Dynamics: report and plans on time step organisation, LBC's and MCUF

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in collaboration with

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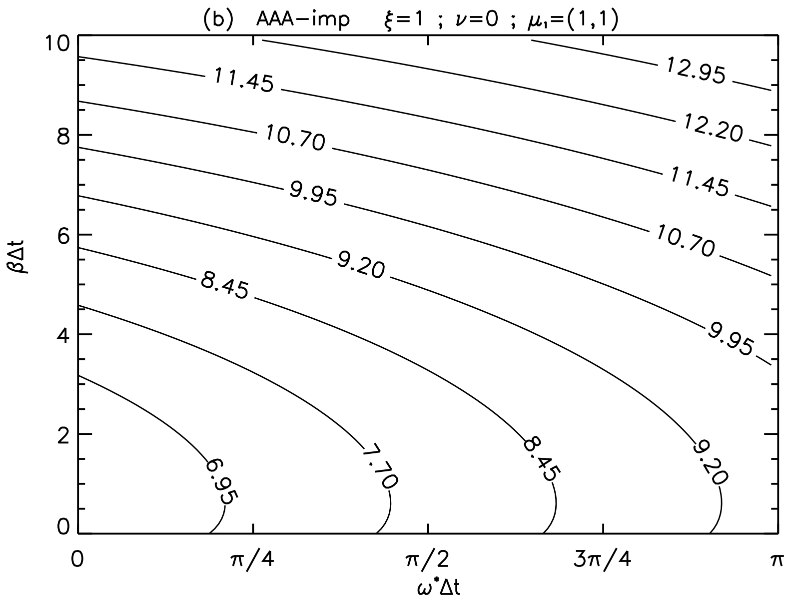
Issues: status and plans

- physics-dynamics coupling
- LBC's
- Monitoring the Coupling Update Frequency (MCUF) and something nasty about DFI

Physics-dynamics coupling

Termonia and Hamdi: $\frac{DF}{Dt} + \mathrm{Dyn} = -\beta F + R\,e^{i(kx+\Omega t)}$

$$\left|\frac{F^{+}}{F^{exact}}\right| =$$

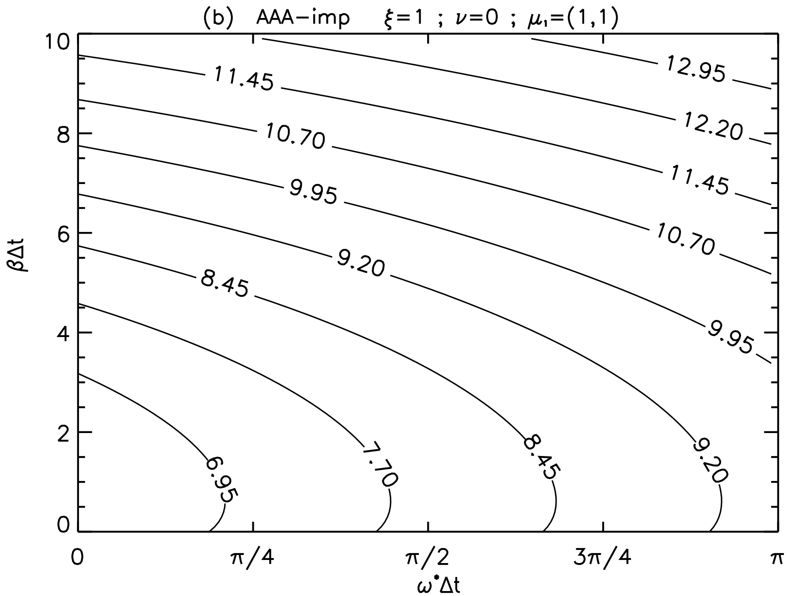


$$\mathrm{AAA} : F_A^{+} - F_{forced}^{exact} = -\frac{iR[(kU+\omega)^2 + 2\beta(\beta+i\Omega) - \omega\Omega - \Omega^2 + \Omega\omega^*]}{2(kU - i\beta + \omega + \Omega)} e^{i(kx+\Omega t)} \Delta t^2 +$$

Physics-dynamics coupling

Termonia and Hamdi: $\frac{DF}{Dt} + \mathrm{Dyn} = -\beta F + R\, e^{i(kx+\Omega t)}$

$$\left| \frac{F^{+}}{F^{exact}} \right| =$$



Why does ARPEGE, ALADIN, AROME work?

$$\mathrm{AAA}: F_A^{+} - F_{forced}^{exact} = -\frac{iR[(kU+\omega)^2 + 2\beta(\beta+i\Omega) - \omega\Omega - \Omega^2 + \Omega\omega^*]}{2(kU - i\beta + \omega + \Omega)} e^{i(kx+\Omega t)} \Delta t^2 +$$

Tuning?

Theory: $\frac{DF}{Dt} + \mathrm{Dyn} = -\beta F + R_1\, e^{i(kx+\Omega_1 t)} + R_2\, e^{i(kx+\Omega_2 t)}$

Model: $\frac{DF}{Dt} + \mathrm{Dyn} = -\beta^M F + R_1^M\, e^{i(kx+\Omega_1^M t)} + R_2^M\, e^{i(kx+\Omega_2^M t)}$

$$F_A^+ - F^{exact} =$$

$$e^{ikx}\Big(-\frac{e^{it\Omega_1} R_1}{\beta + i(kU+\omega+\Omega_1)} + \frac{e^{it\Omega_1^M} R_1^M}{\beta^M + i(kU+\omega+\Omega_1^M)}$$

$$-\frac{e^{it\Omega_2} R_2}{\beta + i(kU+\omega+\Omega_2)} + \frac{e^{it\Omega_2^M} R_2^M}{\beta^M + i(kU+\omega+\Omega_2^M)}\Big)$$

$$+e^{ikx}\Big(-\frac{e^{it\Omega_1} R_1 \Omega_1}{kU - i\beta + \omega + \Omega_1} + \frac{e^{it\Omega_1^M} R_1^M \Omega_1^M}{kU - i\beta^M + \omega + \Omega_1^M}$$

$$-\frac{e^{it\Omega_2} R_2 \Omega_2}{kU - i\beta + \omega + \Omega_2} + \frac{e^{it\Omega_2^M} R_2^M \Omega_2^M}{kU - i\beta^M + \omega + \Omega_2^M}\Big)\Delta t$$

$+$ much more junk you don't want to know ...

Tuning?

Theory: $\frac{DF}{Dt} + \mathrm{Dyn} = -\beta F + R_1\, e^{i(kx+\Omega_1 t)} + R_2\, e^{i(kx+\Omega_2 t)}$
Model: $\frac{DF}{Dt} + \mathrm{Dyn} = -\beta^M F + R_1^M\, e^{i(kx+\Omega_1^M t)} + R_2^M\, e^{i(kx+\Omega_2^M t)}$

Tuning:

$$0 = \begin{aligned} &-\frac{e^{it\Omega_1} R_1}{\beta + i(kU+\omega+\Omega_1)} + \frac{e^{it\Omega_1^M} R_1^M}{\beta^M + i(kU+\omega+\Omega_1^M)} \\ &-\frac{e^{it\Omega_2} R_2}{\beta + i(kU+\omega+\Omega_2)} + \frac{e^{it\Omega_2^M} R_2^M}{\beta^M + i(kU+\omega+\Omega_2^M)} \end{aligned}$$

Eliminate one of the 5 parameters: $\beta^M, R_1^M, R_2^M, \Omega_1^M, \Omega_2^M$ as a function of $\beta, R_1, R_2, \Omega_1, \Omega_2$

Discussion

- Time step organisation provides a sort of skeleton for the time step
- The 1D exercises suggests that the AAA models have a bad “skeleton”
- with correction by “muscular” physics parameteristion
- Is this a problem? If you have 1 model: no. If you replace the parameterisation by one of another model: then this suggests yes.

Plans

- understand tuning in 1D model
- go to 3D: see presentation Martina Tudor

LBC's

- PhD work Fabrice Voitus (GMAP/CNRM/Météo-France)
- find an alternative for the Davies scheme
- "Can one import science obtained in gridpoint models into spectral models?"

LBC's: methodology

- LBC's are a *mine field*
- the only chance to make progress is to cut the problem into pieces and find techniques to *master* each individual one; only then can we hope to *control* the scheme.
- we try to have an *ideal solution* (in the sense that it may seem unfeasibly in real model circumstances), to which we can make compromises later to get something cheap enough.

A desirable feature

It would be benifi cial if would could treat the LBC's independently from the dynamical core of the model, in other words *externalize* the LBC's. Advantages:

- if LBC's are a different module of the model than the dynamics: good for importing science and good for maintenance
- this is needed for spectral models: the Helmholtz equation is solved in spectral space and one can not impose the LBC's in there.

Is this crazy?

LBC methodology

- first master gridpoint space, than spectral space!

LBC methodology

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- first master simple models: 1D, linear shallow water equation with Coriolis

LBC methodology

- first master gridpoint space, than spectral space!
- first master simple models: 1D, linear shallow water equation with Coriolis
- take what is established as a reference: McDonald (2001), MWR

Time step: grid point model C grid

Interior (dynamical core)	lateral boundaries
$u^0_{-\frac{1}{2}}, u^0_{\frac{1}{2}}, u^0_{\frac{3}{2}}, \ldots, u^0_{L-\frac{1}{2}}, u^0_{L+\frac{1}{2}}$ $\longrightarrow$	[redacted]
Explicit Dynamics (no LBC) $R^\Phi_0, R^\Phi_1, \ldots, R^\Phi_{L-1}, R^\Phi_L$	
$u^+_{-\frac{1}{2}}$ & $u^+_{L+\frac{1}{2}}$ $\longleftarrow$	
$\Phi^+_0 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{1}{2}} - u^+_{-\frac{1}{2}}) = R^\Phi_0$ $\Phi^+_1 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{3}{2}} - u^+_{\frac{1}{2}}) = R^\Phi_1$ $\vdots$ $\Phi^+_L + \frac{\Delta t}{2\Delta x}(u^+_{L+\frac{1}{2}} - u^+_{L-\frac{1}{2}}) = R^\Phi_L$	
$\Phi^+_0, \Phi^+_1, \ldots, \Phi^+_L$	

Time step: grid point model C grid

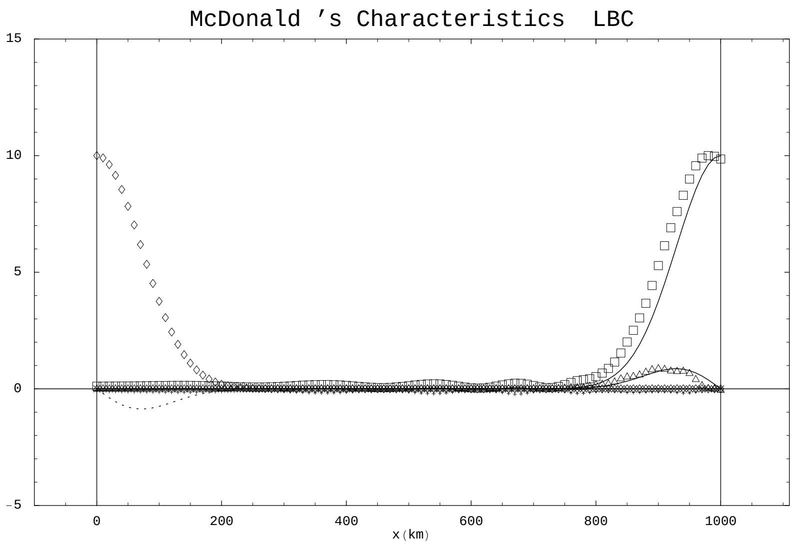
Interior (dynamical core)		lateral boundaries
$u^0_{-\frac{1}{2}}, u^0_{\frac{1}{2}}, u^0_{\frac{3}{2}}, \ldots, u^0_{L-\frac{1}{2}}, u^0_{L+\frac{1}{2}}$	$\longrightarrow$	$u^0_{-\frac{1}{2}}, \ldots,$ & $\ldots, u^0_{L+\frac{1}{2}}$
Explicit Dynamics (no LBC)		Substepping (with LBC)
$R^\Phi_0, R^\Phi_1, \ldots, R^\Phi_{L-1}, R^\Phi_L$		$u^+_{-\frac{1}{2}}, \ldots, u^+_{N+\frac{1}{2}}$ & $u^+_{L-N-\frac{1}{2}}, \ldots, u^+_{L+\frac{1}{2}}$
$u^+_{-\frac{1}{2}}$ & $u^+_{L+\frac{1}{2}}$	$\longleftarrow$	$u^+_{-\frac{1}{2}}$ & $u^+_{L+\frac{1}{2}}$
$\Phi^+_0 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{1}{2}} - u^+_{-\frac{1}{2}}) = R^\Phi_0$ $\Phi^+_1 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{3}{2}} - u^+_{\frac{1}{2}}) = R^\Phi_1$ $\vdots$ $\Phi^+_L + \frac{\Delta t}{2\Delta x}(u^+_{L+\frac{1}{2}} - u^+_{L-\frac{1}{2}}) = R^\Phi_L$		
$\Phi^+_0, \Phi^+_1, \ldots, \Phi^+_L$		

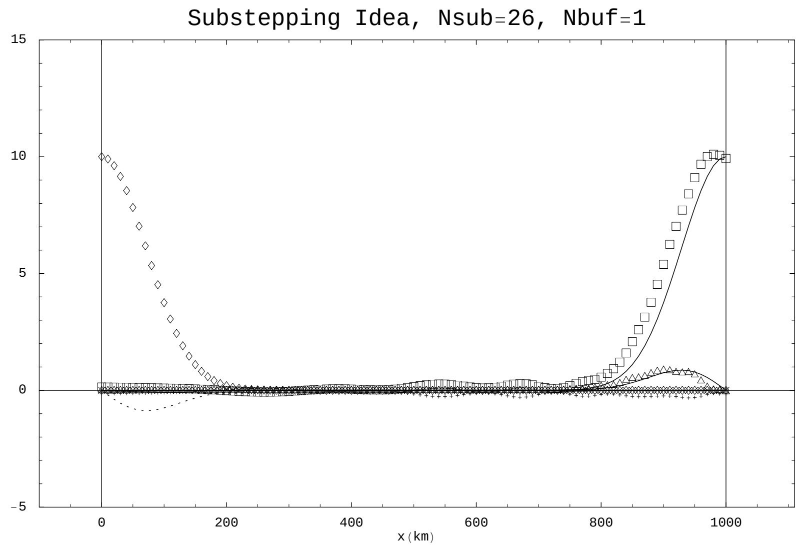
Time step: grid point model C grid

Interior (dynamical core)	lateral boundaries
$u^0_{-\frac{1}{2}}, u^0_{\frac{1}{2}}, u^0_{\frac{3}{2}}, \ldots, u^0_{L-\frac{1}{2}}, u^0_{L+\frac{1}{2}}$ $\longrightarrow$	[redacted]
Explicit Dynamics (no LBC) $R^\Phi_0, R^\Phi_1, \ldots, R^\Phi_{L-1}, R^\Phi_L$	
$u^+_{-\frac{1}{2}}$ & $u^+_{L+\frac{1}{2}}$ $\longleftarrow$	
$\Phi^+_0 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{1}{2}} - u^+_{-\frac{1}{2}}) = R^\Phi_0$ $\Phi^+_1 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{3}{2}} - u^+_{\frac{1}{2}}) = R^\Phi_1$ $\vdots$ $\Phi^+_L + \frac{\Delta t}{2\Delta x}(u^+_{L+\frac{1}{2}} - u^+_{L-\frac{1}{2}}) = R^\Phi_L$	
$\Phi^+_0, \Phi^+_1, \ldots, \Phi^+_L$	

substepping

A. McDonald, MWR (2001) + substepping



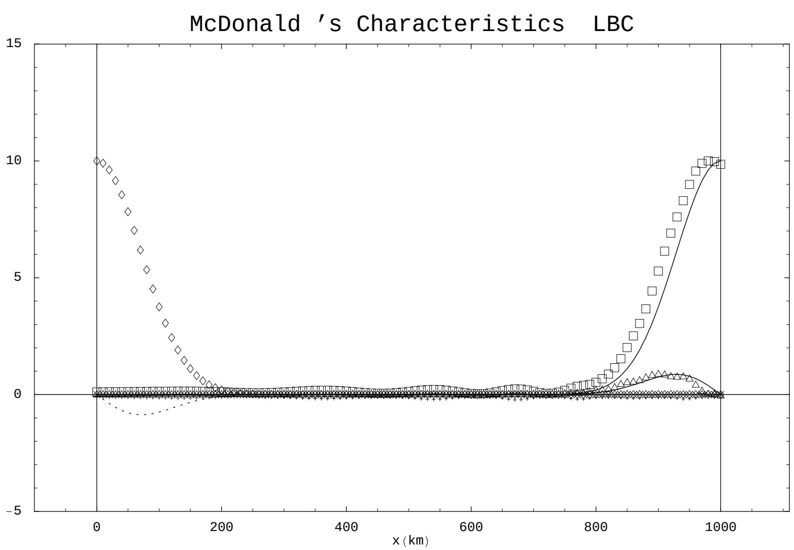


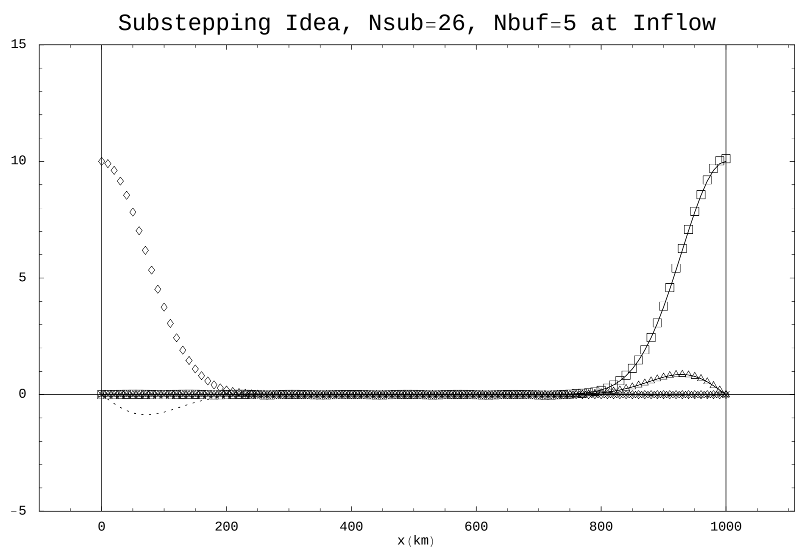
Time step: grid point model C grid

Interior (dynamical core)		lateral boundaries
$u^0_{-\frac{1}{2}}, u^0_{\frac{1}{2}}, u^0_{\frac{3}{2}}, \ldots, u^0_{L-\frac{1}{2}}, u^0_{L+\frac{1}{2}}$	$\longrightarrow$	$u^0_{-\frac{1}{2}}, \ldots,$ & $\ldots, u^0_{L+\frac{1}{2}}$
Explicit Dynamics (no LBC) $R^\Phi_0, R^\Phi_1, \ldots, R^\Phi_{L-1}, R^\Phi_L$		Substepping (with LBC) $u^+_{-\frac{1}{2}}, \ldots, u^+_{N+\frac{1}{2}}$ & $u^+_{L-N-\frac{1}{2}}, \ldots, u^+_{L+\frac{1}{2}}$
$u^+_{-\frac{1}{2}}$ & $u^+_{L+\frac{1}{2}}$	$\longleftarrow$	$u^+_{-\frac{1}{2}}$ & $u^+_{L+\frac{1}{2}}$
$\Phi^+_0 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{1}{2}} - u^+_{-\frac{1}{2}}) = R^\Phi_0$ $\Phi^+_1 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{3}{2}} - u^+_{\frac{1}{2}}) = R^\Phi_1$ $\vdots$ $\Phi^+_L + \frac{\Delta t}{2\Delta x}(u^+_{L+\frac{1}{2}} - u^+_{L-\frac{1}{2}}) = R^\Phi_L$		
$\Phi^+_0, \Phi^+_1, \ldots, \Phi^+_L$		
$\Phi^+_0, \ldots, \Phi^+_N, \Phi^+_{N+1}, \ldots$	$\longleftarrow$	$\Phi^+_0, \ldots, \Phi^+_N$

substepping with extra buffer

A. McDonald, MWR (2001) + substepping and substep buffer





Time step: spectral

Interior (dynamical core)

$$u^0_{-\frac{1}{2}}, u^0_{\frac{1}{2}}, u^0_{\frac{3}{2}}, \ldots, u^0_{L-\frac{1}{2}}, u^0_{L+\frac{1}{2}} \longrightarrow$$

Explicit Dynamics (no LBC)

$$R^\Phi_0, R^\Phi_1, R^\Phi_2 \ldots, R^\Phi_{L-2}, R^\Phi_{L-1}, R^\Phi_L$$

$$R \equiv \tilde{R}^\Phi_0, \tilde{R}^\Phi_1, R^\Phi_3 \ldots, R^\Phi_{L-2}, \tilde{R}^\Phi_{L-1}, \tilde{R}^\Phi_L \longleftarrow$$

periodicity + FFT $[R]$

$$\begin{pmatrix} u \\ v \\ \Phi \end{pmatrix}^+ = \left[1 - \frac{1}{2}\Delta t \mathcal{L}\right]^{-1}_{SP} \begin{pmatrix} R^u \\ R^v \\ R^\Phi \end{pmatrix}^+$$

[redacted]

Time step: spectral

Interior (dynamical core)		lateral boundaries
$u^0_{-\frac{1}{2}}, u^0_{\frac{1}{2}}, u^0_{\frac{3}{2}}, \ldots, u^0_{L-\frac{1}{2}}, u^0_{L+\frac{1}{2}}$	$\longrightarrow$	$u^0_{-\frac{1}{2}}, \ldots,$ & $\ldots, u^0_{L+\frac{1}{2}}$
Explicit Dynamics (no LBC) $R^\Phi_0, R^\Phi_1, R^\Phi_2 \ldots, R^\Phi_{L-2}, R^\Phi_{L-1}, R^\Phi_L$		Substepping (with LBC) $u^+_{-\frac{1}{2}}, u^+_{\frac{1}{2}}, u^+_{\frac{3}{2}}$ & $u^+_{L-\frac{1}{2}}, u^+_{L-\frac{1}{2}}, u^+_{L+\frac{1}{2}}$
$R \equiv \tilde{R}^\Phi_0, \tilde{R}^\Phi_1, R^\Phi_3 \ldots, R^\Phi_{L-2}, \tilde{R}^\Phi_{L-1}, \tilde{R}^\Phi_L$	$\longleftarrow$	$\left.\begin{aligned} \tilde{R}^\Phi_0 &= \Phi^+_0 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{1}{2}} - u^+_{-\frac{1}{2}}) \\ \tilde{R}^\Phi_1 &= \Phi^+_1 + \frac{\Delta t}{2\Delta x}(u^+_{\frac{3}{2}} - u^+_{\frac{1}{2}}) \end{aligned}\right\} = \left[1 - \frac{1}{2}\Delta t \mathcal{L}\right]^\Phi_{GP} \begin{pmatrix} u^+ & v^+ & \Phi^+ \end{pmatrix}^T$
periodicity + FFT $[R]$		
$\begin{pmatrix} u \\ v \\ \Phi \end{pmatrix}^+ = \left[1 - \frac{1}{2}\Delta t \mathcal{L}\right]^{-1}_{SP} \begin{pmatrix} R^u \\ R^v \\ R^\Phi \end{pmatrix}^+$		

The operators?

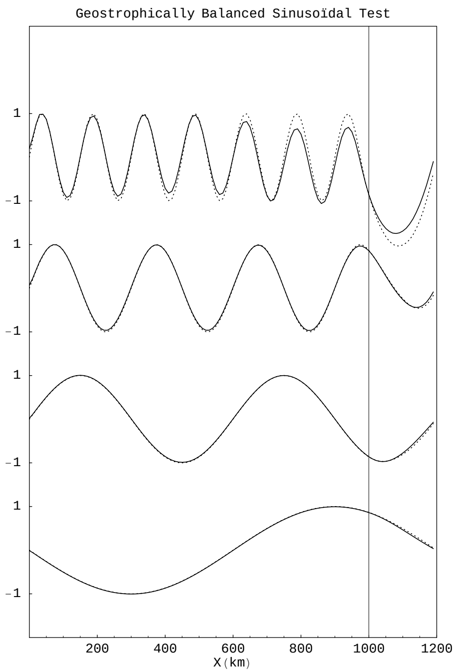
Accuracy:

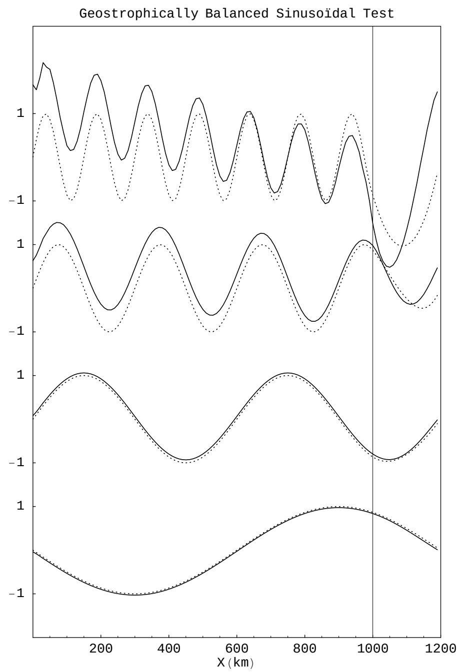
$$\left[1-\frac{1}{2}\Delta t\mathcal{L}\right]^{-1}_{SP}\left[1-\frac{1}{2}\Delta t\mathcal{L}\right]_{GP}=1+\mathcal{O}[\Delta x^p,\Delta t^q]$$

Stability:

$$Eigenvalues\left\{\left[1-\frac{1}{2}\Delta t\mathcal{L}\right]^{-1}_{SP}\left[1-\frac{1}{2}\Delta t\mathcal{L}\right]_{GP}\right\}\leq 1$$

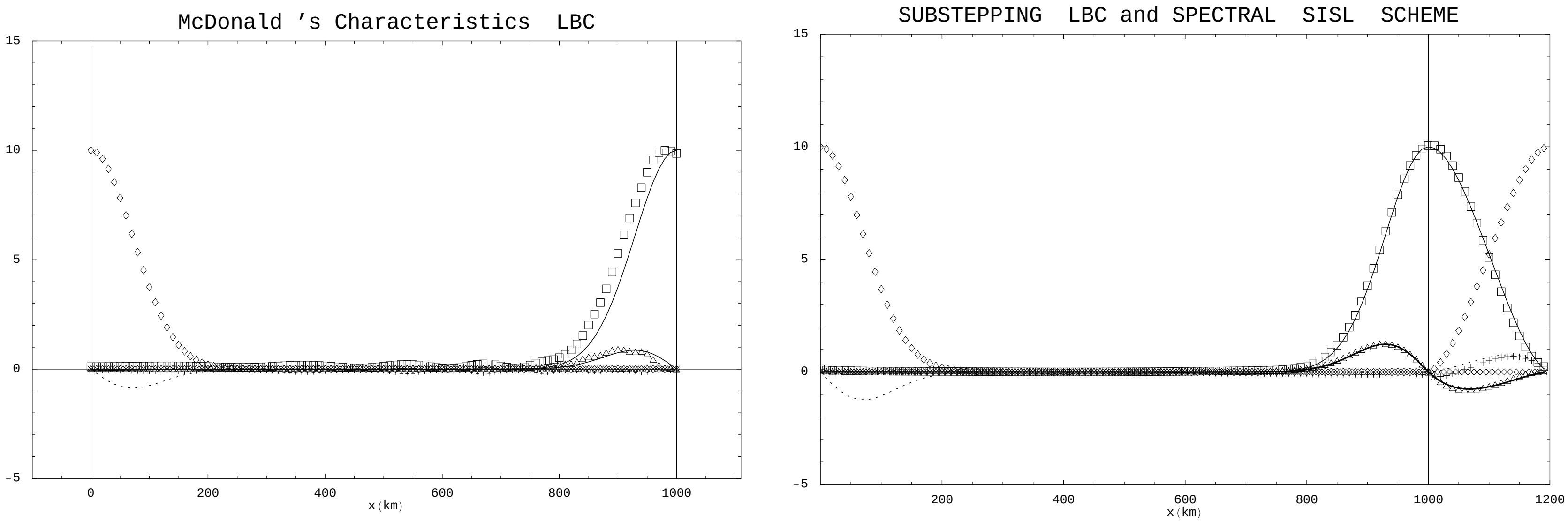
Inaccurate operator





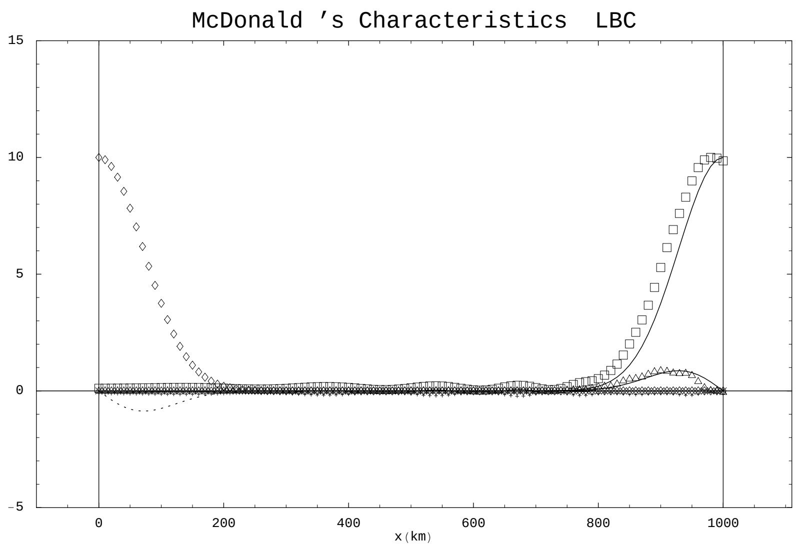
spectral ISL

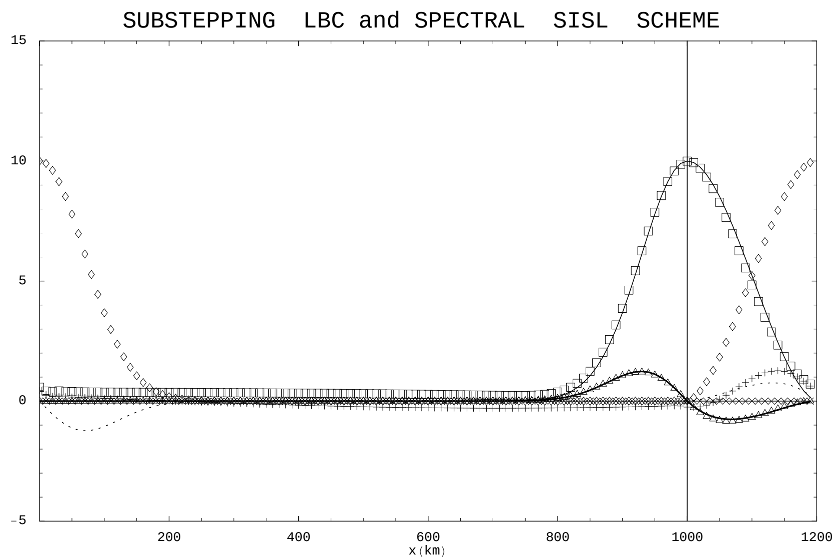
A. McDonald, MWR (2001) + spectral ISL ($\Delta t = 100.s$)



spectral ISL

A. McDonald, MWR (2001) + spectral ISL ($\Delta t = 416.s$)





Iterative schemes

Interior (dynamical core)

FFT

$$X^0 \leftarrow X_{LBC}$$

$\partial X^0|_{\partial\Gamma}$ in grid point

Explicit Dynamics

$$R = \left[1 + \tfrac{1}{2}\Delta t \mathcal{L}\right]_{GP} X^0$$

$$\Lambda = \mathcal{L}_{GP}^{LBC} - \mathcal{L}_{SP}$$

$$\delta R^{(k)} = \left[\left(1 - \tfrac{1}{2}\Delta t \Lambda\right)^{-1} - 1\right]\left[R + \tfrac{1}{2}\mathcal{L}_{SP} X^{(k)}\right]$$

$$R^{(k)} = R + \delta R^{(k)}$$

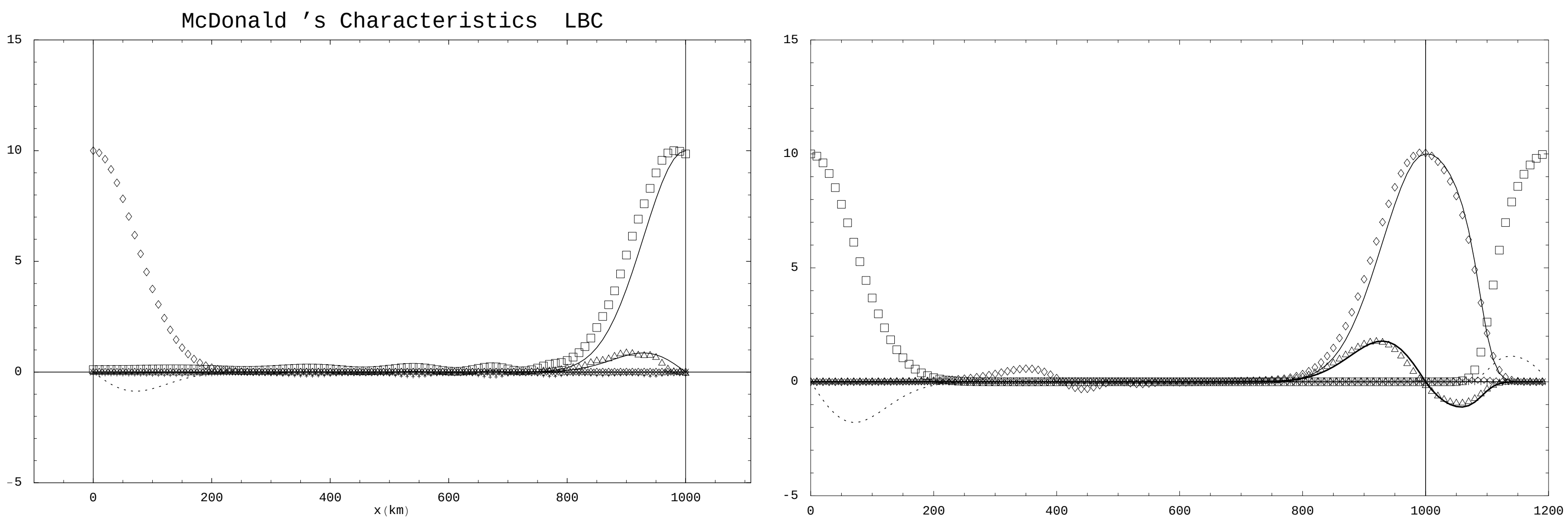
periodicity + FFT $[R^{(k)}]$ FFT$^{-1}[X^{(k+1)}]$

$$X^{(k+1)} = \left[1 - \tfrac{1}{2}\Delta t \mathcal{L}\right]_{SP}^{-1} R^{(k)}$$

This is compatible with the ICI schemes proposed by Bénard (2003), MWR

Iterative scheme

A. McDonald, MWR (2001) + iterative scheme ($\Delta t = 400.s$, 4 iterations)



Problem of SL LBC's

- Something has to be done when the trajectories go out of the domain in SL schemes: trajectory truncation
- not a problem within substepping because CFL restricts the trajectories
- 3 iterations (1 predictor + 3 correctors) are necessary

pro's and cons

	substepping	iterative
pro	- allows "externalization" - CFL removes traj. problem	- solves problem of inverse operators at the outflow
con	- problem of inverse operator (outflow) - cost	- traj. trunc (inflow) - intrinsic in dynamics - we need 3 iteration of the corrector - not straightforward if not used with ICI

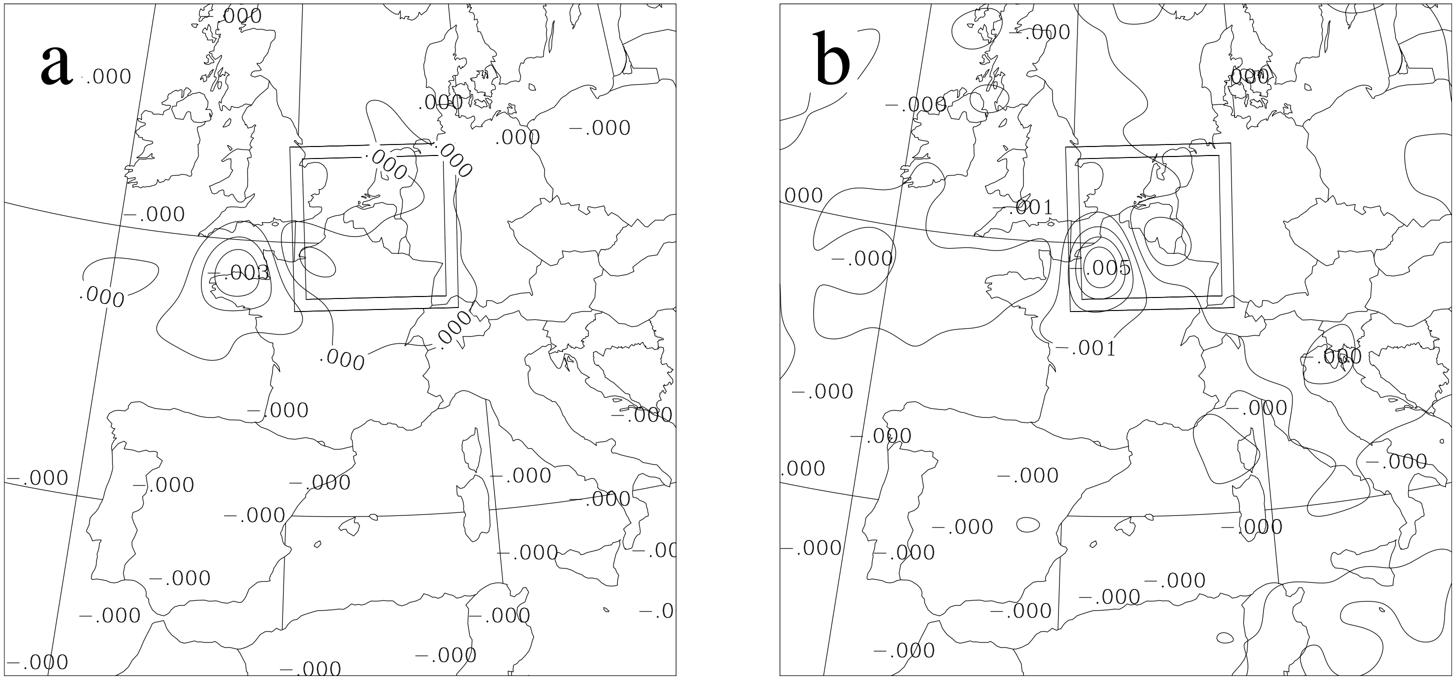
Hybrid solution: use substepping at inflow and iterative method at outflow

LBC's: conclusions

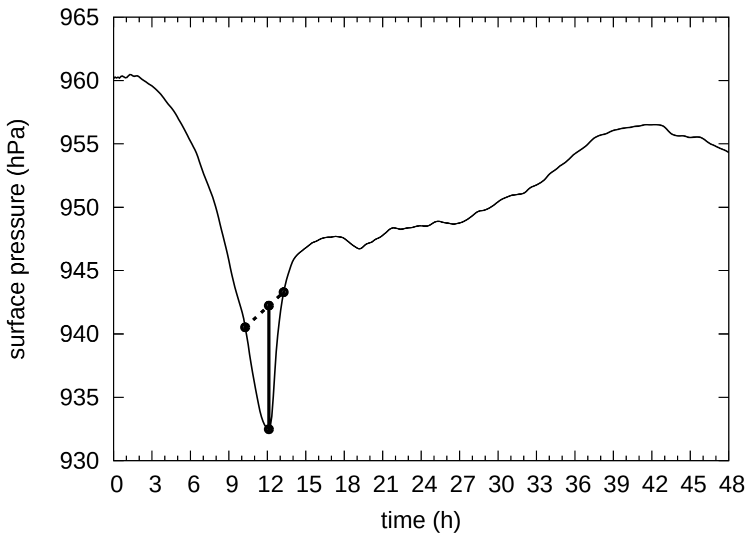
- bad news: don't expect to find a quick fix.
- good news: actually we can master the invidual pieces
- The idea of *externalizing* the LBC's works in the 1D model!
- Spectral models pose an extra problem: the the discrepancy between the gridpoint and the spectral operators requires extra research.

MCUF

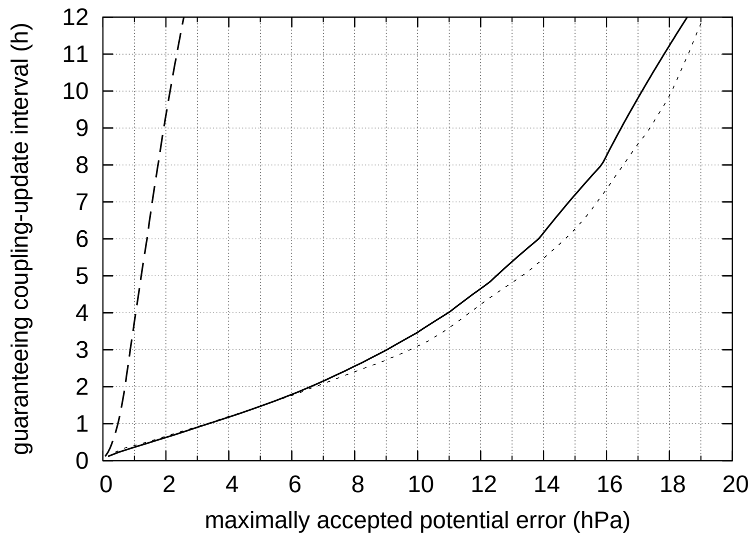
Termonia (2004), *Mon. Wea. Rev.*



MCUF



965
960
955
950
945
940
935
930
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48
surface pressure (hPa)
time (h)

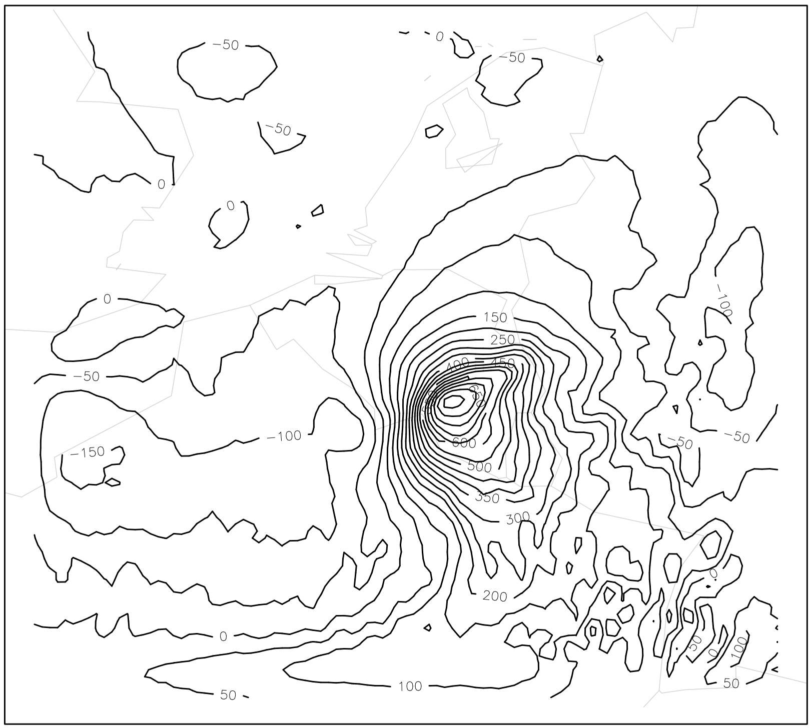


12
11
10
9
8
7
6
5
4
3
2
1
0
0 2 4 6 8 10 12 14 16 18 20
guaranteeing coupling-update interval (h)
maximally accepted potential error (hPa)

DFI

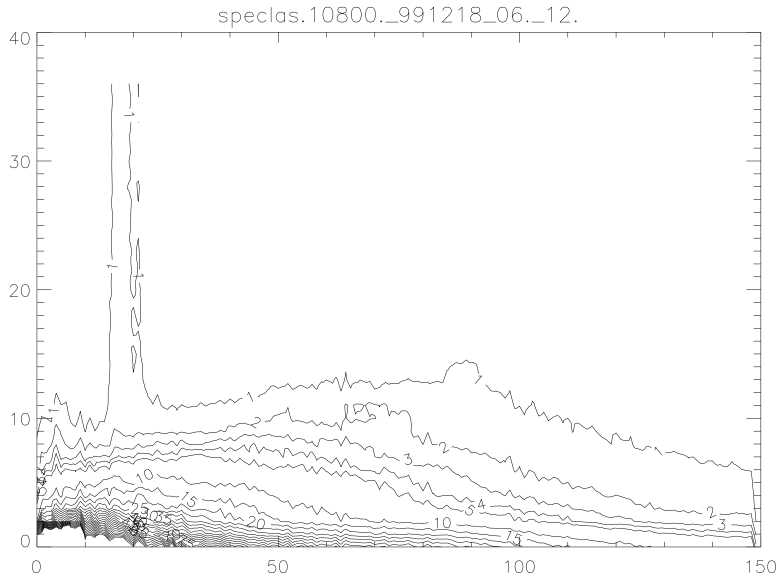
Work in progress:

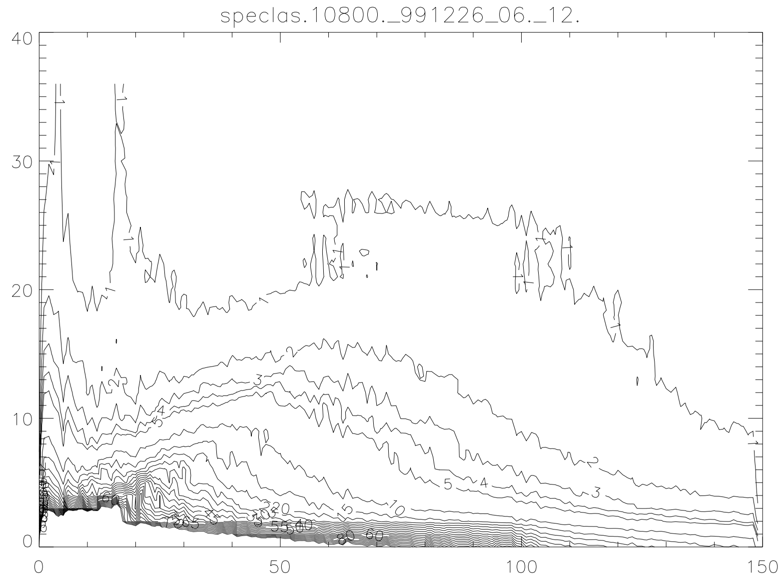
MSLP 26/12/1999 +9h
DFI(3h) − no DFI



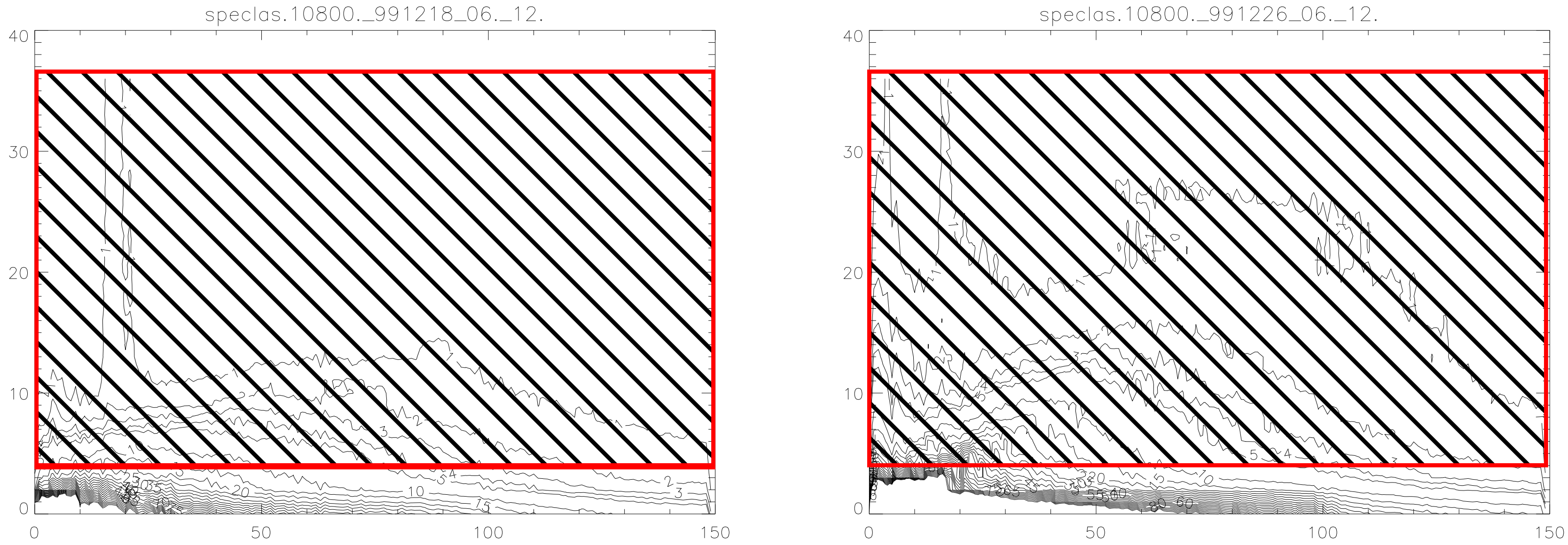
max difference of about 8.5 hPa!

MCUF

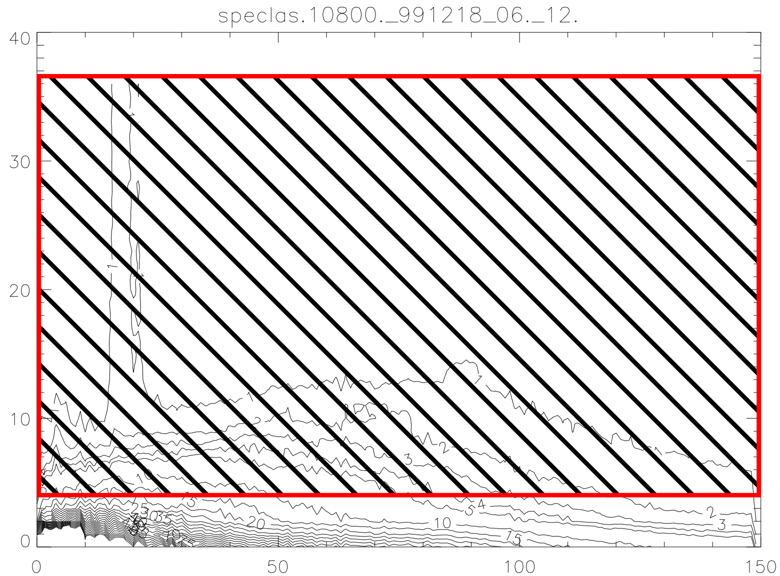


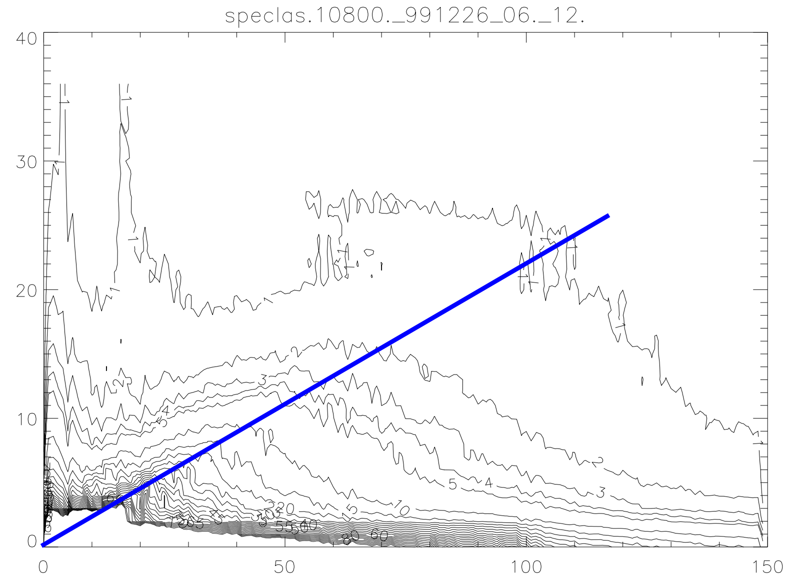


MCUF

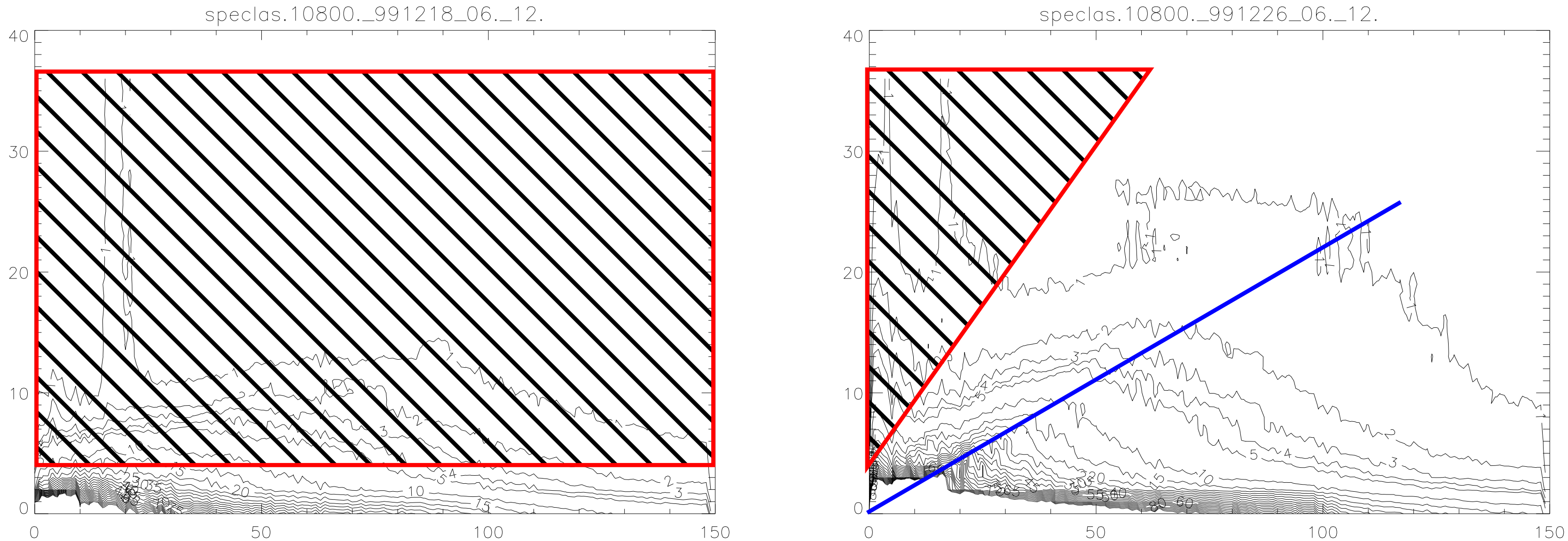


MCUF





MCUF



MCUF and DFI: conclusions

- DFI filters storms!
- Also, using the MCUF field does not make sense *before* this is solved

substepping