Part IV
Subgrid Dynamic Processes

1 Subgrid orographic (gravity wave) drag

This quite abusively named "gravity-wave" drag flux aims actually at parameterizing all subgrid orography interactions with the large scale flow.
It is a vertical flux of horizontal momentum, given by the vector (PSTRDU, PSTRDV).
It should include following effects:

- Gravity waves may arise from the flow passing above the obstacle: those waves can occur only where the large scale flow is stable. They can extend very high in the atmosphere.
- The low level flow may also be blocked in front of an obstacle high enough, inducing a flow separation round the mountain flanks. The acting force has two components:
  - the *form drag*, opposite to the flow velocity;
  - the *lift effect*, perpendicular to the mean flow velocity.

  The occurrence of a blocked part depends on the obstacle height and atmosphere stability.
- Meso scale flow blocking inside the valleys of massive mountains causes areas of complex terrain to act on the large scale flow as if they had a larger height than that prescribed by the mean orography: this part can only be addressed by using some kind of envelope orography.

Until recently, the blocked part was – unsatisfyingly – addressed through the use of an *envelope orography*. Increasing the height and volume of the orography, the envelope induces an enhanced drag, but it has detrimental effect on assimilation — leading to the systematic rejection of observation stations situated below the level of the envelope — and also tends to give excessive precipitation especially from convection generated by the elevated heating of the enhanced orography.
Some new ideas on this subject talk about making a "constant mass envelope" by lowering the troughs and increasing the slopes at the border of mountain ridges, through quite heuristic grid point computations.
Research must go on.

1.1 Surface interaction

$$\vec{\tau}_s = K_g \; \rho_s N_s \mathbf{v}_{fs} h_s$$

where

- $\vec{\tau}_s \equiv (\mathsf{ZSTUS}, \mathsf{ZSTVS})$ is the vertical flux of (vector) horizontal momentum;
- $\rho_s \equiv$ PGWDCS is the surface density computed by ACHMT (see chapter III);
- N_s and $\mathbf{v}_{fs}$ are the effective Brunt-Vaisäla frequency and fictive wind speed at the surface (see below);
- $g \cdot h_s \equiv$ PGETRL is the standard deviation of the unresolved surface geopotential;
- $K_g \equiv$ GWDSE is an adimensional tuning coefficient.

As the surface wind is always zero, and the use of the lowest model level wind would introduce a harmful dependency on the vertical discretization, an effective surface wind value is computed as follows.

The effective obstacle height in pressure coordinates is

$$H \quad \equiv \quad \mathsf{ZOBST} = \min\left\{p^{\overline{L}} - p^1 \quad , \quad \max\left\{\mathsf{HOBST}\cdot\rho_s\cdot gh_s \quad , \quad p^{\overline{L}} - p^L\right\}\right\}$$

where HOBST is a tunable parameter. ZOBST is the pressure difference between the ground and at least the lowest model level (L), at most the highest model level 1.

$$\mathsf{ZWGHT} = \min\left\{0 \quad , \quad \frac{p^{l+1}-(p^{\overline{L}}-H)}{H} \quad , \quad \frac{p^{l+1}-p^{l}}{H}\right\}$$

$$\mathsf{ZSUMU} = \sum_{L}^{1}\mathsf{ZWGHT}\cdot u^{\overline{l}} \qquad \mathsf{ZSUMV} = \sum_{L}^{1}\mathsf{ZWGHT}\cdot v^{\overline{l}} \qquad \mathsf{ZSUMF} = \sum_{L}^{1}\mathsf{ZWGHT}\cdot\left(\frac{N^2}{\rho^2 g^2}\right)^{\overline{l}}$$

and the upwards summations may be stopped as soon as ZWGHT becomes zero.
$N/(\rho g) \equiv$ PNBVNO are the Brunt-Vaisäla frequencies computed by ACHMT and ACCOEFK.
We write then

$$\mathbf{v}_s \equiv (\mathsf{ZSUMU}, \mathsf{ZSUMV}) \qquad \left(\frac{N^2}{\rho^2 g^2}\right)_s \equiv \mathsf{ZSUMF}$$

As the subgrid obstacle may raise across several model levels (the lowest interface vertical level follows the *resolved* orography), we modify the wind and Brunt-Vaisäla frequency profiles below H, to avoid discontinuities perturbing the computations:

$$\mathbf{V}_l = \mathbf{v}_s\left(1-\tfrac{\triangle p_l}{H}\right)+\mathbf{V}_l\tfrac{\triangle p_l}{H} \equiv (\mathsf{ZU},\mathsf{ZV}) \qquad \text{with} \quad \triangle p_l = p^{\overline{L}} - p^{l}$$

$$\left(\tfrac{N}{\rho g}\right)_l = \left(\tfrac{N}{\rho g}\right)_s\left(1-\tfrac{\triangle p_l}{H}\right)+\left(\tfrac{N}{\rho g}\right)_l\tfrac{\triangle p_l}{H} \equiv \mathsf{ZNFNO} \qquad \text{with} \quad \triangle p_l = p^{\overline{L}} - p^{\overline{l}}$$

For the surface wind itself, we should take into account the orographic anisotropy, represented by

- $\gamma^2 \equiv$ PVRLAN: anisotropy coefficient, ratio of the two eigenvalues of the tensor of unresolved orographic variance;
- $\psi \equiv$ PVRLDI is the direction of the first eigenvector of this tensor (angle between the direction of low level wind and the axis of the gradient of the orography)

This anisotropy makes that the surface tension is not parallel to the surface effective wind. We compute a fictive surface wind collinear with the surface tension, and which is equal to the effective surface wind in the case there is no anisotropy:

$$\mathsf{ZUSUR} \equiv u_{fs} = Au_s + D(u_s\cos 2\psi + v_s\sin 2\psi) \qquad \mathsf{ZVSUR} \equiv v_{fs} = Av_s + D(u_s\sin 2\psi - v_s\cos 2\psi)$$

$$A = \gamma^2 + \frac{4(1-\gamma)(1+\alpha_1\gamma)-\gamma(1+\alpha_2)\ln\gamma)}{2\pi} \qquad D = \frac{4(1-\gamma)(1+\delta_1\gamma)+3\gamma(1+\delta_2)\ln\gamma)}{2\pi}$$

$$\alpha_1 \simeq 1.44 \qquad \alpha_2 \simeq 0.22 \qquad \delta_1 \simeq 0.67 \qquad \delta_2 \simeq 0.44$$

We introduce then the variables:

$$\mathsf{ZUSR} \equiv \|\mathbf{v}_{fs}\| \qquad \mathsf{ZSUSR} \equiv \mathbf{v}_s\cdot\mathbf{v}_{fs} \qquad \mathsf{ZAUSR} \equiv \mathbf{V}^{\overline{l}}\cdot\mathbf{v}_{fs} \qquad \mathsf{ZFACS} \equiv \frac{N_s}{\rho_s g(\mathbf{v}_s\cdot\mathbf{v}_{fs})^3}$$

1.2 Upper levels interactions

For a wave of amplitude a and length λ, the momentum flux is constant along the vertical as long as it stays linear ($a \ll \lambda$), so that there is no interaction with the large scale flux (saturation or breaking):

$$\tau = \rho NU\frac{a^2}{\lambda} \qquad \text{with } U = \frac{\mathbf{V}\cdot\mathbf{v}_{fs}}{\|\mathbf{v}_{fs}\|}$$

The wave breaks at the ground, and also by ***saturation*** when the Richardson number of the resulting flux, perturbed by the wave, falls below a critical value (close to 1). Let's rewrite the expressions for $\mathcal{R}i$ and N.

$$N^2 \equiv \frac{g}{\theta}\frac{\partial\theta}{\partial z} \sim g\frac{\triangle s}{c_p T\triangle z}$$

$$\mathcal{R}i \equiv \frac{\text{buoyancy}}{\text{vertical shear}} \sim \frac{g\triangle s/(c_p T\triangle z)}{(\triangle\mathbf{V})^2/(\triangle z)^2} = N^2\frac{(\triangle z)^2}{(\triangle\mathbf{V})^2}$$

The velocity shear experienced by the gravity wave when meeting the large scale flux is simply the large scale horizontal velocity $\triangle \mathbf{V} \sim U$, over a vertical thickness equal to the wave amplitude $\triangle z \sim a$.
So when we approach neutrality or unstable atmosphere, we have

$$\mathcal{R}i \sim 1 \Longrightarrow N^2 \sim \frac{U^2}{a^2} \Longrightarrow \lambda\tau = \rho g N U a^2 \sim \frac{\rho U^3}{N}$$

The profile of the wave length λ at saturation is such that the ratio of the phase velocity to the large scale velocity is constant over the whole vertical: $\lambda N = C \cdot U$.

Defining

$$\mathsf{ZPRRAP} \equiv \Upsilon = \frac{\rho}{N}\frac{N_s}{\rho_s}\left(\frac{U}{U_s}\right)^3 = \frac{\mathsf{ZFACS} \cdot \mathsf{ZAUSR}^3}{\mathsf{ZNFNO}}$$

Υ is the ratio of $\left(\frac{\lambda\tau}{\lambda_s\tau_s}\right)$ at saturation, provided the momentum flux takes the same expression at the surface as in the upper levels. This way we may state that saturation occurs when $\tau = \Upsilon\tau_s$, hence $\Upsilon\tau_s$ represents the maximum value of the momentum vertical flux:

$$\begin{aligned} \tau_l &= \min(\Upsilon\tau_s, \tau_{l+1}) \\ \mathsf{ZRAPP} \equiv \Gamma_l &= \frac{\tau_l}{\tau_s} \longrightarrow \left\{ \begin{array}{l} \Gamma_L = 1 \\ \Gamma_l = \left\{ \begin{array}{ll} \min(\Upsilon_l, \Gamma_{l+1}) & \text{if } > 0 \\ 0 & \text{otherwise} \end{array} \right. \end{array} \right. \end{aligned}$$

If Γ_l gets negative or null,we reset it to zero for the upper levels, as the wave has been totally absorbed and the flux disappeared. This occurs when the wind angle with the surface wind overrides 90^o, or anyway at the top of the atmosphere ($\rho = 0$).
If the stratification becomes neutral or unstable, N changes its sign, but as it is at the denominator we must take care to determine the critical level where this occurs:

- above this level, we must set $\Gamma_l = 0$;
- below, we have also to modify the profile of Γ_l by subtracting a linear function of the pressure, null at the surface and ensuring $\Gamma_l = 0$ at the critical level.

1.3 Resonance effects

The wave does not necessarily drop all its momentum at the saturation level, and is actually reflected downwards; after several reflexions resonance could occur. The practical parameterization in ARPÈGE-ALADIN has been done by MLYNARZ [1990].
Following PELTIER and CLARK [1979], resonant amplification takes place when the reflected wave is in phase with the incident wave, which is realized when the height of saturation z is such that

$$\begin{aligned} z &= (\frac{3}{4} + k)\lambda \quad , \qquad \text{with k integer} \\ \text{or} \qquad \frac{N}{U}z &= \frac{3\pi}{2} + 2k\pi \end{aligned}$$

When this condition is not fulfilled, the incident and reflected waves interfere destructively, attenuating the stress in the lower layers of the atmosphere.

The first level with saturation z_{crit} is the lowest one where $\Upsilon^l < 1$. We compute then the phase angle

$$\theta = \int_0^{z_{\text{crit}}} \frac{N}{U} dz \qquad \text{into} \qquad \mathsf{ZSUM} = \sum \frac{N}{\rho g}\frac{\triangle p}{U}$$

and we define a periodic function f for the resonance, having an average value of 1 which is maximum for $\theta = 3\pi/2$ (amplification) and minimum for $\theta = \pi/2$ (destruction); we than modify Γ_l as follows:

$$f(\theta) = \left|1 - iK_a e^{i\theta} - K_a^2 e^{2i\theta} + \cdots\right| = \left(1 + 2K_a \sin\theta + K_a^2\right)^{-\frac{1}{2}} \doteq \mathsf{ZSTM}$$

$$\Longrightarrow f(\frac{3\pi}{2}) = \frac{1}{1-K_a} \qquad f(\frac{\pi}{2}) = \frac{1}{1+K_a}$$

$$\Gamma_l = \begin{cases} f(\theta) & \text{at the ground} \\ \text{linear correction with } p & 0 < z < z_{\text{crit}} \\ \Gamma^{\text{crit}} = \min(1, f(\theta)) & z = z_{\text{crit}} \\ \min(\Gamma_{\text{crit}}, \Gamma_l) & z > z_{\text{crit}} \end{cases}$$

where $K_a \equiv \mathsf{GWDAMP} < 1$ is an empirical parameter. Only reflexions at the lowest critical level are taken into account.
In the case the stratification gets neutral or unstable below z_{crit}, no resonance can take place, so we reset $f(\theta) \equiv 1$. This is done via variables ZPR1 (pressure level of the lowest deposition) and ZPR2 (pressure level of the first reflexion), which have been computed by examining Γ and N respectively.

1.4 Blocked-flow drag

Mere introduction of gravity wave effects does not tackle completely for subgrid scale orographic effects. Subgrid orography can produce some blocking of the flow below a certain level, and often this effect was handled by using an *envelope orography*, obtained by adding to the mean (resolved) orography, a fraction of the standard deviation of the unresolved orography. As this treatment was far from satisfactory, especially with high resolution models, Lott and Miller [1997] proposed to use a second conceptual model based on ideas introduced by Baines and Palmer [1990] considering that part of the flow can skirt round the orography instead of being forced above it.
The part of the flow passing round the orography does not produce gravity waves, but instead produces an additional surface drag.

For each level below the blocked layer top Z_C a force per unit volume is applied:

$$D_b(z) = -\rho_0\, C_d\, l(z)\, \frac{\mathbf{V}|\mathbf{V}|}{2}$$

where $l(z)$ represents the horizontal width of the obstacle as seen by the flow at an upstream height z, and $\mathsf{GWDCD} \equiv C_d \sim 1$ tunes the blocked flow drag amplitude. $l(z)$ depends on the subgrid orography standard deviation σ, its anisotropy γ, its slope μ and orientation ψ through the angle between the incident flow and the normal ridge direction; and on the aspect ratio r of the obstacle as seen by the incident flow.
As Arpège-Aladin doesn't use the slope, we have to replace it by some rough evaluation.
To get the corresponding stress we write:

$$\tau_b = C_d \int_0^{Z_C} -\rho_0 l(z) \frac{\mathbf{V}|\mathbf{V}|}{2} dz$$

In Arpège-Aladin, our computation being focused on the *drag* rather than the waves themselves, a simple adaptation was done through multiplying the Γ_l coefficients (hence the corresponding gravity wave drag fluxes) by a factor linked to this effect.

Orography skirting occurs when the normalized mountain height

$$\mathcal{H} = H \frac{N_s \|\mathbf{v}_{fs}\|}{U_s^2}$$

overrides a critical empirical value $\mathcal{H}_c \equiv 1/\mathsf{GWDBC}$ (we keep here the pressure height H [Pa] in the definition). The flow that is closer to the top than this critical value passes over, while the part of the flow below $\mathcal{H} - \mathcal{H}_c$ goes around.
Defining

$$b_d = \begin{cases} \frac{\mathcal{H}-\mathcal{H}_c}{\mathcal{H}} & \text{if } \mathcal{H} > \mathcal{H}_c \\ 0 & \text{otherwise} \end{cases}$$

$\triangle P_c = b_d \cdot H$ is the critical surface-level pressure difference below which skirting occurs.

We write:

$$\Gamma_l = \Gamma_l \cdot \left\{ 1 + a_d \sqrt{\frac{\left(1 - \frac{\triangle P^l}{b_d H}\right)^3}{1 + \frac{\triangle P^l}{H}}} \right\} \qquad \text{if } \triangle P^l < b_d H$$

with

$$\triangle P^l = p^{\overline{L}} - p^{\overline{l}} \qquad , \qquad a_d \equiv C_d \frac{\|\mathbf{v}_{fs}\|}{H\ N_s} \cdot b_d$$

So the coefficients a_d and b_d are not null (and the tensions are modified) only if $\mathcal{H} > \mathcal{H}_c$.

In the code, we have actually:

$$\begin{aligned}
\mathsf{ZTUNE} &= \mathsf{GWDBC} \cdot \left(\frac{\|\mathbf{v}_{fs}\|^2}{\mathbf{v}_s \cdot \mathbf{v}_{fs}}\right)^2 = \frac{1}{\mathcal{H}_c} \frac{\|v_{fs}\|^2}{U_s^2} \\
b_d \equiv \mathsf{ZZBB} &= \begin{cases} 1 - \frac{\|\mathbf{v}_{fs}\|}{\mathsf{ZTUNE} \cdot \mathsf{HOBST} * g h_s \rho_s N_s} = 1 - \frac{\|\mathbf{v}_{fs}\| U_s^2 \mathcal{H}_c}{\|v_{fs}\|^2 \cdot H N_s} = 1 - \frac{\mathcal{H}_c}{\mathcal{H}} & \text{if } > 0 \\ 0 & \text{otherwise} \end{cases} \\
a_d \equiv \mathsf{ZZAA} &= \mathsf{GWDCD} \cdot \mathsf{ZTUNE} \cdot \mathsf{ZZBB}(1 - \mathsf{ZZBB}) = C_d \frac{1}{\mathcal{H}_c} \frac{\|v_{fs}\|^2}{U_s^2} \left(1 - \frac{\mathcal{H}_c}{\mathcal{H}}\right) \frac{\mathcal{H}_c}{\mathcal{H}} = C_d \frac{\|\mathbf{v}_{fs}\|}{H\ N_s} \cdot b_d \\
\mathsf{ZRZ} &= \frac{p^{\overline{L}} - p^{\overline{l}}}{\mathsf{ZZBB} \cdot H} \qquad \text{and} \qquad \Gamma^l = \Gamma^l (1 + \mathsf{ZZAA} \sqrt{\frac{(1 - \mathsf{ZRZ})^3}{1 + \mathsf{ZRZ} \cdot \mathsf{ZZBB}}})
\end{aligned}$$

1.5 Final recombination and fluxes

Γ^l is recomputed by combining the different effects obtained above. The gravity wave drag flux is then obtained by

$$(\mathsf{PSTRDU}, \mathsf{PSTRDV}) = \mathsf{ZRAPP} \cdot (\mathsf{ZSTUS}, \mathsf{ZSTVS})$$

1.6 Numerical aspects

An implicit treatment of the evolution of the horizontal momentum due to the orographic drag is advisable. But as the routine simply return fluxes, some tricks are performed so that the final recombination in CPTEND and CPUTQY are equivalent to an implicit formulation.

1.6.1 Surface wind dependence

The surface wind is a notion purely local to routine ACDRAG; but its calculation from the lower levels velocities and the surface characteristics could hide an evolution equation, which, if not handled cautiously, could become unstable.
All fluxes are proportional to $U_s = \frac{\mathbf{v}_s \cdot \mathbf{v}_{fs}}{\|\mathbf{v}_{fs}\|}$, the projection of the surface wind on the fictive wind. As

$$\mathbf{v}_s = \frac{1}{H} \int_0^H \mathbf{V} dp \Longrightarrow \frac{\partial U_s}{\partial t} = -g \frac{\partial \tau_s}{\partial p} \propto \frac{\partial}{\partial p} \int_0^H U_s dp \text{ or } \frac{\partial U_s}{\partial t} = -K U_s$$

For large time steps, in presence of altitude gradients possibly bigger than those at the lowest level, this system could become unstable: an implicit formulation is then needed. Writing

$$\frac{U_s^+ - U_s}{\Delta t} = -K U_s^+ \Longrightarrow U_s^+ (1 + K \Delta t) = U_s \Longleftrightarrow \frac{U_s^+ - U_s}{\Delta t} = -\frac{K}{1 + K \Delta t} U_s$$

To get this (split implicit formulation), we just have to multiply all proportional relations between τ_l and U_s by

$$\frac{1}{1+K\triangle t} = \frac{1}{1-\frac{\triangle t}{U_s}\frac{\partial U_s}{\partial t}}$$

where $\frac{\partial U_s}{\partial t} \equiv \mathsf{ZSUMD}$ is obtained by integrating the flux vertical derivative over H.

1.6.2 Large scale wind dependence

Discretization of

$$\frac{\partial U}{\partial t} = -g\frac{\partial \tau}{\partial p} \qquad \text{with} \qquad U = \frac{\mathbf{V}\cdot\mathbf{v}_{fs}}{\|\mathbf{v}_{fs}\|}$$

but the dependency on U is not linear because $\tau_l = \Gamma_l(U)\tau_s(U_s)$.
Considering the problem as a vertical advection of horizontal momentum, we write in the explicit form

$$\tau_l = \frac{1}{g}U_l\mathcal{M}_l \qquad \Rightarrow \frac{U_l^+ - U_l}{\Delta t} = -\frac{U_l\mathcal{M}_l - U_{l-1}\mathcal{M}_{l-1}}{\delta p_l} \Rightarrow U_l^+ - U_l = -A_lU_l + A_{l-1}U_{l-1}$$

where $\mathcal{M}$ plays the role of a vertical velocity or mass flux, and defining $A_l \equiv \frac{\mathcal{M}_l\triangle t}{\delta p_l}$ (analogous with a CFL vertical condition).
If we discretize implicitly downwards, the tendency staying null above level $l = 1$:

$$\begin{aligned} U_l^+ - U_l &= -(A_lU_l^+ - A_{l-1}U_{l-1}^+) \\ (1+A_l)(U_l^+ - U_l) &= -A_lU_l + A_{l-1}U_{l-1} + A_{l-1}(U_{l-1}^+ - U_{l-1}) \end{aligned}$$

This leads to replace the flux τ by $\dot\tau$ so that:

$$\begin{aligned} \frac{U_l^+ - U_l}{\Delta t} &= -g\frac{\partial\dot\tau}{\partial p} \\ \dot\tau_0 = 0 \qquad & \dot\tau_l = \dot\tau_{l-1} - \frac{1}{1+A_l}(\dot\tau_{l-1} - \tau_l) \end{aligned}$$

1.7 Orographic lift and blocking

Orographic lift effect as assessed by LOTT [1997b] has been surreptitiously introduced end of 1997 at the very end of the routine.

Mountain forces exerted on the flow include

- The gravity waves drag at the upper part;
- At the lower part, the so-called blocked part of the flow implies
  - a *drag* opposite to the wind, which was parameterized in section 1.4,
  - the *lift* is the component which modifies the direction of the flow but does not decelerate it locally, and does not work against it.

(These denominations refer to the forces exerted on wing profiles, even though the "lift" is here essentially a horizontal effect).
The lift effect induces for instance *vortex stretching*, and should be of more importance for large scale models. As it acts perpendicularly to the flow, its treatment is similar to a local modification of the Coriolis parameter f.

The representation applies a force per unit volume L which is perpendicular to the wind at each level below the *mountain maximum height* $Z_{\max}$ or H in pressure units:

$$L = -\rho\, C_l\, f\, \frac{Z_{\max} - Z}{Z_{\max}}\, \mathbf{k}\wedge\mathbf{V}$$

This induces a stress

$$\tau_{\text{lift}} = -C_l\ f \int_0^{Z_{\max}} \rho\ \frac{Z_{\max} - Z}{Z_{\max}}\ \mathbf{k} \wedge \mathbf{V} dz\ \doteq\ -\rho\ C_l\ f\ \frac{Z_{\max}}{2}\ \mathbf{k} \wedge \mathbf{V}$$

in the case of a uniform incident wind over the vertical. This is equivalent to the lift stress exerted by an obstacle of mean height $Z_{\max}/2$ on a quasi-geostrophic flow — i.e. when the maximum mountain height is twice its mean height.
ARPÈGE-ALADIN's orographic lift coding compute a pseudo Coriolis factor $f^* \sim \mathsf{ZFORCL}$:

$$\mathsf{ZFORCL} = \frac{\mathcal{L}_t}{(2\ln 2-1)} \frac{\triangle t}{\mathsf{HOBST}} f \frac{1-\mathsf{ZALTI}}{1+\mathsf{ZALTI}} \qquad \text{with} \qquad \mathsf{ZALTI} = \begin{cases} \frac{p^L - \overline{p^l}}{H} & \text{if } < 1 \\ 1 & \text{elsewhere} \end{cases}$$

$$A \equiv \frac{\mathsf{ZFORCL}}{1+\left(\frac{\mathsf{ZFORCL}}{2}\right)^2} \qquad B \equiv \frac{\mathsf{ZFORCL}^2/2}{1+\left(\frac{\mathsf{ZFORCL}}{2}\right)^2}$$

$$\tau_u^{l\prime} = \tau_u^{l-1\prime} + (\tau_u^l - \tau_u^{l-1}) + \frac{\triangle p}{g\triangle t}(Bu^l - Av^l) \quad \tau_v^{l\prime} = \tau_v^{l-1\prime} + (\tau_v^l - \tau_v^{l-1}) + \frac{\triangle p}{g\triangle t}(Au^l + Bv^l)$$

The factor $2\ln 2 - 1$ was introduced so that the value $\mathcal{L}_t \equiv \mathsf{GWDLT} = 1$ corresponds to an effective global lift force equivalent to the actual Coriolis effect.
The formulation also ensures that the lift force does not perform any work, affecting only the direction of the wind and not its intensity.

2 Convective gravity wave drag

Similarly to the flow over subgrid orography, subgrid convective systems experience subgrid vertical velocities that could induce gravity waves into the stable layers situated above.
This part is however switched off in the operational model, setting the model key LGWDC to .FALSE.. The roughness of the parameterisation and the fact that its validity requires a very good performance from the deep convection scheme (exactitude in time and space) justify probably this abandon.

It is assumed that the intensity of the phenomenon is linked to the surface precipitation flux (which is a rather rough hypothesis), while the interaction with the large scale flow originates at the cloud top, in the direction of the large scale wind at this level; and the referential for the waves is taken at the base of the convective slab.
The positions of base and top of the convective region were previously computed in routine ACNEBC, which is no longer used operationally: it is therefore not possible to run this when using the new cloudiness routine ACNEBN.

$$\vec{\tau}_t = K_c \mathcal{P}_c \frac{\mathbf{V}_t}{\|\mathbf{V}_t\|} \equiv (\mathsf{ZSTUS}, \mathsf{ZSTVS}) = (\mathsf{PSTRCU(IUPC)}, \mathsf{PSTRCV(IUPC)})$$

where $\mathcal{P}_c$ is the total convective precipitation, and level $t \equiv \mathsf{IUPC} \doteq \mathsf{PNEBH(JLON,2)}$ is the top of the convectively active slab.
Saturation is treated in a similar way to the orographic gravity waves:

$$\mathsf{ZPRRAP} \equiv \Upsilon = \frac{\rho}{N} \frac{N_t}{\rho_t} \left(\frac{U}{U_t}\right)^3 \qquad \text{with} \qquad U = \frac{(\mathbf{V} - \mathbf{V}_b) \cdot \mathbf{V}_t}{\|\mathbf{V}_t\|}$$

U is the projection on the wind at the cloud top, of the wind relative to the wind at the base of the convective slab ($b \equiv \mathsf{IDWC} \doteq \mathsf{PNEBH(JLON,3)}$).

3 Mesospheric "drag"

This part does not exactly concern a "dynamic" process.
Model key LRRMES is set to .FALSE. in the operational model, so this part is not called.
It is intended for models including levels in the mesosphere (above $50Km$), where none of the physical parameterizations intervenes. It simply imposes a linear relaxation of the wind to zero and the temperature to the temperature of a standard mesosphere T_{sta}, defined in module SUSTA. Module SUTOPH computes relaxation coefficients for the wind and for the temperature, varying with the level.