# Part I
# Physics-Dynamics interface

## 1 Equations

### 1.1 Coordinate systems

In the physical grid point routines, the equations are generally written in hydrostatic pressure or geopotential $\phi = gz$ coordinates. The conversion rules between different vertical coordinates are given in $\pi$ or $\phi$ coordinates, so it is useful to remind the rules for changing from one system to the other.

- Partial derivatives along x, y, t:

$$\begin{aligned}\left(\frac{\partial A}{\partial x}\right)_\zeta &= \left(\frac{\partial A}{\partial x}\right)_z + \left(\frac{\partial z}{\partial x}\right)_\zeta \frac{\partial A}{\partial z} \\ \left(\frac{\partial A}{\partial t}\right)_\zeta &= \left(\frac{\partial A}{\partial t}\right)_z + \left(\frac{\partial z}{\partial t}\right)_\zeta \frac{\partial A}{\partial z}\end{aligned}$$

- Partial derivatives along the vertical:

$$\frac{\partial A}{\partial \zeta} = \frac{\partial A}{\partial z}\frac{\partial z}{\partial \zeta}$$

- Combining, we can express horizontal gradient and divergence, and the tendency:

$$\nabla_z = \nabla_\zeta - \frac{\partial \zeta}{\partial z}(\nabla_\zeta z)\frac{\partial}{\partial \zeta} \qquad \left(\frac{\partial}{\partial t}\right)_z = \left(\frac{\partial}{\partial t}\right)_\zeta - \frac{\partial \zeta}{\partial z}\left(\frac{\partial z}{\partial t}\right)_\zeta \frac{\partial}{\partial \zeta}$$

- The total derivative has a similar form in any coordinate:

$$\frac{dA}{dt} = \left(\frac{\partial A}{\partial t}\right)_\zeta + \mathbf{V}\cdot\nabla_\zeta A + \dot{\zeta}\frac{\partial A}{\partial \zeta}$$

The relevant coordinates yield:

- Hydrostatic pressure coordinate:

$$\frac{\partial \pi}{\partial z} = -\rho g \qquad \frac{\partial z}{\partial \pi} = -\frac{1}{\rho g} \qquad \frac{\partial \pi}{\partial \phi} = -\rho \qquad \frac{\partial \phi}{\partial \pi} = -\frac{1}{\rho} \qquad \dot{\pi} \equiv \omega \qquad \dot{z} \equiv w$$

  yields:

$$\nabla_z = \nabla_\pi + \rho g(\nabla_\pi z)\frac{\partial}{\partial \pi} \qquad \left(\frac{\partial}{\partial t}\right)_z = \left(\frac{\partial}{\partial t}\right)_\pi + \rho g(\nabla_\pi z)\frac{\partial}{\partial \pi}$$

- $\eta$ coordinate:

$$\pi(x,y,\eta,t) = A(\eta)\pi_{top} + B(\eta)\pi_s(x,y,t) \qquad \frac{\partial \pi}{\partial \eta} \equiv m = A'(\eta)\pi_{top} + B('\eta)\pi_s(x,y,t)$$

$$\Longrightarrow \frac{\partial \eta}{\partial \phi} = -\frac{\rho}{m}$$

  yields:

$$\nabla_z = \nabla_\eta + \frac{\rho g}{m}(\nabla_\eta z)\frac{\partial}{\partial \eta} \qquad \nabla_\pi = \nabla_\eta - \frac{1}{m}(\nabla_\eta \pi)\frac{\partial}{\partial \eta}$$

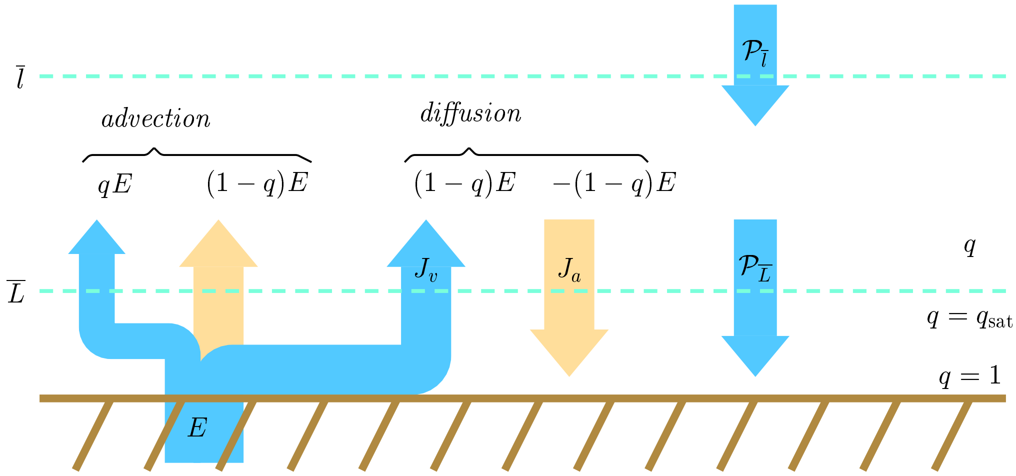


**Figure 1:** Surface budget with $\delta_m = 1$. Blue arrows represent water, beige arrows represent dry air

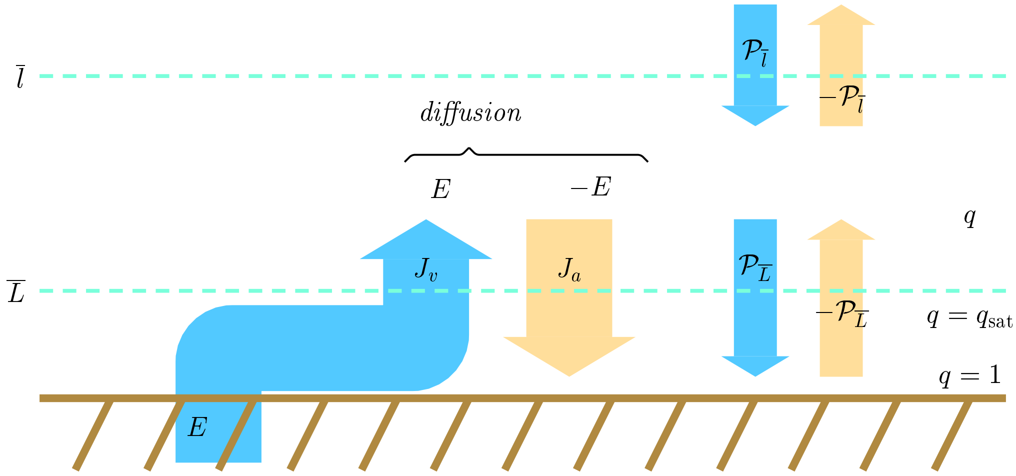


**Figure 2:** Surface budget with $\delta_m = 0$.

## 1.2 Variable Mass option

Parameter NDPSFI $\equiv \delta_m$ controls mass conservation:

- $\delta_m = 0$: Total mass is conserved, water rained out is replaced by *dry* air, while the vapour produced by evaporating precipitation is compensated by a removal of dry air.
- $\delta_m = 1$: Total mass is not conserved, mass changes are controlled by the precipitation-evaporation budget at the surface. Condensation produces a local mass deficit, while precipitation evaporation produces a local mass increase.

The budget for both situations is illustrated on figures 1 and 2.
Presently, the operational model is run with $\delta_m = 0$, while the more refined case $\delta_m = 1$ is used in longer range climatological runs.

As we do not want to model explicitly the surface viscous layer ($\sim 1mm$ thickness) where the mixing between evaporating water ($q = 1$) and atmospheric air ($q \ll 1$) takes place, we will write the different mass fluxes at the top of the viscous layer and use them as boundary condition at the ground.

Within the surface layer, both mass and water are conserved.
In the atmosphere, mass is conserved if $\delta_m = 0$, but water is never conserved, as there are exchanges with the ground reservoirs.

Considering the case $\delta_m = 1$ (fig. 1):

- the net evaporation flux $E$ from the soil produces an equal mass vertical advection of moist air characterized by the local specific humidity $q$. This implies to advect $qE$ of water and $(1-q)E$ of dry air from the surface layer.

- Moisture steady state in the viscous layer requires the diffusion of $J_v = (1 - qE)$ of water, replaced by the diffusion of $J_a = -(1 - qE)$ of dry air.

- the precipitation flux $\mathcal{P}_{\overline{L}}$ leaving at the lower interface implies a loss of mass of the atmosphere.

while in the case total mass is conserved ($\delta_m = 0$, fig. 2), the evaporation flux $E$ implies that the same mass of dry air be removed from the atmosphere. Similarly, the precipitation flux $\mathcal{P}$ is compensated by an opposite dry air flux, keeping the mass constant.
Hence the different fluxes at the top of the viscous layer write

- Water vapour advection: $\dot{\eta}\frac{\partial p}{\partial \eta}q = \delta_m E q$

- Water vapour diffusion: $gJ_v = gE(1 - \delta_m q)$

- Dry air advection: $\dot{\eta}\frac{\partial p}{\partial \eta}q_a = gE\delta_m(1 - q)$

- Dry air diffusion: $gJ_a = -gE(1 - \delta_m q)$

So the total water flux from ground evaporation entering the atmosphere is always

$$\frac{1}{g}\left[\dot{\eta}\frac{\partial p}{\partial \eta}q + J_v\right] = E \tag{1}$$

while we have:

- $J_v + J_a = 0$: no net mass transfer by diffusion. This remains valid along the whole vertical, i.e we assume that the vertical flux linked to turbulence is mass-conserving in the atmosphere.

- $gJ_a + \dot{\eta}\frac{\partial p}{\partial \eta}q_a = \begin{cases} 0 & \text{if} \quad \delta_m = 1 \\ gJ_a = -gE & \text{if} \quad \delta_m = 0 \end{cases}$
  so a dry air flux exiting the atmosphere ensures mass conservation in the case $\delta_m = 0$.

The surface budget has also an impact on the budget of the other layers, higher in the atmosphere:

- With $\delta_m = 0$, the local vertical divergence of the precipitation flux in layer $l$ is compensated by an equivalent mass of dry air, added or removed from the atmosphere.

- On the contrary, with $\delta_m = 1$, the vertical divergence of the precipitation flux makes the mass of layer $l$ change.

The vertical mass flux (counted positively downwards) writes, in $\eta$ vertical coordinate:

$$\frac{1}{g}\dot{\eta}\frac{\partial p}{\partial \eta}$$

The surface budget affects primarily the surface pressure $\pi_s$. The mass flux at interface level $\bar{l} \neq \overline{L}$ is affected by the mass change due to $\delta_m = 1$ as following:

$$\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{\bar{l}}^{\delta_m=1} = \left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{\bar{l}}^{\delta_m=0} + \delta_m g\left[\frac{\partial p_{\bar{l}}}{\partial \pi_s}\left(\frac{\partial \pi_s}{\partial t}\right)_{\text{s.b.}} - \mathcal{P}_{\bar{l}}\right] = \left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{\bar{l}}^{\delta_m=0} + \delta_m g\left[B_{\bar{l}}(\mathcal{P}_{\overline{L}} + E) - \mathcal{P}_{\bar{l}}\right]$$

where $\left(\frac{\partial \pi_s}{\partial t}\right)_{\text{s.b.}}$ is the partial tendency of the surface pressure due to the local vertical budget, and $\mathcal{P}$ is the precipitation flux.

## 1.3 Continuity equation

In $\eta$ coordinates, where $\nabla_\eta$ and $\mathbf{V}_\eta$ remind that the "horizontal" derivatives are computed along a *constant* $\eta$ surface,

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}\right) = -\nabla_\eta \cdot \left(\mathbf{V}_\eta \frac{\partial p}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) - g\frac{\partial F_m}{\partial \eta} \tag{2}$$

with the turbulent vertical mass flux

$$F_m \equiv F_{\mathcal{P}} + J_v + J_a = F_{\mathcal{P}}$$

as $J_v + J_a \equiv 0$, and noting $F_{\mathcal{P}}$ the vertical mass flux due to the precipitation flux.
At the top of the atmosphere, we have $F_m = 0 = \left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)$. Using

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}\right) = B'(\eta)\frac{\partial \pi_s}{\partial t}$$

and integrating vertically (2) from the top ($\eta = 0$) to level $\eta$:

$$B(\eta)\frac{\partial \pi_s}{\partial t} = -\int_0^\eta \nabla_\eta \cdot \left(\mathbf{V}_\eta \frac{\partial p}{\partial \eta}\right) d\eta - \dot{\eta}\frac{\partial p}{\partial \eta} - gF_{\mathcal{P}}(\eta)$$

If we integrate down to the surface we have

$$\left.\begin{array}{lcl} \left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) & = & \delta_m g E \\ gF_{\mathcal{P}} & = & \delta_m g \mathcal{P}_{\overline{L}} \end{array}\right\} \text{at } \eta = 1$$

and as $B(1) = 1$ we obtain the evolution equation for the surface pressure:

$$\frac{\partial \pi_s}{\partial t} = -\int_0^1 \nabla_\eta \cdot \left(\mathbf{V}_\eta \frac{\partial p}{\partial \eta}\right) d\eta - g\delta_m(E + \mathcal{P}_{\overline{L}}) \tag{3}$$

For levels above, the vertical mass advection term is zero, and the *generalised vertical velocity* writes

$$\dot{\eta}\frac{\partial p}{\partial \eta} = -B(\eta)\frac{\partial \pi_s}{\partial t} - \int_0^\eta \nabla_\eta \cdot \left(\mathbf{V}_\eta \frac{\partial p}{\partial \eta}\right) d\eta - gF_{\mathcal{P}} \tag{4}$$

The pressure vertical velocity:

$$\begin{aligned} \omega \equiv \dot{p} &= \frac{\partial p}{\partial t} + (\mathbf{V}\cdot\nabla)_\eta p + \dot{\eta}\frac{\partial p}{\partial \eta} = (\mathbf{V}\cdot\nabla)_\eta p + \int_0^\eta \frac{\partial}{\partial \eta}\left(\frac{\partial p}{\partial t} + \dot{\eta}\frac{\partial p}{\partial \eta}\right) d\eta \\ &= (\mathbf{V}\cdot\nabla)_\eta p - \int_0^\eta \left[\nabla_\eta \cdot \left(\mathbf{V}_\eta \frac{\partial p}{\partial \eta}\right) + g\frac{\partial F_{\mathcal{P}}}{\partial \eta}\right] d\eta \end{aligned} \tag{5}$$

## 1.4 Moisture equation

Conservation of moisture has a form similar to (2) (tendency=advection + turbulent source), where the turbulent fluxes are the diffusion flux $J_v$ and the precipitation flux $\mathcal{P}$.

$$\frac{\partial}{\partial t}\left(q\frac{\partial p}{\partial \eta}\right) = -\nabla_\eta \cdot \left(\mathbf{V}_\eta \frac{\partial p}{\partial \eta} q\right) - \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta} q\right) - g\frac{\partial}{\partial \eta}(J_v + \mathcal{P})$$

subtracting $q*(2)$ and multiplying by $\frac{\partial \eta}{\partial p}$ yields

$$\begin{aligned} \frac{dq}{dt} &\equiv \frac{\partial q}{\partial t} + (\mathbf{V}\cdot\nabla)_\eta q + \dot{\eta}\frac{\partial q}{\partial \eta} = -g\frac{\partial \eta}{\partial p}\frac{\partial}{\partial \eta}(J_v + \mathcal{P} - qF_{\mathcal{P}}) \\ &= -g\left\{(1-\delta_m q)\frac{\partial \mathcal{P}}{\partial p} + \frac{\partial J_v}{\partial p}\right\} \equiv \left(\frac{\partial q}{\partial t}\right)_\Phi \end{aligned} \tag{6}$$

labelling $()_\Phi$ the partial tendencies provided by the physics.
With the boundary condition at the surface: $\mathcal{P} = \mathcal{P}_{\overline{L}}$ and $J_v = E(1 - \delta_m q)$, while the total vertical water flux produced by evaporation is given by (1): $\left(\dot{\eta}\frac{\partial p}{\partial \eta} q + J_v\right)_{\text{surf}} = gE$

## 1.5 Momentum equations

Labelling the 3-D vectors with a $\vec{\ }$,

$$\frac{d\vec{V}}{dt} = -f\vec{k} \times \vec{V} - \vec{\nabla}\phi - RT\vec{\nabla}\ln p \underbrace{-g\delta_m \mathcal{P}\frac{\partial \mathbf{V}}{\partial p} + \text{Vertic diff}}_{\left(\frac{\partial \mathbf{V}}{\partial t}\right)_\Phi} + \text{Horiz diff} \tag{7}$$

The term $\left(\frac{\partial \mathbf{V}}{\partial t}\right)_\Phi$ being the partial tendency of the horizontal momentum provided by physics. The boundary condition at the ground is $\left(\dot{\eta}\frac{\partial p}{\partial \eta}\mathbf{V}\right)_{\text{surf}} = 0$

## 1.6 Thermodynamic equation

We have:

$$\begin{aligned} dE = dQ + dW &= c_v dT \\ dW = -pdV \Longrightarrow c_v dT + pdV &= dQ + dE_{cin} + dE_{pot} \\ p = \rho RT \Longrightarrow c_p dT = RT\frac{dp}{p} &+ dQ + dE_{cin} + dE_{pot} \\ \frac{dT}{dt} = \frac{RT}{c_p}\frac{\omega}{\rho} + \frac{1}{c_p}\{F_{\text{vertic}} &+ F_{\text{horiz}} + dE_{cin} + dE_{pot}\} \end{aligned}$$

where $F_{\text{vertic}}$ represents all subgrid heat production and vertical transfers, $F_{\text{horiz}}$ represents horizontal heat transfers - for instance horizontal diffusion. Practically physics provides subgrid contributions not only of heat but also of potential and kinetic energy conversions. Hence the material large scale heating writes:

$$\frac{dT}{dt} = \frac{RT}{c_p}\frac{\omega}{p} + \underbrace{\frac{1}{c_p}\left[L_v g\frac{\partial \mathcal{P}}{\partial p} - g\mathcal{P}c_\ell\frac{\partial T}{\partial p} - \delta_m g\mathcal{P}\frac{\partial \phi}{\partial p} + Q + \text{Vertic diff}\right]}_{\left(\frac{\partial T}{\partial t}\right)_\Phi} + \text{Horiz diff} \tag{8}$$

so the physical tendency $\left(\frac{\partial T}{\partial t}\right)_\Phi$ includes

- heat source terms
  - $L_v g\frac{\partial \mathcal{P}}{\partial p}$ comes from the evaporation of the precipitation;
  - $-g\mathcal{P}c_\ell\frac{\partial T}{\partial p}$ is the heating of the liquid precipitation;
  - $Q$ is the radiative heating
- energy conversion terms:
  - $-\delta_m g\mathcal{P}\frac{\partial \phi}{\partial p}$ is the conversion of the potential energy of the precipitation;
- vertical transfer terms:
  - "Vertic diff": includes all vertical exchanges: turbulent vertical diffusion, convection, kinetic energy variations...

## 1.7 Communication between physics and dynamics

Actual communication between physics and dynamics proceeds in a quite different manner from what was outlined above.
Computational constraints lead to a time step algorithm using a semi-implicit integration scheme.

### 1.7.1 Principles

We could represent model evolution as

$$\frac{\partial X}{\partial t} = \mathcal{M}(X) \qquad \begin{array}{lll} \textit{explicit} & CFL & c\frac{\Delta t}{\Delta x} < 1 \\ \textit{implicit} & & \Delta t \rightarrow \text{precision} \end{array}$$

where $X$ is the model state, $\Delta x$ and $\Delta t$ the space mesh and time step, $c$ the velocity of the fastest waves (Lamb's waves, i.e. sound velocity). An implicit discretization remains stable for much longer time steps, but requires solving nonlinear equations which would burden the computation.
$\mathcal{M}$ describes a variety of processes such as for example energy conversions, advection by the wind or wave propagation. Some of these processes are well approximated by a linear operator $\mathcal{L}$.
The semi-implicit resolution scheme computes a first guess by solving the model equation explicitly in real grid point space, and then applies a semi-implicit correction stabilizing the system. The stability criterion is then linked to the maximum wind speed instead of the fastest wave speed. It writes:

$$\begin{aligned} X^+ - X^- &= 2\Delta t \mathcal{M}(X) + 2\Delta t \beta \mathcal{L}\left(\frac{X^+ + X^-}{2} - X\right) \\ X^+(1 - \Delta t \beta \mathcal{L}) &= \underbrace{2\Delta t \mathcal{M}(X) + X^-}_{\substack{X^+_{\text{expl}} \\ \text{first guess}}} + \underbrace{\Delta t \beta \mathcal{L}(X^- - 2X)}_{\text{semi}-\text{implicit correction}} \end{aligned}$$

where parameter $\beta$ sets the degree of implicitness. The linear implicit part, implying the inversion of a Laplacian, takes place in spectral space, while the nonlinear explicit part is computed in real grid point space, under the control of routine CPG. Physics contribution to this computation appear in CPG as tendencies of the model variables.

### 1.7.2 Eulerian time scheme

Noting $\psi$ a model state variable, the Eulerian time scheme computes the *advective tendency* as

$$\mathcal{A}(\psi) \equiv \left(\frac{\partial \psi}{\partial t}\right)_{\text{adv}} = -\mathbf{V} \cdot \nabla \psi - \dot{\eta} \frac{\partial \psi}{\partial \eta}$$

Total tendency of $\psi$ is given by

$$\frac{\partial \psi}{\partial t} = \underbrace{\mathcal{A}(\psi)}_{\text{advection}} + \underbrace{\underbrace{\mathcal{L}(\psi)}_{\text{linear part}} + \underbrace{\mathcal{N}(\psi)}_{\text{nonlinear part}}}_{\text{remaining dynamics}} + \underbrace{\Phi(\psi)}_{\text{physic tendencies}}$$

In the leap-frog scheme, all explicit terms are evaluated at time t in order to perform a jump from $t - \Delta t$ to $t + \Delta t$. As it is taking into account three time levels simultaneously, its characteristic equation is quadratic and there are two numerical modes, the *physical* mode corresponding to the meteorological signal, while the second computational mode represents a $2\Delta t$ oscillation polluting the solution. To alleviate this problem, a temporal filter has to be applied (GPTF1, GPTF2).
A decentered temporal filter is applied in two steps:

$$\begin{aligned} \widetilde{\psi}^0 &= \psi^0 + \epsilon_1(\overline{\psi}^- - \psi^0) - \epsilon_2 \psi^0 \\ \overline{\psi}^0 &= \widetilde{\psi}^0 + \epsilon_2 \psi^+ \end{aligned}$$

The semi-implicit discretization used in ARPÈGE-ALADIN writes:

$$\frac{\psi^+ - \psi^-}{2\Delta t} = \mathcal{A}(\psi^0) + \mathcal{L}\left(\frac{\psi^+ + \psi^-}{2}\right) + \mathcal{N}(\psi^0) + \Phi(\psi^-)$$

Operator $\mathcal{L}$ linearizes the model around a reference state, with an isothermal atmosphere at $300K$ and a surface pressure of $1000hPa$.

### 1.7.3 Semi-Lagrangian time schemes

The semi-Lagrangian scheme treats the advective part by solving the equation:

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{V}\cdot\nabla\psi + \dot{\eta}\frac{\partial\psi}{\partial\eta} = 0$$

and computing the values at grid point $G$ as resulting from advection of values at point $O$, by the (3D) wind at medium point $M$. As neither $O$ nor $M$ have to be on the model grid, 3D interpolations are needed.
The semi-implicit algorithm writes now:

$$\frac{\psi_G^+ - \psi_O^-}{2\triangle t} = \mathcal{L}(\frac{\psi_G^+ + \psi_O^-}{2}) + \mathcal{N}(\psi_M^0) + \Phi(\psi_{O/G}^-)$$

The CFL criterion no longer determines the length of the time step, but it is replaced by the fact that trajectories $(O - M - G)$ should not cross each other.
Other specific dynamic considerations lead to decenter the leapfrog scheme.
A choice must be made about where to compute the physic tendencies: in ARPÈGE-ALADIN it is computed at $t - \triangle t$ at the origin point $O$, and thus interpolated.

## 2 Grid Point calculations flowchart: CPG

CPG essentially computes the physical and dynamical tendencies.
Grid point calculations are performed on packets of horizontal rows by separate calls of the subroutine chain below SCAN2H, which eases the explicit multitasking. This is perfect for the Eulerian version of the time step where all computations are purely local.
But the semi-Lagrangian version also implies values at the origin and middle point of the trajectory, which need to be interpolated from values at neighbouring grid points. To do this, semi-Lagrangian buffers (SLB) were introduced, and the grid point computations splitted as next:

- the non-lagged part (CPG):
  - evaluation of all grid point tendencies from physics (APLPAR, CPTEND...);
  - evaluation of grid point quantities (LACDYN).
  - extension to the C+I domain
  - storage in SLB1 of all inputs for trajectory construction and interpolation:
  - storage in SLB2 of quantities used for evaluation at the arrival points of the trajectories
- the lagged part starts in CPGLAG, using one SLB2 packet (arrival point) and several SLB1 packets for interpolation. It calls LAPINE which is the main interface to the semi-lagrangian computations.

CPG proceeds as follows.
With the Eulerian time scheme:

- SC2RDT9: read work buffers of time $t - \triangle t$;
- GPTF2: apply time filter part 2;
- various initialisations for auxiliary variables;
- Call physic parameterisations: APLPAR
- Compute tendencies: CPFHPFS, CPTEND
- Compute $T$, $q$, wind at time $t + \triangle t$: CPUTQY
- Compute surface tendencies: CPTENDS, CPWTS
- Compute surface $T$, $q$, wind ad time $t + \triangle t$: CPUTQYS
- CPDYN: compute dynamic tendencies;
- Call Diagnostics
- Store cumulated and instantaneous fluxes
- GPTF1: apply time filter part 1;
- SC2WRT9: write to work files.

With a Semi-Lagrangian time scheme:

- SC2RDT9: read work buffers of time $t - \triangle t$;
- GPTF2: apply time filter part 2;
- various initialisations for auxiliary variables;
- get and initialize semi-Lagrangian buffers;
- Call physical parameterisations: APLPAR
- Compute tendencies: CPFHPFS, CPTEND
- Compute $T$, $q$, wind at time $t + \triangle t$: CPUTQY
- Compute surface tendencies: CPTENDS, CPWTS
- Compute surface $T$, $q$, wind ad time $t + \triangle t$: CPUTQYS
- LACDYN: evaluate grid point quantities at $t$ and $t - \triangle t$ for subsequent SL computations
- Call Diagnostics
- Store cumulated and instantaneous fluxes
- GPINISLB9: fill semi-Lagrangian buffers for lagged physics;
- GPTF1: apply time filter part 1;
- SC2WRT9: write to work files.

# 3 Physical tendencies and time step calculations

The local physical tendencies include the vertical divergence of vertical physical fluxes, and a vertical pseudo-advection term linked to the precipitation flux in the case mass is not conserved.
This pseudo-advection is applied to large scale momentum and large scale enthalpy (or dry static energy and kinetic energy). Concerning moisture, it would be dubious to consider advection of large scale moisture by the precipitation flux, as the moisture in the neighbourhood of the precipitation flux has little relation with the large scale moisture, and is closer to the wet bulb moisture associated to the large scale temperature. So the pseudo-advection term does not appear in the expression of the moisture tendency.

## 3.1 Model levels tendencies and time step CPTEND, CPUTQY

### 3.1.1 Momentum

$$\left(\frac{\partial \mathbf{V}}{\partial t}\right)_\Phi = -g\frac{\partial}{\partial p}\left\{J_{\mathbf{V}}^{\mathrm{turb}} + J_{\mathbf{V}}^{\mathrm{gwd}} + J_{\mathbf{V}}^{\mathrm{med}} + J_{\mathbf{V}}^{\mathrm{conv}}\right\} - \delta_m g\mathcal{P}\frac{\partial \mathbf{V}}{\partial p} \tag{9}$$

with

- PSTRTU, PSTRTV $\equiv J_{\mathbf{V}}^{\mathrm{turb}}$: turbulent momentum flux
- PSTRCU, PSTRCV $\equiv J_{\mathbf{V}}^{\mathrm{conv}}$: convective momentum flux
- PSTRDU, PSTRDV $\equiv J_{\mathbf{V}}^{\mathrm{gwd}}$: gravity wave drag
- PSTRMU, PSTRMV $\equiv J_{\mathbf{V}}^{\mathrm{med}}$: mesospheric drag
- $\mathcal{P} \equiv$ PFPLCL + PFPLCN + PFPLSL + PFPLSN: total precipitation flux

Tendency is stored into PTENDU, PTENDV.

The physical time step is performed by CPUTQY by modifying the values at $t + \triangle t$ through the addition of the physical tendencies.

$$\mathbf{V}_1 = \mathbf{V}_1 + \triangle t_{\mathrm{phys}} \cdot \left(\frac{\partial \mathbf{V}}{\partial t}\right)_\Phi \tag{10}$$

and stored into PUT1, PVT1.

### 3.1.2 moisture

$$\left(\frac{\partial q}{\partial t}\right)_{\Phi} = -g\frac{\partial}{\partial p}\left\{J_q^{\text{turb}} + J_q^{\text{conv}}\right\} - g(1-\delta_m q)\frac{\partial \mathcal{P}}{\partial p} \tag{11}$$

with

- PDIFTQ $\equiv J_q^{\text{turb}}$: turbulent moisture flux
- PDIFCQ $\equiv J_q^{\text{conv}}$: convective moisture flux

we consider here the turbulent and convective moisture fluxes, and the effect of precipitation evaporation and condensation. We cannot consider a pseudo-advection of the large scale moisture. The precipitation flux is a moisture flux, and the air is always close to saturation in its neighbourhood: it would be meaningless to introduce an advection of the large scale moisture by the precipitation flux. On the other hand, whatever the mass conservation parameter, variation of the precipitation flux has always an impact on the specific moisture. The last term of the RHS represents the partial tendency of specific moisture due to precipitation evaporation/recondensation. This expression was already derived in equation (6) and is shown more intuitively in the chapter about energetic aspects.
Tendency is stored into PTENDQ.

The physical time step is performed by CPUTQY by modifying the value at $t + \triangle t$:

$$q_{v1} = q_{v1} + \triangle t_{\text{phys}} \cdot \left(\frac{\partial q}{\partial t}\right)_{\Phi} \tag{12}$$

and stored into PQT1.

### 3.1.3 moisture equations with prognostic condensed phases

The operational model does not yet consider prognostic (large scale) condensed phases, but the structure is already present to handle them in the dynamics and within the budgets.
Model key LCONDWT activates the condensed phases: in this case, we have to consider the fluxes representing the different phase changes. we split equation (11) between vapour, solid and liquid phases:

- Vapour:
$$\left(\frac{\partial q}{\partial t}\right)_{\Phi} = -g\frac{\partial}{\partial p}\left\{J_q^{\text{turb}} + J_q^{\text{conv}}\right\} - g\frac{\partial F_{\text{cond}}}{\partial p} + g\delta_m q\frac{\partial \mathcal{P}}{\partial p} \tag{13}$$
- Solid water:
$$\left(\frac{\partial \ell_i}{\partial t}\right)_{\Phi} = -g\frac{\partial}{\partial p}\left\{J_{\ell_i}^{\text{turb}} + J_{\ell_i}^{\text{conv}}\right\} - g\frac{\partial \mathcal{P}_i - F_{\text{cond\_i}}}{\partial p} + g\delta_m \ell_i\frac{\partial \mathcal{P}}{\partial p} \tag{14}$$
- Liquid water:
$$\left(\frac{\partial \ell_w}{\partial t}\right)_{\Phi} = -g\frac{\partial}{\partial p}\left\{J_{\ell_w}^{\text{turb}} + J_{\ell_w}^{\text{conv}}\right\} - g\frac{\partial \mathcal{P}_w - F_{\text{cond\_w}}}{\partial p} + g\delta_m \ell_w\frac{\partial \mathcal{P}}{\partial p} \tag{15}$$

where

- $\mathcal{P}_{i|w}$ represent the total (convective + large scale) solid and liquid precipitation fluxes,
- $F_{\text{cond}}$ the total (convective + large scale, ice and water) condensation flux linked to the precipitation flux — if LCONDWT = .FALSE., this flux is equal to the total precipitation flux, and we find back equation 11;
- $F_{\text{cond\_w|i}}$ are the liquid and solid parts of the same flux.

The *"condensation flux"* corresponds actually to the total tendency of the condensate present above a given level, while the *precipitation flux* is the part of it travelling downwards (and it is only this part that implies a net advection under $\delta_m = 1$).

### 3.1.4 enthalpy and temperature

Enthalpy is not a model variable, but we need it to compute the temperature evolution. We define the *pseudo-enthalpy* as

$$\begin{aligned} \hbar &\equiv c_p T + \phi + \frac{u^2+v^2}{2} = c_p T + \phi + E_{\text{kin}} \\ \text{with } c_p &= c_{pa} + (c_{pv} - c_{pa})q + (c_w - c_{pa})\ell_w + (c_i - c_{pa})\ell_i \end{aligned}$$

The physical tendency PTENDH is

$$\left(\frac{\partial \hbar}{\partial t}\right)_\Phi = -g\frac{\partial}{\partial p}\left\{F_\odot + F_\dagger + J_h^{\text{meso}} + J_s^{\text{turb}} + J_s^{\text{conv}} + F_{s\mathcal{P}}\right\} - \delta_m g \mathcal{P}\frac{\partial \hbar}{\partial p} \tag{16}$$

where

- PFRSO $\equiv F_\odot$ and PFRTH $\equiv F_\dagger$ are the enthalpy fluxes from the solar and thermal radiation
- PFRMH $\equiv J_h^{\text{meso}}$ is the enthalpy flux from the mesosphere (this is used only in specific climatological runs of the ARPÈGE model)
- PDIFTS $\equiv J_s^{\text{turb}}$ is the turbulent diffusion dry static energy flux
- PDIFCS $\equiv J_s^{\text{conv}}$ is the convective dry static energy flux
- PFHP $\equiv F_{s\mathcal{P}}$ represents the dry static energy flux associated to the total precipitation flux at the given level (convective and large scale, liquid and solid) or associated to total precipitation *and* condensation fluxes under LCONDWT: this flux is computed by routine CPFHPFS.
  With no large scale condensate we have:

  $$F_{s\mathcal{P}} \equiv \mathsf{PFHP} = \mathsf{PFHPCL} + \mathsf{PFHPCN} + \mathsf{PFHPSL} + \mathsf{PFHPSN} = -L_{\text{flux}}\mathcal{P}$$

  Under LCONDWT we must distinguish

  - the part linked to condensation: $-L_{v-w|i}(0K)\cdot F_{\text{cond\_w|i}}$ (with the latent heats of the different phase changes, taken at $0K$)
  - the sensible heat fluxes linked to the precipitation flux: $\mathcal{P}_{w|i}\cdot T\cdot(c_{w|i} + c_{pa}(\delta_m - 1))$,
  - the pseudo enthalpy flux born by the precipitation under $\delta_m = 1$: $-\delta_m \cdot c_p \cdot T \cdot \mathcal{P}$

The physical time step is performed by CPUTQY as follows. Enthalpy at $t_9 \equiv t - \triangle t$ is

$$\hbar_9 = c_{p9}T_9 + \frac{u_9^2 + v_9^2}{2}$$

First we evaluate the evolution of moisture, specific heat , kinetic energy, enthalpy, due to *physics only* (labelled as $t_{9+\Phi}$):

$$\begin{aligned} q_{v9+\Phi} &= q_{v9} + \triangle t\cdot\mathsf{PTENDQ} \qquad q_{l9+\Phi} = q_{l9} + \triangle t\cdot\mathsf{PTENDQL} \qquad q_{i9+\Phi} = q_{i9} + \triangle t\cdot\mathsf{PTENDQI} \\ c_{p9+\Phi} &= \begin{cases} c_{pa} - (c_{pa} - c_{pv})q_{v9+\Phi} \\ c_{pa}\cdot(1 - q_{v9+\Phi} - q_{l9+\Phi} - q_{i9+\Phi}) + c_{pv}q_{v9+\Phi} + c_w\cdot q_{l9+\Phi} + c_i\cdot q_{i9+\Phi} & \text{if } \mathsf{LCONDWT} \end{cases} \\ \hbar_{9+\Phi} &= \hbar_9 + \triangle t\cdot\mathsf{PTENDH} \\ E_{\text{kin}\,9+\Phi} &= \frac{u_{9+\Phi}^2 + v_{9+\Phi}^2}{2} \end{aligned}$$

We have then:

$$T_1 = T_1 + \frac{\hbar_{9+\Phi} - E_{\text{kin}\,9+\Phi}}{c_{p9+\Phi}} - T_9$$

A Kinetic energy dissipation flux PFDIS, zero at the top, is obtained by:

$$J_{\text{diss}}^l = J_{\text{diss}}^{l-1} + \frac{\delta p(E_{\text{kin}\,9+\Phi} - E_{\text{kin}\,9})}{g\triangle t} \tag{17}$$

## 3.2 Variable mass effect on surface pressure and vertical velocity: CPMVVPS

### 3.2.1 surface pressure

Computed according to equation(3). The integrated divergence PSDIV is input, and the $\delta_m$ term is added to it to output the surface pressure tendency as PSDIV:

$$-\mathsf{PSDIV(KLEV)} = \frac{\partial \pi_s}{\partial t} = -\int_0^1 \nabla \cdot \left(\mathbf{V}\frac{\partial p}{\partial \eta}\right) d\eta - \underline{g\delta_m(F_{\mathrm{evn}} + F_{\mathrm{evl}} + \mathcal{P}_s)} \tag{18}$$

where

- PFEVL $= F_{\mathrm{evl}}$ is the evaporation over water or moist soil,
- PFEVN $= F_{\mathrm{evn}}$ is the evaporation flux over snow or ice.

### 3.2.2 vertical velocity

Equation(4) gives the generalised vertical velocity in $\eta$ coordinates, PEVEL:

$$\dot{\eta}\frac{\partial p}{\partial \eta} = -B(\eta)\frac{\partial \pi_s}{\partial t} - \int_0^\eta \nabla \cdot \left(\mathbf{V}\frac{\partial p}{\partial \eta}\right) d\eta - \underline{g\delta_m \mathcal{P}}$$

and (5) gives the vertical velocity in pressure coordinates, PVVEL:

$$\frac{\omega}{p} = \; = \mathbf{V}\cdot\nabla_\eta \ln(p) - \frac{1}{p}\int_0^\eta \nabla_\eta \cdot \left(\mathbf{V}\frac{\partial p}{\partial \eta}\right) d\eta - \underline{\frac{1}{p}\int_0^\eta g\delta_m \frac{\partial \mathcal{P}}{\partial \eta} d\eta}$$

The discretization uses

$$\frac{1}{X}\int X d\eta = \left(\frac{\partial p}{\partial \eta}\right)^{-1}\left(\frac{\partial}{\partial \eta}\left(\ln(p)\int X d\eta\right) - X\cdot\ln(p)\right)$$

and also $p_l = e^{-\alpha_l}\cdot p_{\bar{l}}$ if LAPRXPK=.FALSE..

## 3.3 Surface tendencies and physical time step: CPTENDS, CPWTS

Definitions, fluxes, tendencies, and evolution of ground variables are covered in the chapter about the soil-vegetation scheme.

# 4 Physical Parameterisations Call: APLPAR

The different physical parameterisations called by routine APLPAR are described in the following chapters. They are organized as follows:

5. Auxiliary tools and initialisations
   - Energetics
   - Thermodynamical functions
   - Wet bulb computation
   - Initialisations
6. PBL and turbulent processes
   - ACHMT: Turbulent fluxes between surface and lowest model level
   - ACCOEFK: Turbulent fluxes between model levels
   - ACDIFUS:Vertical turbulent diffusion

- Anti-Fibrillation scheme

7. Subgrid dynamical and other processes
   - ACDRAG: Orographic gravity wave drag
   - ACDRAC: Convective gravity wave drag
   - ACDRME: Mesospheric drag
8. Soil processes
   - ISBA soil-vegetation scheme
   - Practical implementation
     - ACSOL, ACSOLW: Surface characteristics
     - ACVEG: Vegetation characteristics
     - ACDROV: Water transfers inside the soil-vegetation scheme
     - ACHMT, ACDIFUS
     - APLPAR
     - CPTENDS, CPWTS
9. Cloudiness and Large scale precipitation
   - ACNEBN: Combined cloudiness scheme
   - ACPLUIE: Large scale Precipitation
10. Radiation and chemicals
    - ACRANEB: Economic radiation scheme
    - ACOZONE: Ozone and chemicals
11. ACCVIMP, ACCVIMPD: Deep convection – this chapter is detailed in part III.

# 5 Diagnostics

Namelist NAMZDI allows to order 4 different kinds of zonal diagnostics (i.e. performing an average over all longitudes, for a range of latitudes), each one with its own frequency. They are:

- LZDIDY (routine ZODIDY): global diagnostics on dynamic fields:
  - maximal values on the sphere (and their location): horizontal wind, vertical velocity, horizontal and vertical CFL numbers, critical semi-Lagrangian divergence.
- LZDIFI (routine ZODIFI): zonal mean diagnostics on fields: wind, vorticity, divergence, $\Delta p$, $c_p * T$, specific and relative humidity, cloudiness, solid and liquid moisture.
- LZDIGI (routine ZODIGI): zonal mean diagnostics on invariants: mass, potential and kinetic energy, kinetic momentum (3 components), entropy, potential temperature, potential vorticity.
- LZDIFF (routine ZODIFF): zonal mean diagnostics on physical fluxes:
  - Convective and turbulent fluxes of moisture, enthalpy, horizontal momentum;
  - gravity wave drag flux of horizontal momentum;
  - convective and large scale rain and snow;
  - enthalpy flux from precipitation and from kinetic energy dissipation;
  - solar and thermal radiation fluxes;
  - fictive flux of negative specific humidity;
  - mesospheric fluxes of horizontal momentum and enthalpy;
  - ozone flux.

Beside this system, the DDH package provides a more flexible way to generate various diagnostics over arbitrary horizontal domains, through namelist NAMDDH. Refer to [Piriou, 1999].