Part VIII
Radiation and chemicals

1 ACRANEB economic radiation scheme

1.1 Interaction Radiation/Material

We study the interaction of a monochromatic radiation I_ν of frequency ν with the material.

Scattering by aerosols and molecular or *Rayleigh* diffusion (i.e. diffusion of visible light by molecules without change of frequency) are perceived by the *macroscopic interface approach* as a series of direction changes, each of them classified either as reflexion or transmission.
Over a path $d\ell$ we write:

$$dI_\nu = -k_{\nu\mathrm{diff}}\ \rho\ I_\nu\ d\ell$$

Absorption and *emission* are molecular effects, which we may write the same way:

$$\begin{aligned} \text{absorption :} \qquad & dI_\nu = -k_{\nu\mathrm{abs}}\ \rho\ I_\nu\ d\ell \\ \text{emission :} \qquad & dI_\nu = -k_{\nu\mathrm{emiss}}\ \rho\ B_\nu(T)\ d\ell \end{aligned}$$

where $B_\nu(T)$ is the Planck function giving the black body radiation for frequency ν at temperature T; Kirchoff's law states that the emission coefficient is equal to the absorption coefficient.

When considering the incidence angle θ, which differs for diffused and reflected beams from the direct beam angle θ_0, we have $d\ell = dz/cos\theta \equiv dz/\mu$.
Combination of attenuation effects is christened *extinction*:

$$\begin{aligned} k_{\nu\mathrm{ext}} &= k_{\nu\mathrm{diff}} + k_{\nu\mathrm{abs}} \\ dI_\nu &= -k_{\nu\mathrm{ext}}\ \rho\ I_\nu\ d\ell \\ \Longrightarrow \quad I_\nu &= I_{\nu_0}\ e^{-\delta_\nu/\mu} \qquad\qquad \frac{\delta_\nu}{\mu} = \int k_{\nu\mathrm{ext}}\ \rho\ d\ell \end{aligned}$$

δ_ν/μ is called the *optical thickness*; δ_ν is the *optical depth*, counted > 0 downwards.
For *Rayleigh diffusion*, the dependency to the wave length λ (or the frequency $\nu = c/\lambda$) is

$$\delta_{\ Rayleigh} = 0.00879\ \lambda^{-4.09}$$

so the shorter wavelengths are more affected than the longer ones, hence the blue hue of the sky; as the phase of the diffusion is nearly isotropic (peanut-shaped phase function) when several diffusions occur, we meet this same hue everywhere.

The transmissivity writes

$$\tau = e^{-\frac{\delta}{\mu}} \qquad\qquad \mu \equiv \cos\theta$$

We define the *simple diffusion albedo*

$$\varpi_\nu \quad \equiv \quad \frac{k_{\nu\mathrm{diff}}}{k_{\nu\mathrm{ext}}}$$

1.2 Radiative Transfer Equation

Previous paragraph gives us the extinction and emission parts of the radiation transfer equation; including the azimuthal angle ϕ, yields

$$\begin{aligned} \mu\frac{\partial I_\nu(\nu,\mu,\phi)}{\partial\delta_\nu} &= -I_\nu(\nu,\mu,\phi) - \frac{k_{\nu\mathrm{abs}}}{k_{\nu\mathrm{ext}}}B_\nu(T) + \cdots \\ &= \underbrace{-I_\nu(\nu,\mu,\phi)}_{\text{extinction}} - \underbrace{(1-\varpi_\nu)B_\nu(T)}_{\text{emission}} \end{aligned} \tag{1}$$

$$+\frac{\varpi_\nu}{4\pi}\left\{\underbrace{\left(S_\nu^0 e^{-\frac{\delta_\nu}{\mu_0}}\right)P_{\delta_\nu}(\mu,\phi,-\mu_0,\phi_0)}_{\text{parallel radiation gain}}+\underbrace{\int_0^{2\pi}\int_{-1}^{1}P_{\delta_\nu}(\mu,\phi,\mu',\phi')I_\nu(\delta_\nu,\mu',\phi')d\mu'd\phi'}_{\text{diffuse radiation gain}}\right\}$$

The two *"gains"* are the contribution of the scattering of

- the direct (parallel) radiation S_ν^0, of direction (μ_0,ϕ_0) and
- the diffuse radiation from all *other* directions,

both contributing to the radiation in θ direction.

Resolution requires a quadruple integration of (1): Angular (ϕ,μ), vertical (δ_ν), spectral (ν).

We will suppose a horizontal homogeneity, and vertical axi-symmetry. Boundary conditions are then

- At the top, solar flux I_0 and zenithal angle cosine μ_0;
- At the ground, partial reflexion, defined by the surface albedo supposed isotropic (no phase function for it).

1.2.1 Diffuse radiation

Let's consider an isotropic diffuse radiation, i.e. arriving from all directions on a layer of absorbing material of optical depth δ. The optical thickness for a given beam is δ/μ, increasing with θ.

- Incoming flux: the fraction of I on a unit sphere surface element is $\frac{I}{2\pi}\sin\theta d\phi\, d\theta = -\frac{I}{2\pi}d\mu d\phi$, and to project it on the upper interface of the layer we must multiply by $\cos\theta \equiv \mu$, hence:

$$F=\int_0^{2\pi}\int_0^1 \mu\,\frac{I}{2\pi}\,d\mu d\phi=\frac{I}{2}$$

- Outgoing flux:

$$\begin{aligned} F &= \int_0^{2\pi}\int_0^1 \mu\,\frac{I}{2\pi}\,e^{-\delta/\mu}\,d\mu d\phi = \frac{I}{2}f(\delta)\equiv\frac{I}{2}e^{-\delta/\bar\mu}\equiv\frac{I}{2}e^{-\beta\delta} \\ \text{with } f(\delta) &\equiv \int_0^1 \mu e^{-\delta/\mu}\,d\mu \qquad \Longrightarrow \beta=-\frac{1}{\delta}\ln\left(\int_0^1\mu e^{-\delta/\mu}\,d\mu\right)\end{aligned}$$

β is called the ***diffusivity factor***. The purpose is to represent the transmission through the layer by a single beam with direction $\cos\theta=\bar\mu=1/\beta$.
For a small δ, the average thickness encountered is 2δ, while for a big δ, only the beams close to the vertical will pass:

$$\delta=0\Rightarrow\bar\mu=\frac{1}{2}\Rightarrow\beta=2 \qquad\qquad \delta=\infty\Rightarrow\bar\mu=1\Rightarrow\beta=1$$

So $\beta=2$ represents dominant diffusion effects, while $\beta=1$ represents dominant absorption effects.
We will choose an empirical value within those limits: Rodgers proposed the value $\beta=\sqrt{e}\simeq 1.6487$: this is ARPÈGE-ALADIN's choice. Another frequent choice is $\beta=1.66$.
We define the layer transmission coefficient as

$$\tau=e^{-\beta\delta_{\text{abs gas}}}$$

1.2.2 Parallel radiation

For the direct radiation, a problem may occur when $\mu_0\to 0$, leading to an infinite thickness. This is not reality because the atmosphere is a spheric cap and not an infinite linear slab: so we need a correction for the ***"rotundity"*** of the atmosphere.
if a is the earth radius and H the thickness of the atmosphere, we write a corrected μ_0':

$$\mathsf{EARRT}\equiv\alpha\equiv\frac{a}{H} \qquad\qquad \mu_0'=\frac{1}{\sqrt{\alpha^2\mu_0^2+2\alpha+1}-\alpha\mu_0}$$

This correction should vary with the altitude, and even with other parameters like latitude, season, ... but we do not do it.

1.3 Two streams equations

Eddington's approximation says that isotropy allows us to describe the radiation field merely with a descending flux $F_\nu^\downarrow$ and an ascending flux $F_\nu^\uparrow$. This is rather idealised, as none of the fluxes could be isotropic everywhere, and the discontinuity over the horizontal is not physical.
A third flux S_ν corresponding to the parallel solar flux completes the system.
Separating those fluxes in equation (1) yields:

$$
\begin{array}{lllllll}
\frac{\partial S_\nu}{\partial \delta_\nu} & = & -\frac{S_\nu}{\mu_0'} & & & & \\
\frac{\partial F_\nu^\downarrow}{\partial \delta_\nu} & = & 2\pi(1-\varpi_\nu)B_\nu(T) & +\varpi_\nu(1-\beta_0)\frac{S_\nu}{\mu_0'}e^{-\frac{\delta_\nu}{\mu_0'}} & -2\left\{1-\varpi_\nu(1-\beta)\right\}F_\nu^\downarrow & +2\varpi_\nu\beta F_\nu^\uparrow & \\
\frac{\partial F_\nu^\uparrow}{\partial \delta_\nu} & = & -2\pi(1-\varpi_\nu)B_\nu(T) & -\varpi_\nu\beta_0\frac{S_\nu}{\mu_0'}e^{-\frac{\delta_\nu}{\mu_0'}} & +2\left\{1-\varpi_\nu(1-\beta)\right\}F_\nu^\downarrow & -2\varpi_\nu\beta F_\nu^\uparrow &
\end{array}
\tag{2}
$$

Remember that factors "2" come from the angular integration.

Note the use of the diffusivity factor β to split the contributions of up and down-going diffusion fluxes.
β represents actually the back-scattered fraction of the diffuse radiation, while β_0 is the fraction of the solar radiation that is back-scattered upwards.
As seen in section 1.2.1, β depends on the optical thickness of the different constituents, which themselves can depend on the wave length, so we store β and β_0 separately for the solar and thermal radiations, and for the aerosols, the liquid and solid phases in the clouds.
Actual namelist parameters are as follows:

- $\beta \equiv \mathsf{BSF}\left\{\begin{array}{c}\mathsf{S}\\ \mathsf{T}\end{array}\right\}\left\{\begin{array}{c}\mathsf{A}\\ \mathsf{I}\\ \mathsf{N}\end{array}\right\}$ for 'Back Scattered Fraction', 'Solar | Thermal', 'Aerosols| Ice | water ("Nuages")'.
- $\beta_0 \longrightarrow \left\{\mathsf{ZUSA}=\frac{0.5+\mathsf{USAA}\cdot\mu_0}{1+\mathsf{USBA}\cdot\mu_0},\ \mathsf{ZUSI}=\frac{0.5+\mathsf{USAI}\cdot\mu_0}{1+\mathsf{USBI}\cdot\mu_0},\ \mathsf{ZUSN}=\frac{0.5+\mathsf{USAN}\cdot\mu_0}{1+\mathsf{USBN}\cdot\mu_0}\right\}$ the "up-scattered fractions" respectively for aerosols, ice clouds and water clouds. Namelist coefficients ($\mathsf{USA}\{\mathsf{I}|\mathsf{N}\}$) are related to the corresponding β coefficients ($\mathsf{BSFS}\{\mathsf{I}|\mathsf{N}\}$):

$$\mathsf{USA}\{\mathsf{I}|\mathsf{N}\} = 2\cdot\mathsf{BSFS}\{\mathsf{I}|\mathsf{N}\} - 1$$

- δ_ν: see below, §1.4.1

1.4 Spectral integration

B_ν is given by Planck's formula:

$$B(\lambda,T) = \frac{2\pi h c_0^2}{\lambda^5(e^{hc_0/\lambda kT}-1)}$$

where h is Planck's constant, k is Boltzmann constant, c_0 is the speed of light.
The spectral integration yields Stefan's formula

$$B(T) = \int_0^\infty B(\lambda,T)d\lambda = \sigma T^4$$

where σ is Stefan's constant.
For the vertical integration, we will consider $B_\nu(T)$ constant over the vertical layer, with a discontinuity at the layer interfaces, i.e., we neglect in-layer temperature gradients, which is consistent with the dynamics.

The optical thicknesses and the ϖ_ν coefficients depend on the diffusion coefficients, which correspond to absorption by the grey bodies (aerosols, clouds, Rayleigh diffusion) and the spectral absorption coefficients for the gases.

1.4.1 Absorption/diffusion by grey bodies

The formulation is simple and linear:

$$\triangle\delta_{\triangle\nu} = k_{\triangle\nu} u_{\mathrm{grey}}$$

where $u_{\mathrm{grey}} = q_{\mathrm{grey}}\triangle p$ is the quantity of the given grey body (liquid water, ice, aerosol) and $k_{\triangle\nu}$ is a mean spectral coefficient computed once for all.
$\triangle\nu$ represents either the solar $\odot$ (short waves) domain or the thermal $\dagger$ (long waves) domain.
Namelist coefficients are then

$$k_{\triangle\nu} \longrightarrow \left\{ \begin{array}{ll} \mathsf{EOA}\left\{ \begin{array}{c} \mathsf{S} \\ \mathsf{T} \end{array} \right\} \left\{ \begin{array}{c} \mathsf{A} \\ \mathsf{I} \\ \mathsf{N} \end{array} \right\} & (\text{“Epaisseur Optique d'Absorption''}):\ \text{absorption optical depth coefficient} \\ \mathsf{EOD}\left\{ \begin{array}{c} \mathsf{S} \\ \mathsf{T} \end{array} \right\} \left\{ \begin{array}{c} \mathsf{A} \\ \mathsf{I} \\ \mathsf{N} \end{array} \right\} & (\text{“Epaisseur Optique de Diffusion''}):\ \text{scattering optical depth coefficient} \end{array} \right.$$

and $\varpi_\nu = \frac{\mathsf{EOD}}{\mathsf{EOA+EOD}}$.

For clouds, there are two ways to compute the spectral averaging leading to $k_{\triangle\nu}$ and β coefficients, either ignoring saturation effects, or taking them into account – then leading to much smaller solar absorption coefficients. In practice, we make a linear combination between the two methods:

$$k_{\triangle\nu} = \alpha k'_{\triangle\nu} + (1-\alpha)k''_{\triangle\nu}\ , \qquad \beta_\nu k_{\triangle\nu} = \alpha\beta'_\nu k'_{\triangle\nu} + (1-\alpha)\beta''_\nu k''_{\triangle\nu}$$

with $\alpha = 0.0532$ usually.
Besides, averaging between different cloud types is also important. As we do not have presently a prognostic treatment of condensates, the tuning procedure could use factors multiplying EOxxI and EOxxN.
From the different coefficients, following adimensional parameters may be computed:

$$\begin{aligned} a \equiv (1-\varpi) &= \frac{\mathsf{EOA}}{\mathsf{EOA}+\mathsf{EOD}} \\ A_\infty &= \frac{\varpi\cdot\beta}{(1-\varpi(1-\beta)) + \sqrt{(1-\varpi(1-\beta))^2 - (\varpi\beta)^2}} \\ \varepsilon_E &= \sqrt{(\mathsf{EOA}+\beta\mathsf{EOD})^2 - (\beta\mathsf{EOD})^2} \end{aligned}$$

respectively the absorption parameter, the albedo of homogeneously diffuse radiation for a pure water or ice cloud of infinite thickness, and the effective extinction coefficient.
As a practical example, the last tunings (CYCORA) proceeded as follows:

- albedo was too high $\Rightarrow\searrow$ EODSI, $\searrow$ EODSN or $\searrow A_{\infty\mathsf{SI}} \searrow A_{\infty\mathsf{SN}}$
- ice clouds were too thin in the thermal $\Rightarrow\nearrow$ EODTI $\nearrow$ EOATI or $\nearrow \varepsilon_{E\mathsf{TI}}$
- ice clouds did not absorb enough in the solar $\Rightarrow\nearrow$ EOASI or $\nearrow a_{\mathsf{SI}}\cdot\varepsilon_{E\mathsf{SI}}$
- solar absorption by liquid water should be kept constant $\Rightarrow=$ EOASN or $\simeq a_{\mathsf{SN}}\cdot\varepsilon_{E\mathsf{SN}}$ or $\simeq a_{\mathsf{SN}}\cdot(1-A_{\infty\mathsf{SI}})\cdot\varepsilon_{E\mathsf{SN}}$

1.4.2 Absorption/diffusion by gases

We consider $G=3$ different gases: H_2O, CO_2, O_3, each of them presenting specific absorption spectra.
In our approach we want to consider only two wave ranges, so $\triangle\nu$ represents either the solar (short waves) domain or the thermal (long waves) domain.
For each constituent we have an *equivalent band width* $w^g_{\triangle\nu}$ integrating information about width and intensity of the characteristic rays and the associated reference temperature and pressures.
Those width appear in the code as $\mathsf{ZW}\nu\mathsf{g}$ with ν in $\{\mathsf{S},\mathsf{T}\}$ and g in $\{\mathsf{H},\mathsf{C},\mathsf{O}\}$.

In reality, atmospheric gases present absorption bands, but also more continuous absorption regions. A band model from Malkmus, performing a spectral integration including a continuum term, gives for the equivalent widths:

$$w^g_{\triangle\nu} = \frac{a}{2b}\frac{q^g_{r\triangle\nu}}{q^g_{n\triangle\nu}}\left(\sqrt{1+4b\frac{(q^g_{n\triangle\nu})^2}{q^g_{r\triangle\nu}}} - 1\right) + c\ q^g_{r\triangle\nu}$$

where

- $a \equiv \mathsf{GCA}(6)$, $b \equiv \mathsf{GCB}(6)$, $c \equiv \mathsf{GCC}(6)$ depend on the spectral interval $\triangle\nu$ and the gas g, hence the 6 values for each of them, ranged as short wave values for $H_2O,\ CO_2,\ O_3$, followed by the three long wave values.
  Coefficient a stands for the "weak line" part of the equivalent width, b for the "strong line" part and c for the "continuum" part.
- $q^g_{n\triangle\nu}$ is linked to the crossed quantity of gas g : $u_g = 2\delta p\, q_g$, factor 2 coming from the isotropy hypothesis. For the solar descending flux we must take obliqueness into account so that $q^g_{n\triangle\nu} = u_g/(2\mu'_0)$, while for the other fluxes $q^g_{n\triangle\nu} = u_g$.
- $q^g_{r\triangle\nu}$ is the same mass multiplied by different correction factors:
  - the grey-body diffusion and scattering and multiple absorptions makes that the crossed quantity of gas will be larger, leading to the introduction of the above-mentioned *diffusivity factor* β for which we'll take the empirical value $\sqrt{e}$;
  - temperature and pressure modulate also the rate of unscattered parallel solar flux.

  The expressions are given below:

$\triangle\nu$	Solar $\odot$		Thermal $\dagger$	
	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
$q_{n\triangle\nu}$	u_g	$u_g/(2\mu'_0)$	u_g	u_g
$q_{r\triangle\nu}$	$q_{n\triangle\nu}p\ T^\alpha\beta/2$	$q_{n\triangle\nu}pT^\alpha$	$q_{n\triangle\nu}pT^\varepsilon\beta/2$	$q_{n\triangle\nu}pT^\varepsilon\beta/2$

  Temperature exponents α and ε are different for the different gases (see expressions for $\mathsf{ZR}\nu g$ in the code with ν in $\{\mathsf{S},\mathsf{T}\}$ and g in $\{\mathsf{H},\mathsf{C},\mathsf{O}\}$).
- For water vapour in the thermal domain, we need to introduce a more complex continuum term:
  $$q_{c\triangle\nu} = \frac{q_{r\triangle\nu}}{T^4} + q_{r\triangle\nu}\left(d_1 q_{H_2O}T^{41/2}e^{d_2/T}\right) = \frac{q_{r\triangle\nu}}{T^4}\left(1 + d_1 q_{H_2O}T^{49/2}e^{d_2/T}\right)$$
  the first term linked to the classical continuum and the second to the *e-type continuum*. Namelist parameters are $\mathsf{GCD4} \equiv d_1$ and $\mathsf{GCE4} \equiv d_2$.

A Padé formula combines the different optical thicknesses of the G different gases.
The formula implies the development into N_g terms for gas g:

$$\begin{array}{ll} N_{H2O} = 5,\ N_{CO_2} = 5,\ N_{O_3} = 5 & \text{in short waves} \\ N_{H2O} = 5,\ N_{CO_2} = 5,\ N_{O_3} = 3 & \text{in long waves} \end{array}$$

Each power in the development is multiplied by a constant coefficient ($\mathsf{VNP}^n_{\triangle\nu} \equiv \mathsf{VNP}(5,6)$ and $\mathsf{VDP}^n_{\triangle\nu} \equiv \mathsf{VDP}(5,6)$)

$$\triangle\delta_{\triangle\nu} = \sum_{g=1}^{G} w^g_{\triangle\nu}\frac{1 + \sum_{n=1}^{N_g}\left(w^g_{\triangle\nu}\right)^n \mathsf{VNP}^n_{\triangle\nu}}{1 + \sum_{n=1}^{N_g}\left(w^g_{\triangle\nu}\right)^n \mathsf{VDP}^n_{\triangle\nu}}$$

As the dependency of the optical depth to the gas quantities is highly non-linear, the $\triangle\delta$ of the different levels cannot be summed to find the total depth; to pass around this, we choose a reference level, and define our $\triangle\delta_j$ as

$$\begin{array}{llllll} \text{with} & j_{\text{ref}} & = & 1 & \triangle\delta_j & = & \triangle\delta(U_j) - \triangle\delta(U_{j-1}) \\ \text{with} & j_{\text{ref}} & = & \mathsf{KLEV} & \triangle\delta_j & = & \triangle\delta(U_j) - \triangle\delta(U_{j+1}) \end{array}$$

where U_j is the quantity of matter between the *reference* level and level j. So previous expressions yield the relative contributions of layer j to the optical thicknesses met by beams passing through it, either coming from the top of the atmosphere or from the ground.

- For the solar flux,
  - thicknesses are first computed as increments to the total thickness from the top downwards, allowing to compute the descending parallel ($\delta_j^{\odot\downarrow} \equiv$ ZDEOS); flux.
  - Then we treat the solar beams reflected at the ground, with a computation of the thicknesses as increments to the total thickness from space down to the ground up to the given level: this down+up thicknesses are stored into ZUEOS ($\delta_j^{\odot\uparrow} \equiv$ ZUEOS).

  Both parts are used to computed diffuse fluxes.

- For the thermal radiation, there is an additional difficulty, as beside the input πB at the top of the atmosphere, the temperature gradients in atmospheric layers generate source terms, when putting the system in a more usable form (see below, equations 4).
  If the atmosphere were isothermal, the earth-atmosphere system would transmit and reflect the difference between the black body flux and the actual flux, with a single source term for this difference at the top of the atmosphere. The terms $\frac{\partial B}{\partial \delta}$ in equation (4) vanish, and we have an exact analogue for the case of solar radiation. The idea is then to evaluate the absorber path lengths encountered in an isothermal atmosphere. The flux for each layer, will tbe estimated as if the whole earth-atmosphere system had the particular layer temperature.
  - thicknesses are computed downwards as increments to the total thickness from the top ($\delta_j^{\dagger\downarrow} \equiv$ ZDEOT) in order to compute the *cooling to space*, i.e. the cooling of the atmosphere radiating to space (the optical thickness encountered by a radiation escaping to space is counted between its variable origin level and the top of the atmosphere).
  - We compute then the thicknesses as increments to the total thickness from the ground upwards ($\delta_j^{\dagger\uparrow} \equiv$ ZUEOT).
  - To avoid a overestimation of the flux corresponding to the radiation exchanged between the layers, we keep the smallest of the two computed incremental thicknesses ZUEOT = min(ZDEOT, ZUEOT).

1.5 Solving the two-streams equations

Taking back equation(2), replacing the direct solar flux $S(\delta) = S_0 e^{-\delta/\mu_0'}$, yields for the short waves equations

$$\begin{aligned}
\frac{\partial F_{\odot\uparrow}}{\partial \delta} &= \alpha_1 F_{\odot\uparrow}(\delta) \quad -\alpha_2 F_{\odot\downarrow}(\delta) \quad -\alpha_3 \frac{S(\delta)}{\mu_0'} \\
\frac{\partial F_{\odot\downarrow}}{\partial \delta} &= \alpha_2 F_{\odot\uparrow}(\delta) \quad -\alpha_1 F_{\odot\downarrow}(\delta) \quad +\alpha_4 \frac{S(\delta)}{\mu_0'} \\
\frac{\partial S(\delta)}{\partial \delta} &= -\frac{S(\delta)}{\mu_0'}
\end{aligned} \tag{3}$$

For the long wave equations, we replace the diffuse fluxes F by $F^* \equiv \pi B(T) - F_{\dagger}$, which yields

$$\begin{aligned}
\frac{\partial F_\uparrow^*}{\partial \delta} &= \alpha_1 F_\uparrow^*(\delta) \quad -\alpha_2 F_\downarrow^*(\delta) \quad +\pi \frac{\partial B}{\partial \delta} \\
\frac{\partial F_\downarrow^*}{\partial \delta} &= \alpha_2 F_\uparrow^*(\delta) \quad -\alpha_1 F_\downarrow^*(\delta) \quad +\pi \frac{\partial B}{\partial \delta}
\end{aligned} \tag{4}$$

Considering isotherm layers, $B(T)$ is constant and the term $\pi\frac{\partial B}{\partial \delta}$ disappears, but we'll have to take afterwards the discontinuities between layers into account.

$$\begin{array}{lll}
\alpha_1 = 2(1 - \varpi(1-\beta)) & \alpha_2 = 2\,\varpi\,\beta & \\
\alpha_3 = \varpi\,\beta_0 & \alpha_4 = \varpi(1-\beta_0) & \alpha_5 = \frac{1}{\mu_0'}
\end{array}$$

β_0 represents the up-scattered fraction of the parallel solar beam, β is the backscattered fraction of the diffuse solar beams, ϖ the single scattering albedo.
All fluxes (S, $F_\downarrow$, $F_\uparrow$, $F_\downarrow^*$, $F_\uparrow^*$) are fluxes across horizontal surfaces; do not forget to consider the fluxes entering layer k, i.e the upward fluxes at level $\overline{k}$ and downward fluxes at level $\overline{k-1}$.

Coefficients α_i vary through ϖ, β and β_0. Within each layer we assume the α_i coefficients are constant.

1.5.1 Solar fluxes

We can then integrate equation set (3) analytically over this idealised layer, and express the results as a linear dependency of the outgoing fluxes on the incoming fluxes to this layer j:

$$\left\{ \begin{array}{c} S^b \\ F^b_{\odot\downarrow} \\ F^t_{\odot\uparrow} \end{array} \right\}_j = \begin{bmatrix} a_1 & 0 & 0 \\ a_2 & a_4 & a_5 \\ a_3 & a_5 & a_4 \end{bmatrix}_j \cdot \left\{ \begin{array}{c} S^t \\ F^t_{\odot\downarrow} \\ F^b_{\odot\uparrow} \end{array} \right\}_j \tag{5}$$

Index t stands for the top of the layer, b for the bottom.
a_1 is the transmission for the parallel beam, a_2 its diffuse transmission, a_3 its diffuse reflexion, a_4 the transmission for the diffuse radiation, a_5 its reflexion — all are relative to layer j. Analytical expressions for the a_i in function of the α_{ij}, δ_j and μ_0' are given in appendix A of [Geleyn and Hollingsworth, 1979].
This yields a system of $3N$ linear equations for $3N+3$ unknown fluxes, and we close it with three boundary conditions:

- The Solar flux at the top of the atmosphere is $S(0) = \mu_0' \ I_0$, where I_0 is the solar constant;
- the downwards diffuse flux is null at the top of the atmosphere;
- the upwards diffuse flux at the ground is

$$F_{\odot\uparrow}(\delta_g) = A_g(\mu_0')S(\delta_g) + \tilde{A}_g F_{\odot\downarrow}(\delta_g)$$

  where $A_g(\mu_0')$ and $\tilde{A}_g$ are the ground albedoes for the parallel and diffuse fluxes.

Using $F^{bj} \equiv F^{tj+1}$, (5) implies

$$\begin{array}{lcl lcllll} S^0 & = & \mu_0' I_0 & S^{j+1} & = & a_{1j}S^j & & \\ F^0_{\odot\downarrow} & = & 0 & F^{j+1}_{\odot\downarrow} & = & a_{2j}S^j & +a_{4j}F^j_{\odot\downarrow} & +a_{5j}F^{j+1}_{\odot\uparrow} \\ F^L_{\odot\uparrow} & = & A_g(\mu_0')S(\delta_g) + \tilde{A}_g F_{\odot\downarrow}(\delta_g) & F^j_{\odot\uparrow} & = & a_{3j}S^j & +a_{5j}F^j_{\odot\downarrow} & +a_{4j}F^{j+1}_{\odot\uparrow} \end{array}$$

This system of $3N+3$ equations with $3N+3$ unknowns writes in matricial form:

$$\underbrace{\begin{bmatrix} 1 & 0 & \cdots & & 0 \\ A_{21} & 1 & A_{23} & & \\ & & \ddots & & \\ & & & & A_{3N+2,3N+3} \\ 0 & & & A_{3N+3,3N+2} & 1 \end{bmatrix}}_{[\mathcal{A}]} \cdot \underbrace{\left\{ \begin{array}{c} \left\{ \begin{array}{c} S \\ F_{\odot\downarrow} \\ F_{\odot\uparrow} \end{array} \right\}_0 \\ \vdots \\ \left\{ \begin{array}{c} S \\ F_{\odot\downarrow} \\ F_{\odot\uparrow} \end{array} \right\}_N \end{array} \right\}}_{\{\mathcal{F}\}} = \underbrace{\left\{ \begin{array}{c} \mu_0' I_0 \\ 0 \\ \vdots \\ 0 \end{array} \right\}}_{\{\mathcal{S}\}} \tag{6}$$

Matrix $[\mathcal{A}]$ being band-structured with all diagonal elements equal to 1, the other non-zero elements being either $-a_{ij}$ or $-A_g(\mu_0')$ or $-\tilde{A}_g$.

Linear system (6) can easily be solved by a Gaussian elimination-back-substitution method - provided that average optical properties are given for each layer: we use here the optical thicknesses ZUEOS from the upwards computation, and the direct transfer of the parallel flux is done at the same time with ZDEOS.
The net solar flux is then

$$\mathsf{PFRSO} \equiv F^k_{\odot} = S^k + F^k_{\odot\downarrow} - F^k_{\odot\uparrow} \qquad k = 0 \dots L$$

In the code, integration is done as follows. Analytical integrated system over the optical thickness $\triangle\delta$ of layer k is written:

$$\begin{pmatrix} -R^k & 1 & 0 & -T^k \\ -T^k & 0 & 1 & -R^k \end{pmatrix} \begin{pmatrix} F^{k-1}_{\odot\downarrow} \\ F^{k-1}_{\odot\uparrow} \\ F^k_{\odot\downarrow} \\ F^k_{\odot\uparrow} \end{pmatrix} = \begin{pmatrix} \mathcal{R}^k S^{k-1} \\ \mathcal{T}^k S^{k-1} \end{pmatrix} \tag{7}$$

$$S^k = S^{k-1} T_p$$

with reflexion/ transmission coefficient values computed both for clear(⊗='C') and cloudy (⊗='N') situations (see §1.6):

$$
\begin{array}{lcll}
\mathsf{ZA4}\otimes & \equiv & T^k_\otimes = \frac{E(1-M^2)}{1-E^2M^2} & \text{diffuse transmission} \\
\mathsf{ZA5}\otimes & \equiv & R^k_\otimes \frac{M(1-E^2)}{1-E^2M^2} & \text{diffuse reflexion} \\
\mathsf{ZA1}\otimes & \equiv & T^k_{p\otimes} = e^{-\alpha_5 \triangle\delta^k} & \text{parallel transmission} \\
\mathsf{ZA2}\otimes & \equiv & \mathcal{T}^k_\otimes = G_2(T_{p\otimes} - T_\otimes) - G_1 T_p R_\otimes & \text{parallel/diffuse transmission} \\
\mathsf{ZA3}\otimes & \equiv & \mathcal{R}^k_\otimes = G_1(1 - T_{p\otimes} T_\otimes) - G_2 R_\otimes & \text{parallel/diffuse reflexion}
\end{array}
$$

with

$$
\begin{array}{lcll}
\lambda & = & \sqrt{\alpha_1^2 - \alpha_2^2} & \longrightarrow \mathsf{ZEPS} = \lambda \triangle \delta^k \\
E & = & e^{-\lambda\triangle\delta^k} & \longrightarrow \mathsf{ZTAU} \\
M & = & \frac{\alpha_2}{\alpha_1+\lambda} & \longrightarrow \mathsf{ZRHO} \\
G_1 & = & \frac{\alpha_3 - \mu_0'(\alpha_1\alpha_3+\alpha_2\alpha_4)}{1-\lambda^2\mu_0'^2} & \longrightarrow \mathsf{ZG1} \\
G_2 & = & \frac{-\alpha_4 - \mu_0'(\alpha_1\alpha_4+\alpha_2\alpha_3)}{1-\lambda^2\mu_0'^2} & \longrightarrow \mathsf{ZG2}
\end{array}
$$

and $\triangle\delta = \triangle\delta_{\text{diffusion}} + \triangle\delta_{\text{absorption}}$. For the clear sky situation the optical thicknesses take only into account the aerosols, while for the cloudy situation we add the contributions of ice and water clouds.

As mentioned above, namelist parameters are $\beta_{\nu\zeta} = \mathsf{BSF}\nu\zeta$, optical depth coefficients $k_{a\nu\zeta} = \mathsf{EOA}\nu\zeta$ and $k_{d\nu\zeta} = \mathsf{EOD}\nu\zeta$ and coefficients used in $\beta_{0\zeta}$ computation (where ν stands for S or T and ζ stands for A or I or N).
In the code, the α_i are not explicitly computed, but we have:

$$
\begin{array}{rcl}
\mathsf{ZEO}\nu\zeta & = & k_{D\nu\zeta} + k_{A\nu\zeta} \equiv k_{E\nu\zeta} \\
\mathsf{ZEO2}\nu\zeta & = & 2\beta_{\nu\zeta} k_{D\nu\zeta} = 2\beta_{\nu\zeta}\varpi_{\nu\zeta} k_{E\nu\zeta} = \alpha_{2\nu\zeta} k_{E\nu\zeta} \\
\mathsf{ZEO1}\nu\zeta & = & \mathsf{ZEO2}\nu\zeta + 2k_{A\nu\zeta} = \left(2\beta_{\nu\zeta}\varpi_{\nu\zeta} + 2(1-\varpi_{\nu\zeta})\right) k_{E\nu\zeta} = 2(1-\varpi_{\nu\zeta})(1-\beta_{\nu\zeta}) k_{\mathsf{E}\nu\zeta} = \alpha_{1\nu\zeta} k_{E\nu\zeta}
\end{array}
$$

Then, given standard aerosol thickness profile PDAER(KLEV) $\equiv (u^j_A)$, and noting $k_{\mathrm{R}} \equiv$ EORAY $\sim .8606E-06$ the Rayleigh scattering coefficient (scattering by clean air, without aerosols), we compute, starting at first (highest) level:

$$
\begin{array}{lll}
\mathsf{ZEO1}' &= \delta_j^{\odot\uparrow} + \alpha_{1\mathsf{SA}} k_{\mathsf{ESA}} u_{\mathsf{A}}^j + k_{\mathrm{R}} \triangle p^j & = \alpha_{1C}\triangle\delta \\
\mathsf{ZEO2}' &= \alpha_{2\mathsf{SA}} k_{\mathsf{ESA}} u_{\mathsf{A}}^j + k_{\mathrm{R}} \triangle p^j & = \alpha_{2C}\triangle\delta \\
\mathsf{ZEO3}' &= (k_{\mathsf{DSA}} u_{\mathsf{A}} \beta_{0\mathsf{A}} + 0.5 k_{\mathrm{R}} \triangle p^j) \frac{1}{\mu''_{0C}} & = \alpha_{3C}\triangle\delta \\
\mathsf{ZEO4}' &= (k_{\mathsf{DSA}} u_{\mathsf{A}} (1-\beta_{0\mathsf{A}}) + 0.5 k_{\mathrm{R}} \triangle p) \frac{1}{\mu''_{0C}} & = \alpha_{4C}\triangle\delta \\
\mathsf{ZEO5}' &= \frac{1}{\mu''_{0C}} \mathsf{ZEO}' & = \alpha_{5C}\triangle\delta \\
\mathsf{ZEO}' &= 0.5\delta_j^{\odot\downarrow} + k_{\mathsf{ESA}} u_{\mathsf{A}}^j + k_{\mathrm{R}} \triangle p^j &
\end{array}
$$

$$
\text{resonance treatment :} \quad \frac{1}{\mu_0} - \frac{\mathsf{ZEPS}}{\mathsf{ZEO}'}
\begin{cases}
> 0 \Rightarrow \frac{1}{\mu''_{0C}} = \frac{1}{\mu'_0} \\
\gtrsim 0 \Rightarrow \frac{1}{\mu''_{0C}} = \frac{\mathsf{ZEPS}}{\mathsf{ZEO}'} + \epsilon_3 \\
\lesssim 0 \Rightarrow \frac{1}{\mu''_{0C}} = \frac{\mathsf{ZEPS}}{\mathsf{ZEO}'} - \epsilon_3 \\
< 0 \Rightarrow \frac{1}{\mu''_{0C}} = 2\frac{\mathsf{ZEPS}}{\mathsf{ZEO}'} - \frac{1}{\mu'_0}
\end{cases}
, \qquad \epsilon_3 = 10^{-12}
$$

$$
\begin{array}{lll}
\mathsf{ZEO1} &= \mathsf{ZEO1}' + \triangle p^j (\alpha_{1\mathsf{SN}} k_{\mathsf{ESN}} q_{\mathsf{N}}^j + \alpha_{1\mathsf{SI}} k_{\mathsf{ESI}} q_{\mathsf{I}}^j) & = \alpha_{1N}\triangle\delta \\
\mathsf{ZEO2} &= \mathsf{ZEO2}' \triangle p^j (\alpha_{2\mathsf{SN}} k_{\mathsf{ESN}} q_{\mathsf{N}}^j + \alpha_{2\mathsf{SI}} k_{\mathsf{ESI}} q_{\mathsf{I}}^j) & = \alpha_{2N}\triangle\delta \\
\mathsf{ZEO3} &= \left(\mathsf{ZEO3}' \mu''_{0C} + \triangle p^j (k_{\mathsf{DSN}} q_{\mathsf{N}}^j \beta_{0N} + k_{\mathsf{DSI}} q_{\mathsf{I}}^j \beta_{0I})\right) \frac{1}{\mu''_{0N}} & = \alpha_{3N}\triangle\delta \\
\mathsf{ZEO4} &= \left(\mathsf{ZEO4}' \mu''_{0C} + \triangle p^j ((1-\beta_{0N}) k_{\mathsf{DSN}} q_{\mathsf{N}}^j + (1-\beta_{0I}) k_{\mathsf{DSI}} q_{\mathsf{I}}^j)\right) \frac{1}{\mu''_{0N}} & = \alpha_{4N}\triangle\delta \\
\mathsf{ZEO5} &= \frac{1}{\mu''_{0N}} \mathsf{ZEO} & = \alpha_{5N}\triangle\delta \\
\mathsf{ZEO} &= \mathsf{ZEO}' + \triangle p^j \left(k_{\mathsf{ESN}} q_{\mathsf{N}} + k_{\mathsf{ESI}} q_{\mathsf{I}}\right) &
\end{array}
$$

$$
\text{resonance treatment :} \quad \frac{1}{\mu_0} - \frac{\mathsf{ZEPS}}{\mathsf{ZEO}}
\begin{cases}
> 0 \Rightarrow \frac{1}{\mu''_{0N}} = \frac{1}{\mu'_0} \\
\gtrsim 0 \Rightarrow \frac{1}{\mu''_{0N}} = \frac{\mathsf{ZEPS}}{\mathsf{ZEO}} + \epsilon_3 \\
\lesssim 0 \Rightarrow \frac{1}{\mu''_{0N}} = \frac{\mathsf{ZEPS}}{\mathsf{ZEO}} - \epsilon_3 \\
< 0 \Rightarrow \frac{1}{\mu''_{0N}} = 2\frac{\mathsf{ZEPS}}{\mathsf{ZEO}} - \frac{1}{\mu'_0}
\end{cases}
$$

with $q_{\mathsf{N}} \equiv$ PQLI, $q_{\mathsf{I}} \equiv$ PQICE, so-called "radiative" liquid and solid specific humidities, as computed in ACNEBN (see part VII).
So we have in fact ZEO$i \equiv \alpha_i \triangle\delta$, for $i = 1 \dots 5$.

1.5.2 Thermal fluxes

A similar method can be applied from system (4), considering isotherm layers. Defining

$$
F_{\uparrow}^{*k} \equiv \pi B^k - F_{\dagger\uparrow}^k \qquad F_{\downarrow}^{*k} \equiv \pi B^{k-1} - F_{\dagger\downarrow}^k \qquad B^{k-1} \equiv \sigma T_k^4 \qquad B^L \equiv \sigma T_{\mathrm{surf}}^4
$$

we obtain

$$
\begin{array}{llll}
F_{\downarrow}^{*j} &= a_{4j} F_{\downarrow}^{*j-1} & + a_{5j} F_{\uparrow}^{*j} & + a_{4j}\pi(B^{j-1} - B^j) + a_{5j}\pi(B^{j-1} - B^{j-2}) \\
F_{\uparrow}^{*j-1} &= a_{5j} F_{\downarrow}^{*j-1} & + a_{4j} F_{\uparrow}^{*j} & + a_{5j}\pi(B^{j-1} - B^j) + a_{4j}\pi(B^{j-1} - B^{j-2}) \\
F_{\downarrow}^{*0} &= \pi B^0 & & \\
F_{\uparrow}^{*L} &= (1-\epsilon) F_{\downarrow}^{*L} & & + (1-\epsilon)(\pi B^L - \pi B^{L-1})
\end{array}
\tag{8}
$$

where $\epsilon \equiv$ PEMIS is the surface emissivity.
The resolution is complicated by the fact that we get source terms at all levels. For instance the upwards thermal radiation at level k is the sum of the radiation from the surface, the exchange with layers below k, the exchange with the layers above k, and the radiation escaping to space.
A full computation would require the determination of $L(L+1)/2$ different optical thicknesses which would be prohibitive.

Noting that cooling to space is the predominant effect, we will compute it first, and then accept less accuracy for the exchanges.
Starting at the ground which is the only interface level for which we know the temperature,

- In a first step we prepare the computation of the *cooling to space* term, by adding the contributions of each layer *as if the atmosphere was isothermal* with the temperature of the considered layer: this way there are no exchange terms between the layers to consider, the B terms disappear in system (8). Furthermore we put the source term $\pi B(T) \equiv 1$ in (8), getting normalized fluxes f^*. The optical thicknesses are of course the incremental thicknesses with reference to the top of the atmosphere: $\delta^{\dagger\downarrow} \equiv$ ZDEOT

  The net cooling to space flux (difference of the $\uparrow$ and $\downarrow$ fluxes) is then obtained by

$$\begin{aligned} F_c^L &= (f_\uparrow^{*L} - f_\downarrow^{*L})\pi B^L \\ F_c^{k-1} &= F_c^k + \triangle F_c \\ \triangle F_c &= \left\{(f_\uparrow^{*k-1} - f_\downarrow^{*k-1}) - (f_\uparrow^{*k-1} - f_\downarrow^{*k-1})\right\}\pi B^{k-1} \end{aligned}$$

- To get the exchange flux, we lead 2 parallel resolutions of (8), using *in both* the minimum incremental thicknesses from upwards and downwards estimations (ZUEOT $= \min(\delta^{\dagger\downarrow}, \delta^{\dagger\uparrow})$), and with respectively
  - the actual temperature profile $\longrightarrow F^k = F_\uparrow^{*k} - F_\downarrow^{*k} + (\pi B^{k-1} - \pi B^k)$
  - the isotherm normalized profile $\longrightarrow g_\uparrow^{*k}, g_\downarrow^{*k}$

  to obtain by difference the exchange terms. Using simply these minimum of $\delta_j^{\dagger\uparrow}, \delta_j^{\dagger\downarrow}$ we pass over the computation of thicknesses between intermediate layers. The minimum is chosen to avoid instabilities.

$$\begin{aligned} F_e^L &= F^L - (g_\uparrow^{*L} - g_\downarrow^{*L})\pi B^L \\ F_e^{k-1} &= F_e^k + \triangle F_e \\ \triangle F_e &= \triangle F - \left\{(g_\uparrow^{*k-1} - g_\downarrow^{*k-1}) - (g_\uparrow^{*k-1} - g_\downarrow^{*k-1})\right\}\pi B^{k-1} \end{aligned}$$

- The complete net thermal fluxes are then obtained by

$$\text{PFRTH} \equiv F_\dagger^k = F_c^k + F_e^k \qquad k = 0 \ldots L$$

In the code, integration is performed as follows. Analytical integration over optical thickness $\triangle\delta$ of layer k yields:

$$\begin{pmatrix} -R^k & 1 & 0 & -T^k \\ -T^k & 0 & 1 & -R^k \end{pmatrix} \begin{pmatrix} F_\downarrow^{*k-1} \\ F_\uparrow^{*k-1} \\ F_\downarrow^{*k} \\ F_\uparrow^{*k} \end{pmatrix} = \begin{pmatrix} R^k & T^k \\ T^k & R^k \end{pmatrix} \begin{pmatrix} \pi B^{k-1} - \pi B^{k-2} \\ \pi B^{k-1} - \pi B^k \end{pmatrix} \tag{9}$$

$$\pi B^{k-1} \equiv \sigma {T^k}^4 \qquad \pi B^L = \sigma T_{\text{surf}}^4$$

where reflexion/transmission coefficients are the same as in §1.5.1 (but here only diffuse coefficients are needed).

1.6 Introduction of cloudiness

As clouds contain liquid or solid water, they contribute to the diffusion and absorption as grey bodies, hence with a linear relation between the thicknesses and the absorber amounts.

Considering a partially covered layer we assume that

- each cloud stretches over the whole layer height
- the cloud fraction is constant over the layer
- there is no lateral effect between cloud and clear sky

We can then consider that layer j is characterised by two sets of (δ_j, α_{ij}): one for the cloudy part (subscript $_N$) and one for the cloud-free part (subscript $_C$). This way we get two sets of a_{ij} coefficients, a_{Cij} and a_{Nij}.

With a random cloud distribution (LRNUMX=.FALSE.), we just have to use a linear weighting following the layer cloudiness n^j:

$$a_{ij} = n^j a_{Nij} + (1-n^j) a_{Cij} \qquad\qquad j = 1 \dots L$$

to be replaced in equation sets (5 or 7) and (8 or 9).
If we assume (LRNUMX=.TRUE.) that adjacent layers have the maximum overlap, we must solve equation set (5) separately for cloudy and cloud-free cases, following (for short wave radiation):

$$\left\{\begin{array}{c} S_C^b \\ F_{C\odot\downarrow}^b \\ F_{C\odot\uparrow}^t \end{array}\right\}_j = \left[\begin{array}{ccc} a_{C1} & 0 & 0 \\ a_{C2} & a_{C4} & a_{C5} \\ a_{C3} & a_{C5} & a_{C4} \end{array}\right]_j \cdot \left\{\begin{array}{c} b_1 S_C^t + (1-b_3) S_N^t \\ b_1 F_{C\odot\downarrow}^t + (1-b_3) F_{N\odot\downarrow}^t \\ b_2 F_{C\odot\uparrow}^b + (1-b_4) F_{N\odot\uparrow}^b \end{array}\right\}_j \tag{10}$$

$$\left\{\begin{array}{c} S_N^b \\ F_{N\odot\downarrow}^b \\ F_{N\odot\uparrow}^t \end{array}\right\}_j = \left[\begin{array}{ccc} a_{N1} & 0 & 0 \\ a_{N2} & a_{N4} & a_{N5} \\ a_{N3} & a_{N5} & a_{N4} \end{array}\right]_j \cdot \left\{\begin{array}{c} (1-b_1) S_C^t + b_3 S_N^t \\ (1-b_1) F_{C\odot\downarrow}^t + b_3 F_{N\odot\downarrow}^t \\ (1-b_2) F_{C\odot\uparrow}^b + b_4 F_{N\odot\uparrow}^b \end{array}\right\}_j \tag{11}$$

so that the outgoing "clear" and "cloudy" fluxes are linearly dependent on both "clear" and "cloudy" incoming fluxes. Alternately, if we write this from equation (7), we get the systems:

$$\left(\begin{array}{cccc} -R_C^k & 1 & 0 & -T_C^k \\ -T_C^k & 0 & 1 & -R_C^k \end{array}\right) \left(\begin{array}{c} b_1 F_{C\odot\downarrow}^{k-1} + (1-b_3) F_{N\odot\downarrow}^{k-1} \\ F_{C\odot\uparrow}^{k-1} \\ F_{C\odot\downarrow}^{k} \\ b_2 F_{C\odot\uparrow}^{k} + (1-b_4) F_{N\odot\uparrow}^{k} \end{array}\right) = \left(\begin{array}{c} \mathcal{R}_C^k (b_1 S_C^{k-1} + (1-b3) S_N^{k-1}) \\ \mathcal{T}_C^k (b_1 S_C^{k-1} + (1-b3) S_N^{k-1}) \end{array}\right)$$

and

$$\left(\begin{array}{cccc} -R_N^k & 1 & 0 & -T_N^k \\ -T_N^k & 0 & 1 & -R_N^k \end{array}\right) \left(\begin{array}{c} (1-b_1) F_{C\odot\downarrow}^{k-1} + b_3 F_{N\odot\downarrow}^{k-1} \\ F_{C\odot\uparrow}^{k-1} \\ F_{C\odot\downarrow}^{k} \\ (1-b_2) F_{C\odot\uparrow}^{k} + b_4 F_{N\odot\uparrow}^{k} \end{array}\right) = \left(\begin{array}{c} \mathcal{R}_C^k ((1-b_1) S_C^{k-1} + b3 S_N^{k-1}) \\ \mathcal{T}_C^k ((1-b_1) S_C^{k-1} + b3 S_N^{k-1}) \end{array}\right)$$

The b coefficients are then

$$b_{1j} = \frac{1-\max(n^j, n^{j-1})}{1-n^{j-1}}, \quad b_{2j} = \frac{1-\max(n^j, n^{j+1})}{1-n^{j+1}}, \quad b_{3j} = \frac{\min(n^j, n^{j-1})}{1-n^{j-1}}, \quad b_{4j} = \frac{\min(n^j, n^{j+1})}{1-n^{j+1}}$$

stored respectively into ZB1, ZB2, ZB3, ZB4 and they are set to 1 in case of indeterminacy.
This treatment assumes maximum overlapping of adjacent cloudy layers but total independence in position between cloudy parts separated by cloud free air.
This method is better but more expensive; both methods could give similar results (in a case where clouds are in aggregates, rather than random) under the condition to use less cloudiness for the random approach than for the maximum overlap approach, by defining different critical specific moisture thresholds r_c.

1.7 Moon radiation

Under key LRAYLU, moon radiation is taken into account as soon as the sun is down: moon radiation RIP0LU and zenithal angle cosine PMU0LU replace then the values for the sun RII0 and PMU0.
The values have been computed by routine UPDMOON called by UPDTIM, using parameters in namelist NAMRIP.
Additional gadget routines are also available to handle solar eclipses: POSCT giving the centre of the totality zone and FRASOLU giving the fraction of the sun ZFSO(JLON) visible at horizontal coordinate JLON, both called when NOIR=1.

1.8 Conclusions

The proposed scheme, implemented in routine ACRANEB is very cheap compared to the more complete FMR (Fouquart-Morcrette Radiation) scheme used at ECMWF (RADINT), requiring only in 5% of its execution time. For this reason, it can be used at every location and time-step, in order to describe the full feed-back loops with cloud cover and liquid/ice water contents as well as the surface temperature on land. Of course it is less exact, and different refinements are currently under study.
Distinctive features include:

- Use of one spectral interval in long waves and one in short waves;
- Local *gaseous* optical thicknesses are pre-computed for the short waves, and for the not cooling to space long waves, along idealised non-scattering paths.
- The cooling to space is computed exactly.
- 2-stream equations + "adding method" for SW and LW multiple scattering.
- p and T dependency of gaseous absorption, + all continuum effects.
- Cloud geometry with two possible options: random and maximum overlap.

2 ACOZONE: ozone and chemicals

This module is used under the key LOZONE, presently not in ALADIN operational version.

Seen the complexity of stratospheric photo-chemistry, it is unthinkable to introduce it in a general circulation model. Therefore, we use here linearization source and sinks terms obtained by a 2D-model (latitude-pressure). This model includes 36 latitudes, 40 pressure levels, 168 chemical reactions between 56 constituents + 51 photo-dissociation reactions, and uses the stratospheric zonal circulation. It lead to an equilibrium after an integration over 30 years, and an additional step after a perturbation of about 10% of the ozone contents r_{O_3} yielded the derivative of the ozone production/destruction term $P - L$ with respect to r_{O_3}, and in a similar manner the derivative with respect to T, and with respect to the ozone thickness located above the point, Σ_{O_3}.

The linear model is then:

$$\frac{\partial r_{O_3}}{\partial t} = \overline{P - L} + \frac{\partial \overline{P - L}}{\partial r_{O_3}} \left[r_{O_3} - \overline{r_{O_3}} \right] + \frac{\partial \overline{P - L}}{\partial T} \left[T - \overline{T} \right] + \frac{\partial \overline{P - L}}{\partial \Sigma_{O_3}} \left[\Sigma_{O_3} - \overline{\Sigma_{O_3}} \right]$$

The ozone quantity above each point is computed with

$$\Sigma_{O_3}(l) = \sum_{i=0}^{l-1} \frac{p^{\overline{i}} - p^{\overline{i-1}}}{g} r^i_{O_3} + \frac{p^l - p^{\overline{l-1}}}{g} r^l_{O_3}$$