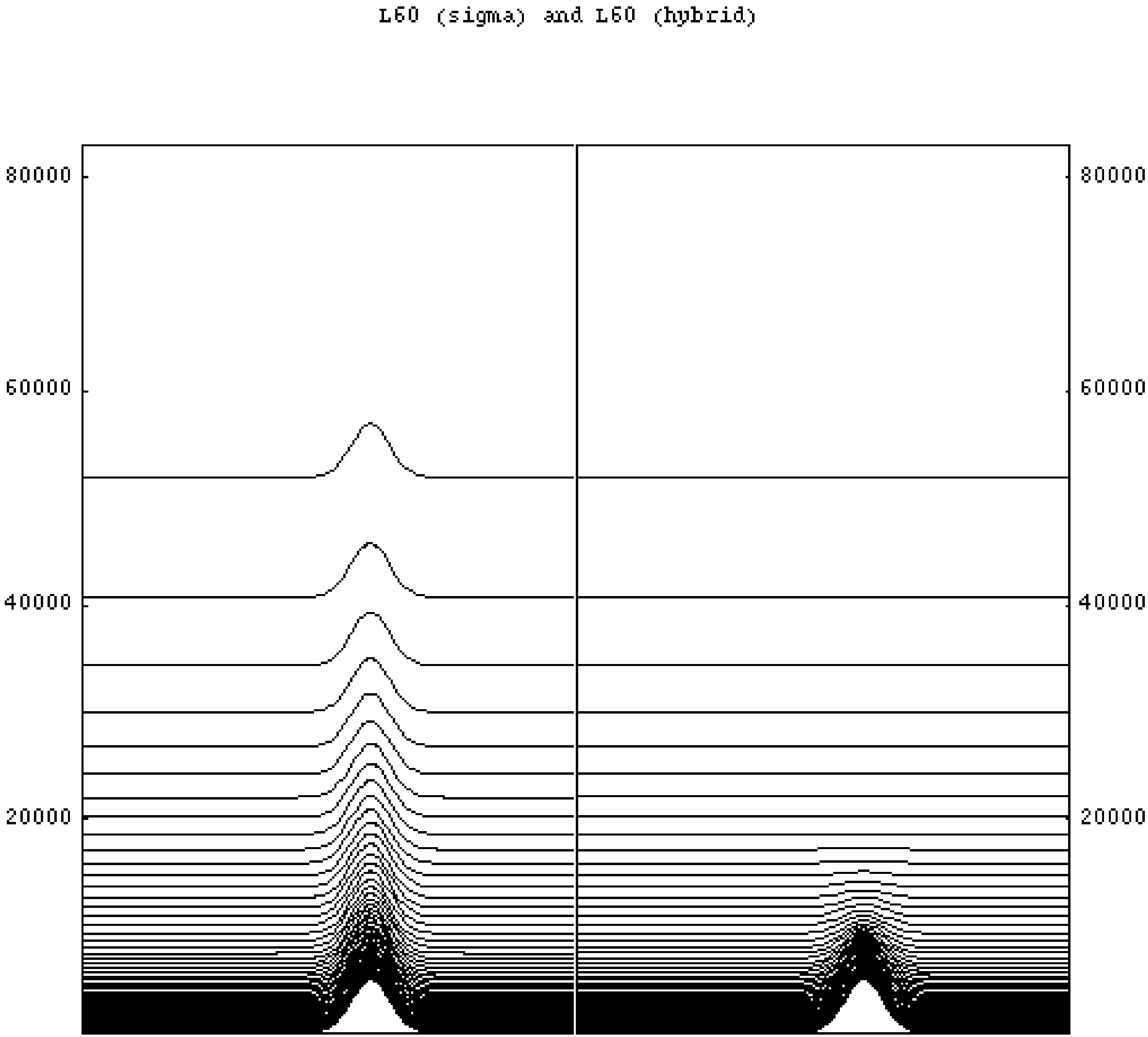
$$\pi(\eta) = A(\eta) + B(\eta)\,\pi_s$$

Where A and B are two specified functions.

Levels where A=0 ➨ pure terrain-following coordinate

Levels where B=0 ➨ pure mass coordinate


L60 (sigma) and L60 (hybrid)
200
400
600
800
1000
200
400
600
800
1000


L60 (sigma) and L60 (hybrid)
80000
60000
40000
20000
80000
60000
40000
20000

NH equations (in η)

The EE system in mass η coordinate:

$$\frac{dv}{dt} = -\frac{RT}{p}\nabla p - \frac{1}{m}\frac{\partial p}{\partial \eta}\nabla \phi$$

$$\frac{d\mathrm{d}}{dt} = -g^2 \frac{p}{mRT}\frac{\partial}{\partial \eta}\left(\frac{1}{m}\frac{\partial \pi P}{\partial \eta}\right) + g\frac{p}{mRT}\frac{\partial V}{\partial \eta}\nabla w + \mathrm{d}(\mathrm{X}\text{-}\mathrm{d}) + \dot{\mathrm{X}}$$

$$\frac{dT}{dt} = -\frac{RT}{C_v}D_3 + \frac{Q}{C_v}$$

$$\frac{dP}{dt} = -(1+P)\left(\frac{C_p}{C_v}D_3 + \frac{\dot{\pi}}{\pi}\right) + (1+P)\frac{Q}{C_v T}$$

NH equations (in η)

The EE system in mass η coordinate (cont'd):

$$\frac{\partial \pi_s}{\partial t} = -\int_0^1 (\nabla . m\mathbf{V})\, d\eta'$$

NH equations (in η)

The NH prognostic variables are changed for stability reasons (explained after)

$$P = \frac{(p - \pi)}{\pi}$$

$$\mathrm{d} = -\frac{gp}{mRT}\frac{\partial w}{\partial z} + \frac{p}{mRT}\nabla\phi.\frac{\partial V}{\partial \eta}$$

Diagnostic relationships :

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V}.\nabla + \dot{\eta}\frac{\partial}{\partial \eta}$$

$$D_3 = \nabla.\mathbf{V} + \mathrm{X} - \frac{gp}{mRT}\frac{\partial w}{\partial \eta}$$

$$m\dot{\eta} = B\int_0^1 (\nabla.m\mathbf{V})\, d\eta' - \int_0^{\eta} (\nabla.m\mathbf{V})\, d\eta'$$

$$\phi = \phi_s + \int_{\eta}^{1} \frac{mRT}{p}\, d\eta'$$

$$\mathrm{X} = \frac{p}{mRT}(\nabla\phi).\left(\frac{\partial V}{\partial \eta}\right)$$

Spatial discretization

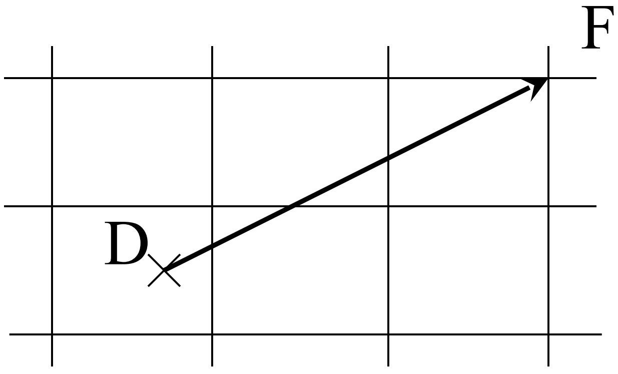
- In the vertical:
  finite differences (most simple, less accurate)
  finite-elements are in progress (more difficult to design)

- In the horizontal:
  spectral (derivatives, inversion of spatial operators)
  cheap and accurate method (but limitations)

Time discretization

•Semi-Lagrangian (SL)

$$\frac{d\psi}{dt} = \frac{\psi_F^{t+\Delta t} - \psi_{D*}^{t}}{\Delta t}$$



The value of ψ must be interpolated at D for each point F
More stable than a traditional Eulerian discretisation
Possibility of controlling diffusivity non-linearly (SLHD)

Time discretization

•Semi-Implicit (SI) $$\frac{dX}{dt} = M(X)$$

$$\frac{X^{t+\Delta t} - X^{t}}{\Delta t} = M\left(X^{t+\Delta t/2} \right) + L^{*}\left(\frac{X^{t+\Delta t} + X^{t}}{2} - X^{t+\Delta t/2} \right)$$

The value of X(t+Δt/2) is obtained through a simple time-extrapolation using (t-Δt,t) states (two-time level scheme)

The implicit treatment of the linear system L* increases the stability of the whole system.

Time discretization

•Iterative Centred-Implicit (ICI) $$\frac{dX}{dt} = M(X)$$

$$\frac{\left[X^{t+\Delta t}\right]^{(k)} - X^{t}}{\Delta t} = M\left(\frac{\left[X^{t+\Delta t}\right]^{(k-1)} + X^{t}}{2}\right) + L^{*}\left(\frac{\left[X^{t+\Delta t}\right]^{(k)} - \left[X^{t+\Delta t}\right]^{(k-1)}}{2}\right)$$

If convergence: implicit treatment of all dynamical sources
Increases stability of the dynamical part of the system.
One interation (after SI) is called "PC scheme"

Stability

The EE system is "hard" to make stable (in comparison with HPE)

It is tough to design a sufficiently stable system for NWP, especially in the spectral context, because L* needs to be horizontally-homogeneous and time-independent (for efficiency)

Studies shows that the choice of prognostic variables is quite critical.

ACADEMIC TESTS

Academic tests

Two kind of tools:

- "off-line" discrete linear analysis for assessing basic stability in very simplified contexts.

- "academic" setting of the normal model, for assessing the relevance of the response.

Off-line linear analyses tool

- Simulates the behaviour of the model for a simplified resting state (dry, adiabatic, motionless, horizontally homogeneous)

-allows thermal or orographic explicitly-treated residuals (those which are the most likely responsible for instabilities)

-in the vertical: reflects the discretization of the true model
-in the horizontal: assumes a single Fourier component

-In practice: computes the space- and time- discrete eigenvalues of the system (using a linear algebra program): assesses "stability"

Discrete linear analysis tool

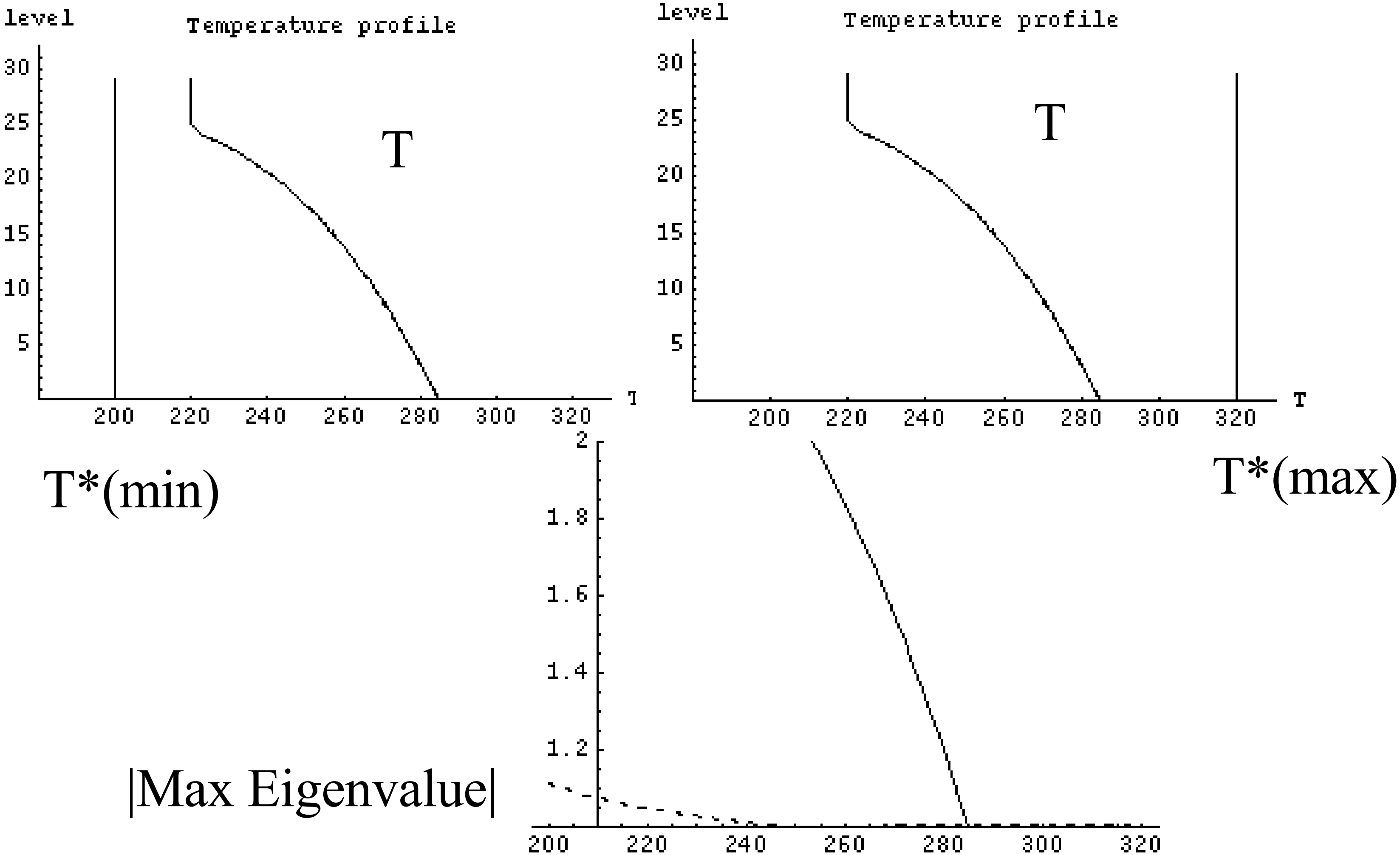
$$\frac{dX}{dt} = M(X)$$

Semi-Implicit (SI) with linear residual

$$\frac{X^{t+\Delta t} - X^{t}}{\Delta t} = \bar{L}\left(X^{t+\Delta t/2} \right) + L^{*}\left(\frac{X^{t+\Delta t} + X^{t}}{2} - X^{t+\Delta t/2} \right)$$

The stability is controlled by the structure of residuals (Lbar- L*) which are treated explicitly.
The stability of the linear part of the model is controlled by the structure of the linear system L*

Exemple: stability of 3-TL scheme with thermal residuals



Academic settings of normal model

- 2D (Vert. Plan) or 3D domains

- Analytic initial and boundary conditions

- ideally, possibility of an analytic reference solution (or well validated with other models)

- usually, stationary response searched

Running 2D (Vert. Plan) or 3D domains

- Common features: (**`LMAP=.F.`**)
  Idealised plane geometry: (map fact. m = 1)
  located nowhere in the world (lat, lon def value)
  Coriolis term = 0 (or constant)
  radiation scheme ??? => adiabatic in practice

- 3D domains (simplest): run normally

- 2D domains (make the "x" width vanishing)
(**`NDLUXG=NDLON=1,  NBZONL=0`**)

Making initial and boundary conditions

- We must prepare a INIT or BC file which mimics a conventional file, in the same format ("FA" file)

- off-line tool in which conditions are specified by hand (T profiles, U, V, orogr,…) : acadfa.f90
-Produces an FA file

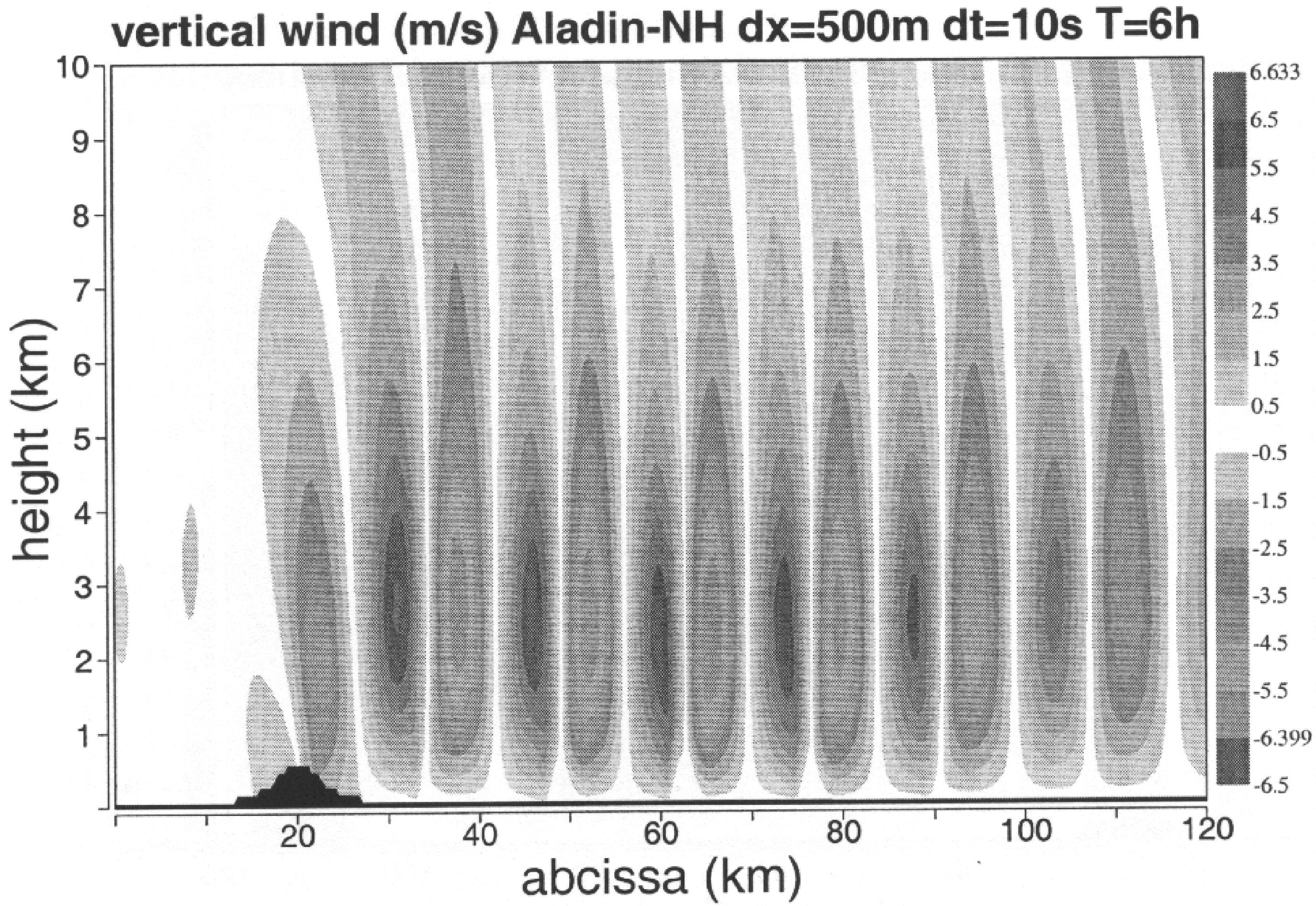
- ICMSHINIT: close to stationary state
- Coupling files: stationary (COUPL=ICMSHINIT)

EXEMPLE (in 2D)

- Stationary response to orographic flow:
  - All fields horizontally homogeneous (except Φs)
  - T and V profiles
  - NH variables set to zero.
- orography restricted near the center of the domain

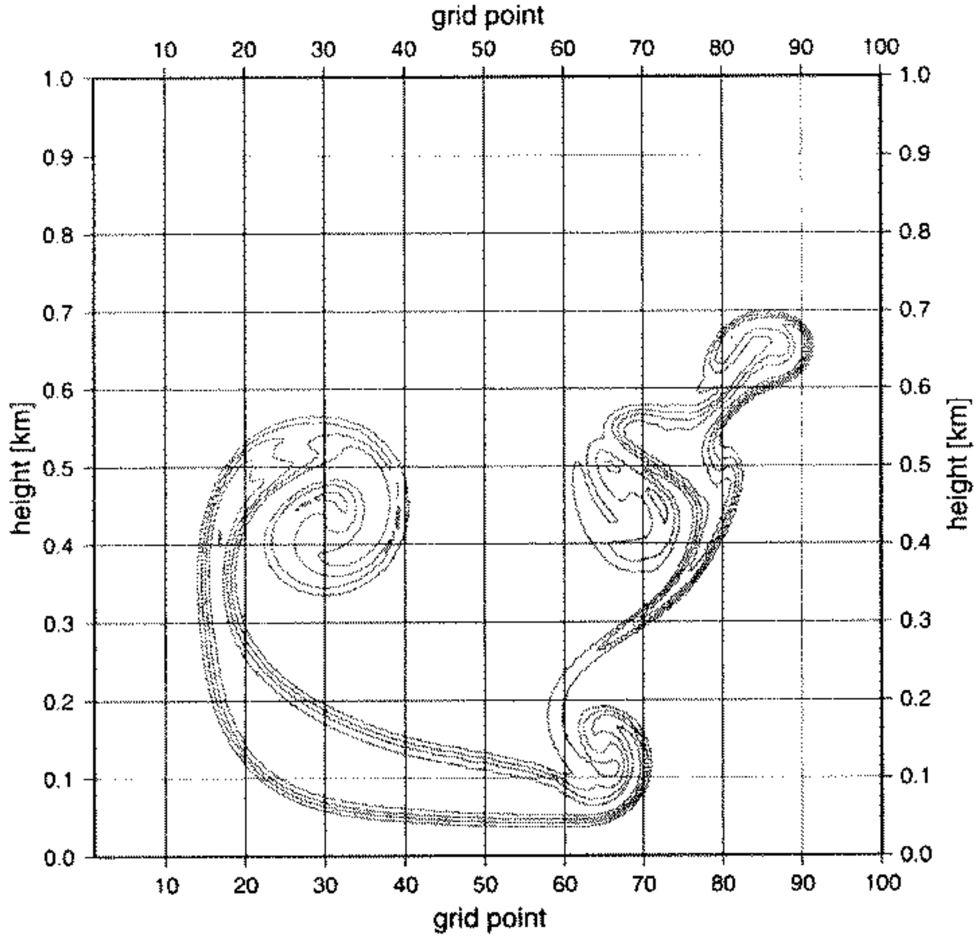
- Initial shock (partially absorbed at top)
- Stationary response

2D Orographic (trapped) wave



Stationary response

2D Cold bubble



Non stationary response

Summary

•Equation System

- Starting from (simple) EE system in z
- leads to (rather more complicated) EE system in η coordinate and new prognostic variables

•Academic tools versions of AROME exist for testing and validation ("easy to use").

- 1D Discrete linear analysis
- 2D vertical plan in various configuratioons
- 3D (LMAP=FALSE) acad orogr waves