New interpolators for SLHD scheme

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Motivation

- SLHD scheme implements non-linear diffusion by exploiting diffusive properties of semi-Lagrangian interpolators
- in current scheme diffusivity is controlled by mixing more accurate high order interpolator with strongly diffusive and less accurate linear one
- undesired side effect of such approach is reduced accuracy in regions with strong diffusion, resulting in loss of mass conservation (seen as positive MSLP bias)
- replacement of cubic Lagrange polynomial by more accurate natural cubic spline was tried, which indeed improved mass conservation in SLHD scheme
- however, it increased temperature and geopotential bias in some parts of troposphere
- is there any workaround this problem?

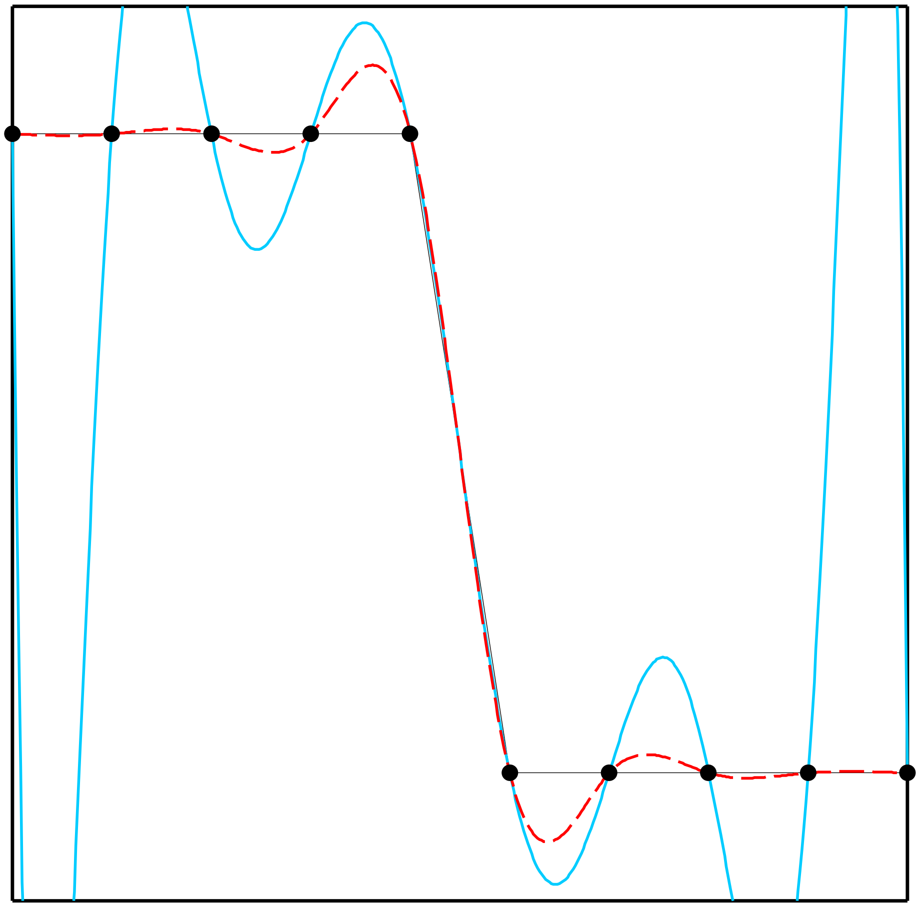
Central problem: diffusivity versus accuracy

- it is intuitive to expect that more diffusive interpolator will be necessarily less accurate
- is this always the case?
- how strongly tied is diffusivity of interpolator to its accuracy?
- when high accuracy is required, is there any maneuvering space left for changing diffusivity?
- in order to answer these questions, suitable family of interpolators must be found, together with way how to measure their diffusivity and accuracy

Part I – bit of theory

Global versus local interpolators

global interpolators



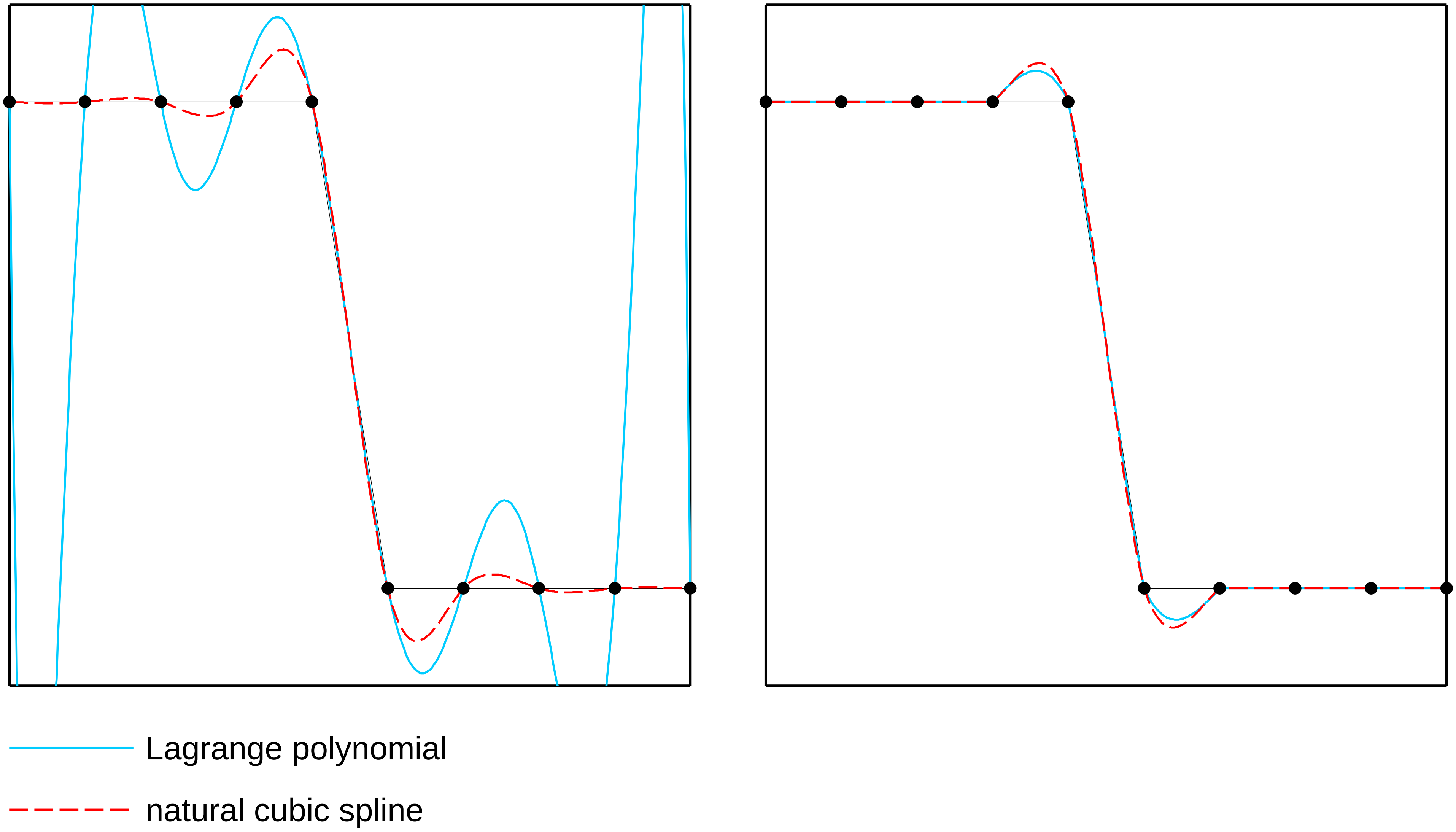
Lagrange polynomial

natural cubic spline

Global versus local interpolators

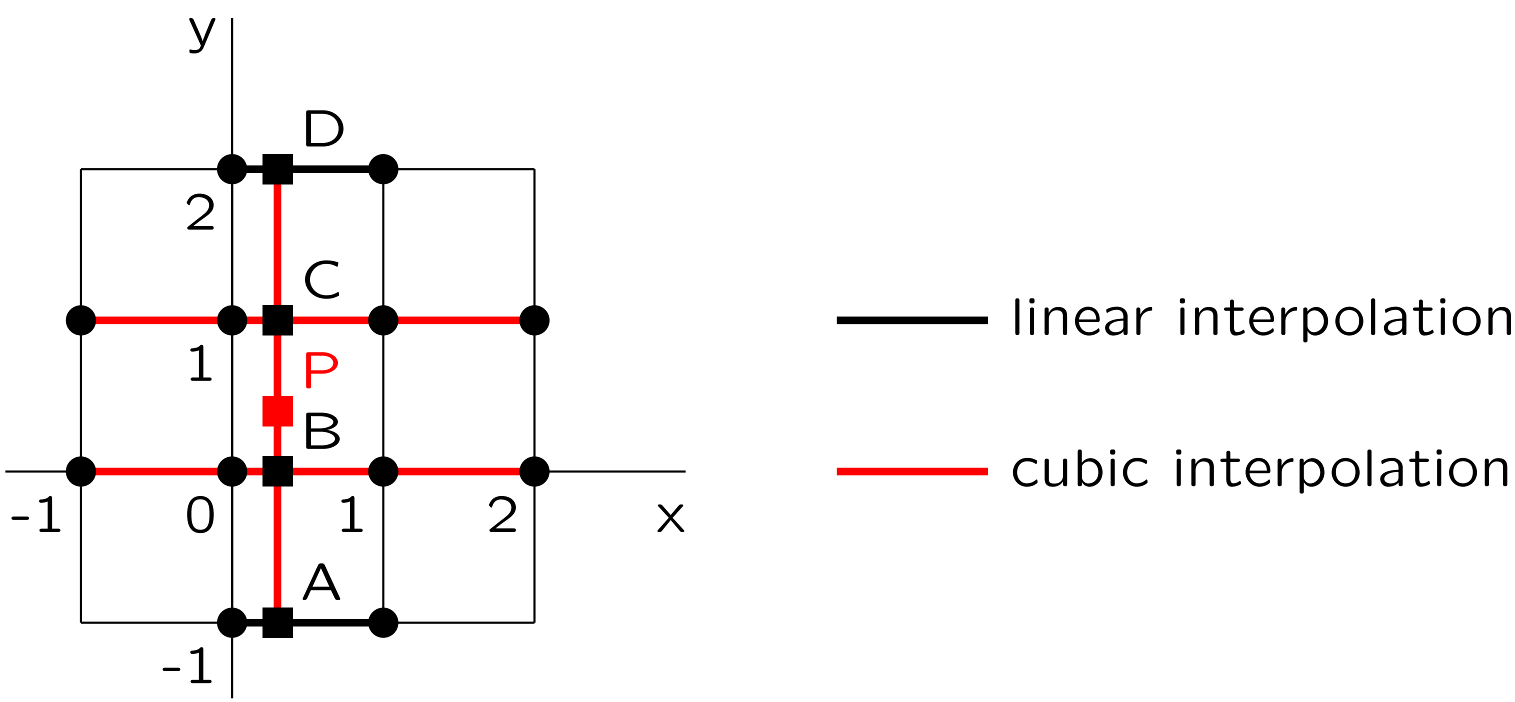
global interpolators

local 4-point interpolators



Multi-dimensional case

- high order 2D interpolators in ALADIN are composed of 3 high order and 2 low order 1D interpolators
- this approach leads to 12-point stencil:



- similar approach is used in 3D case, employing 32-point stencil

Requirements on decent 4-point interpolator

1. linearity with respect to $\mathbf{y} = (y_{-1}, y_0, y_1, y_2)$:

$$F(x, \mathbf{y}) = w_{-1}(x)y_{-1} + w_0(x)y_0 + w_1(x)y_1 + w_2(x)y_2$$

2. invariace with respect to horizontal mirroring:

$$F(1-x, y_2, y_1, y_0, y_{-1}) = F(x, y_{-1}, y_0, y_1, y_2)$$

3. invariace with respect to vertical shift:

$$F(x, \mathbf{y} + c) = F(x, \mathbf{y}) + c$$

4. reproducing of values y_0, y_1:

$$F(0, \mathbf{y}) = y_0$$
$$F(1, \mathbf{y}) = y_1$$

5. reproducing of linear function $y = x$:

$$F(x, -1, 0, 1, 2) = x$$

Family of cubic 4-point interpolators

- when weights w_{-1}, w_0, w_1, w_2 are constrained to polynomials of degree at most 3, interpolator F is restricted to the form:

$$F(x, \mathbf{y}) = u(x)y_{-1} + v(x)y_0 + v(1-x)y_1 + u(1-x)y_2$$
$$u(x) = a_1 x + a_2 x^2 - (a_1 + a_2)x^3$$
$$v(x) = 1 + (a_2 - 1)x - (3a_1 + 4a_2)x^2 + 3(a_1 + a_2)x^3$$

$$a_1, a_2 \in \mathbb{R}$$

- every decent 4-point cubic interpolator can be represented by point in (a_1, a_2) plane

- requirement that F reproduces also quadratic function $y = x^2$ (which implies second order accuracy) defines straight line:

$$6a_1 + 2a_2 = -1$$

How to measure accuracy?

- accuracy of (a_1, a_2) interpolators was evaluated on sample of harmonic test functions:

$$y_m(x) = \sin(2\pi m x/N) \qquad x \in [0, N] \qquad m = 1, 2, \ldots, M$$

- source grid had $N = 100$ intervals, linear truncation with $M = 49$ was chosen

- each test function was interpolated onto 20 times finer target grid

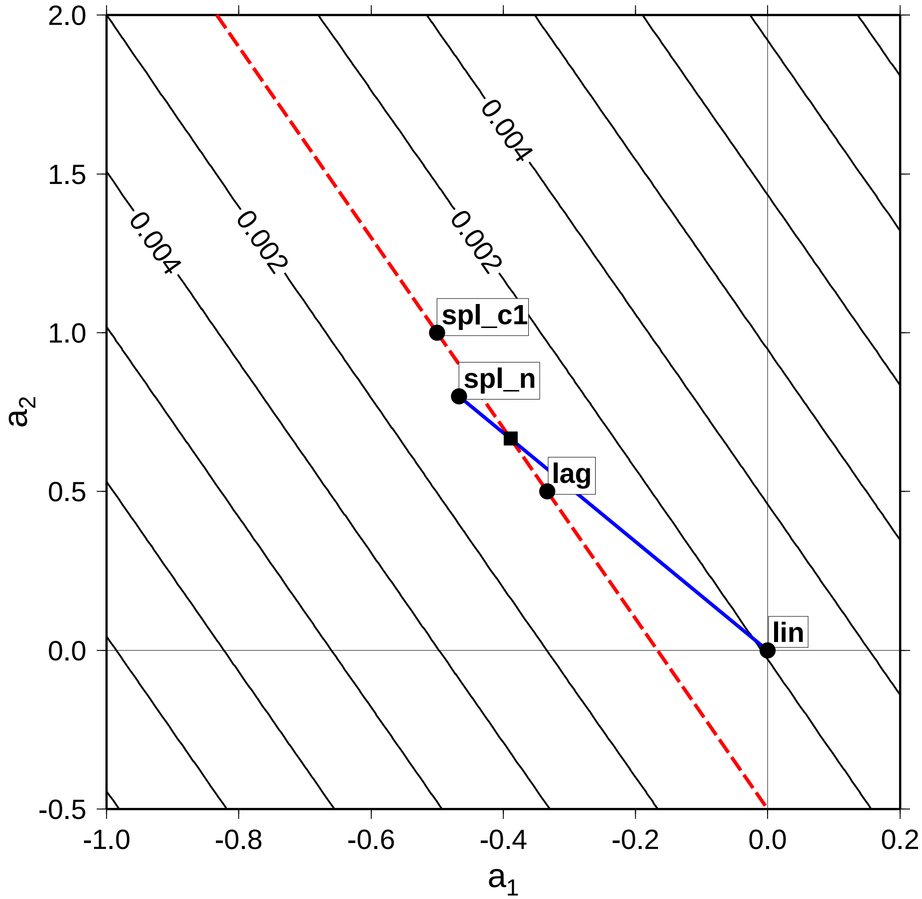
- overall accuracy of interpolator was measured by MAE weighted by function $\exp(-\beta m/M)$

- parameter β was used to control significance of shortest waves

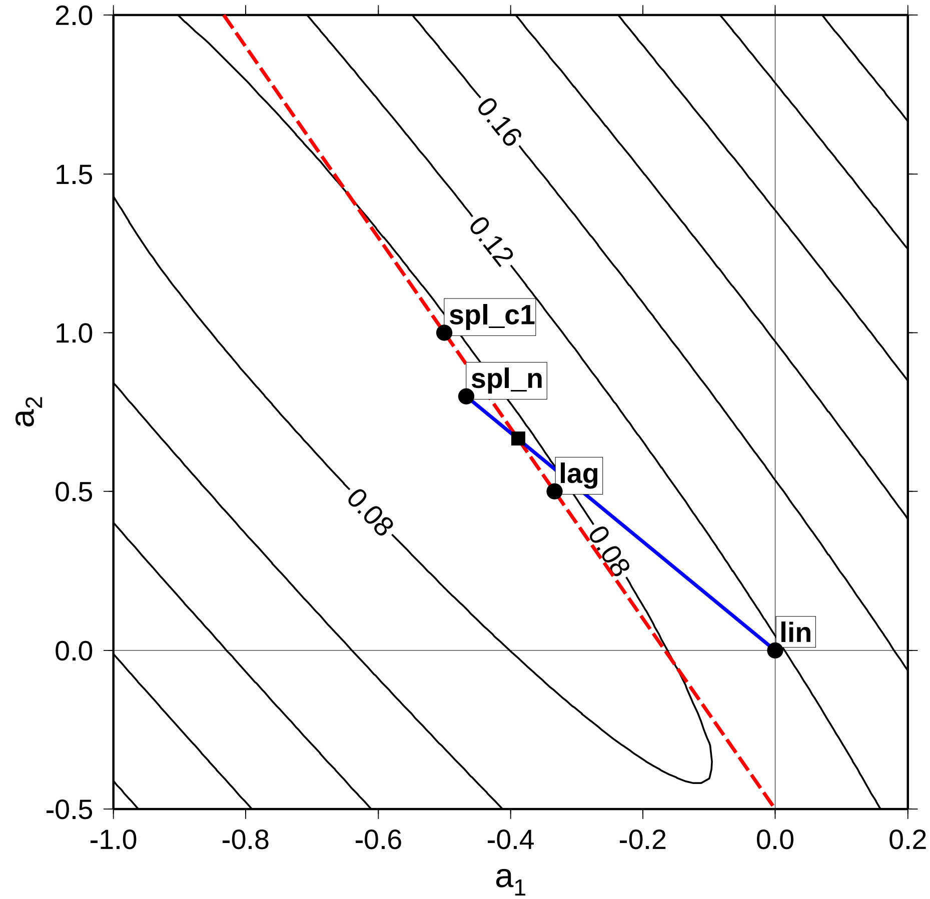
Accuracy maps

accuracy measured by weighted MAE

weight function $\exp(-25m/M)$



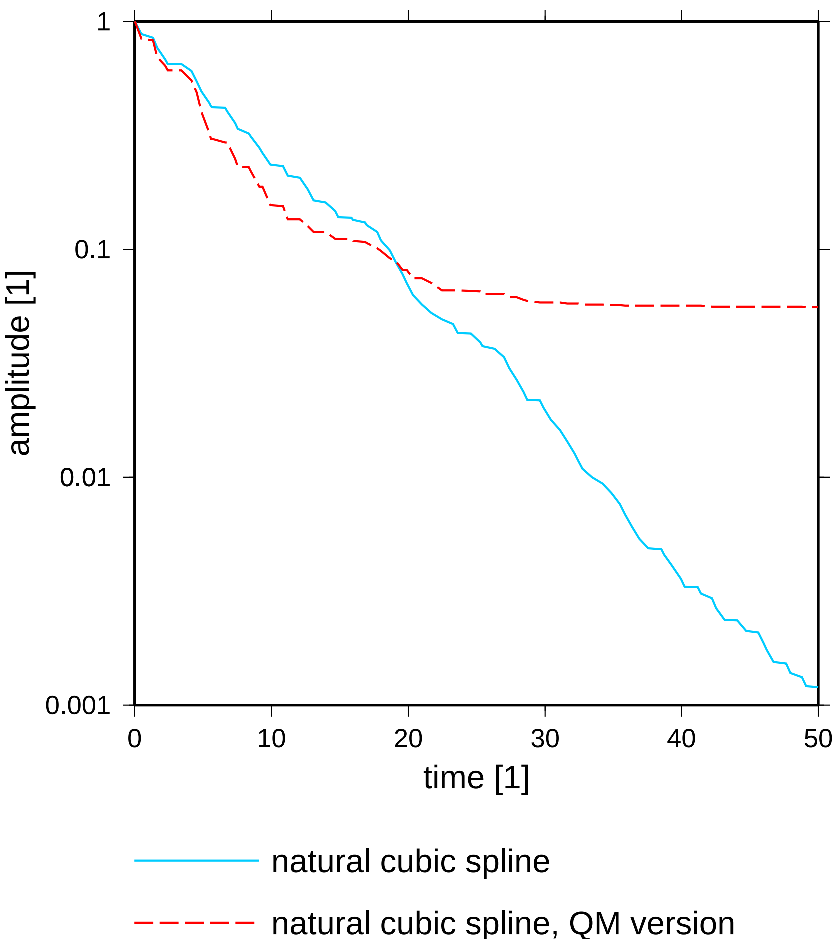
weight function $\exp(-m/M)$



How to measure diffusivity?

advection of harmonic wave

amplitude evolution for
wave with $\lambda = 3.7\Delta x$



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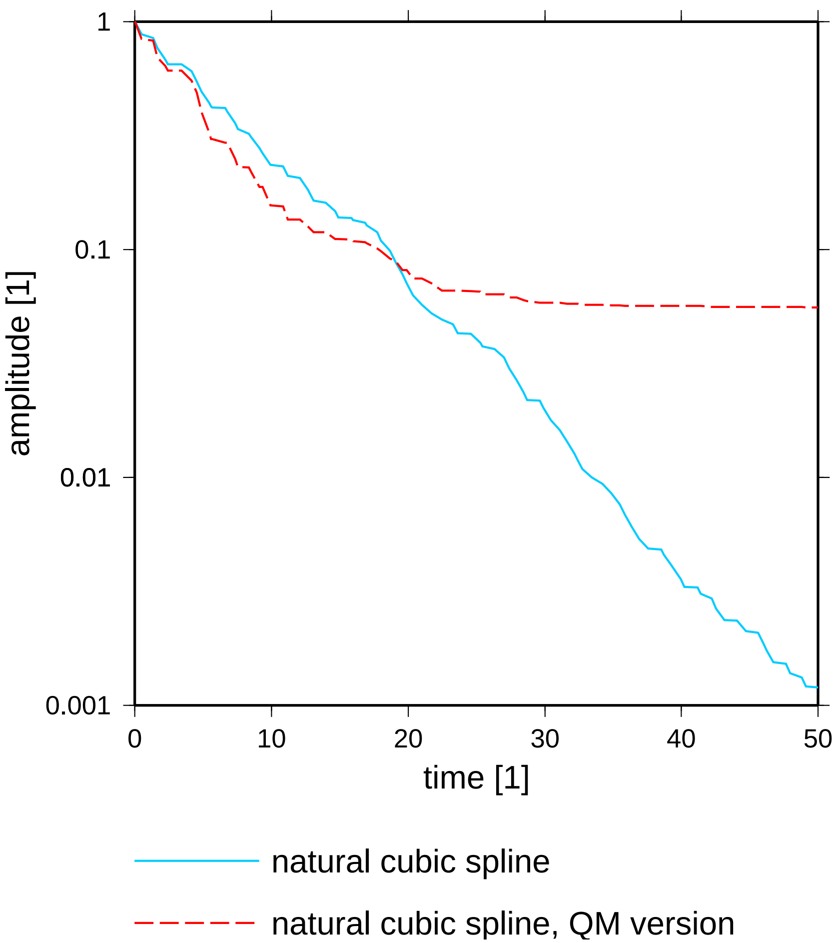
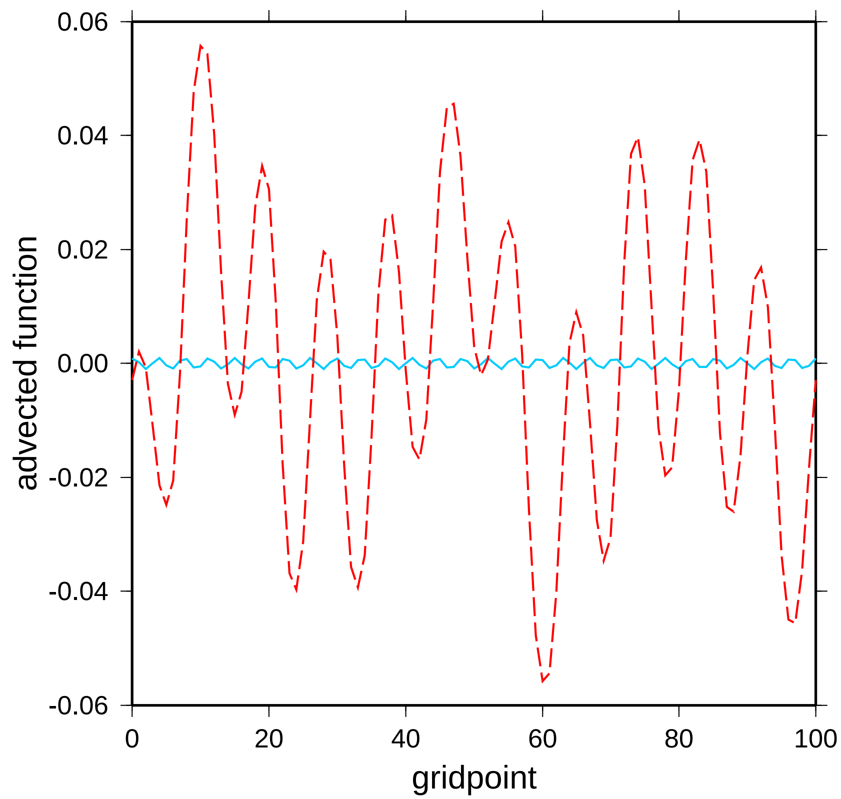


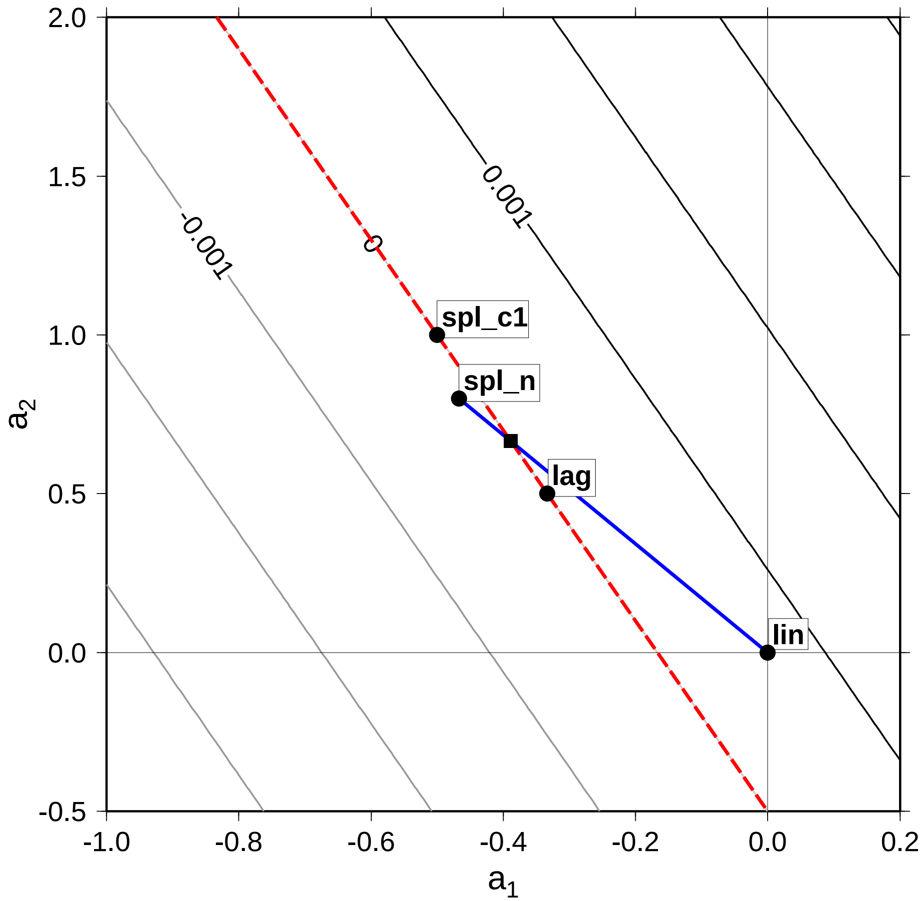
wave after 100 timesteps, QM case dominated by $\lambda = 9.1\Delta x$, $33.3\Delta x$

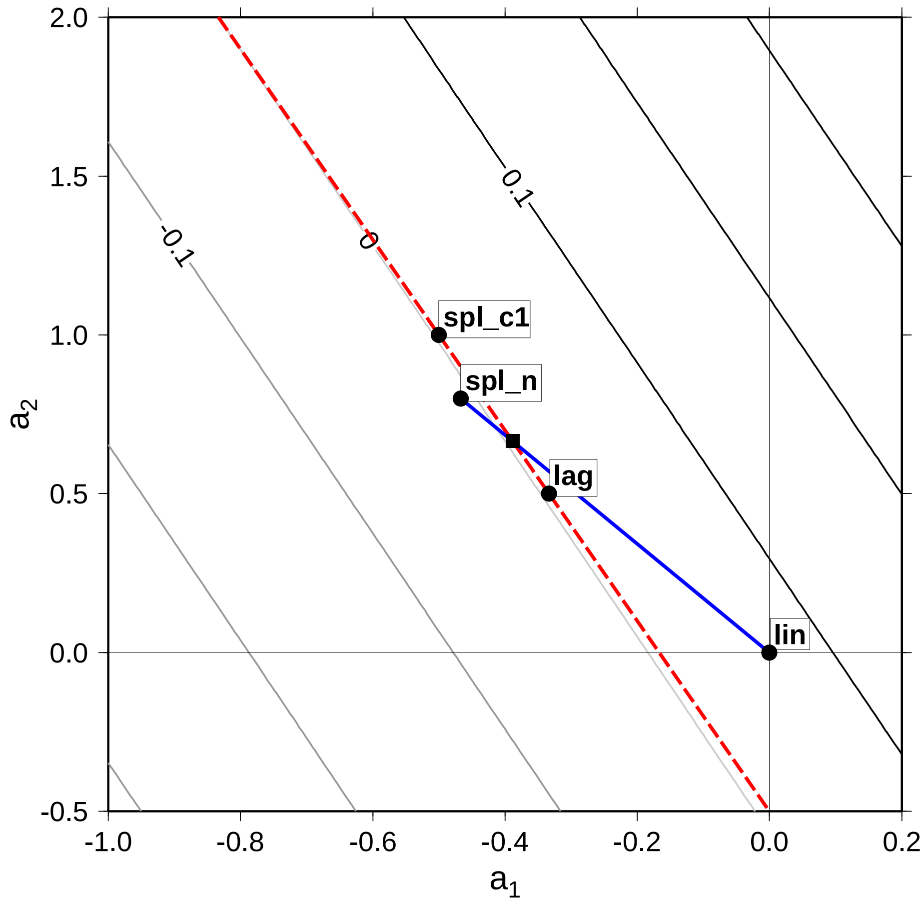


Diffusivity maps (1)

dimensionless damping rate

$\lambda = 100\Delta x$ $\qquad$ $\lambda = 10\Delta x$

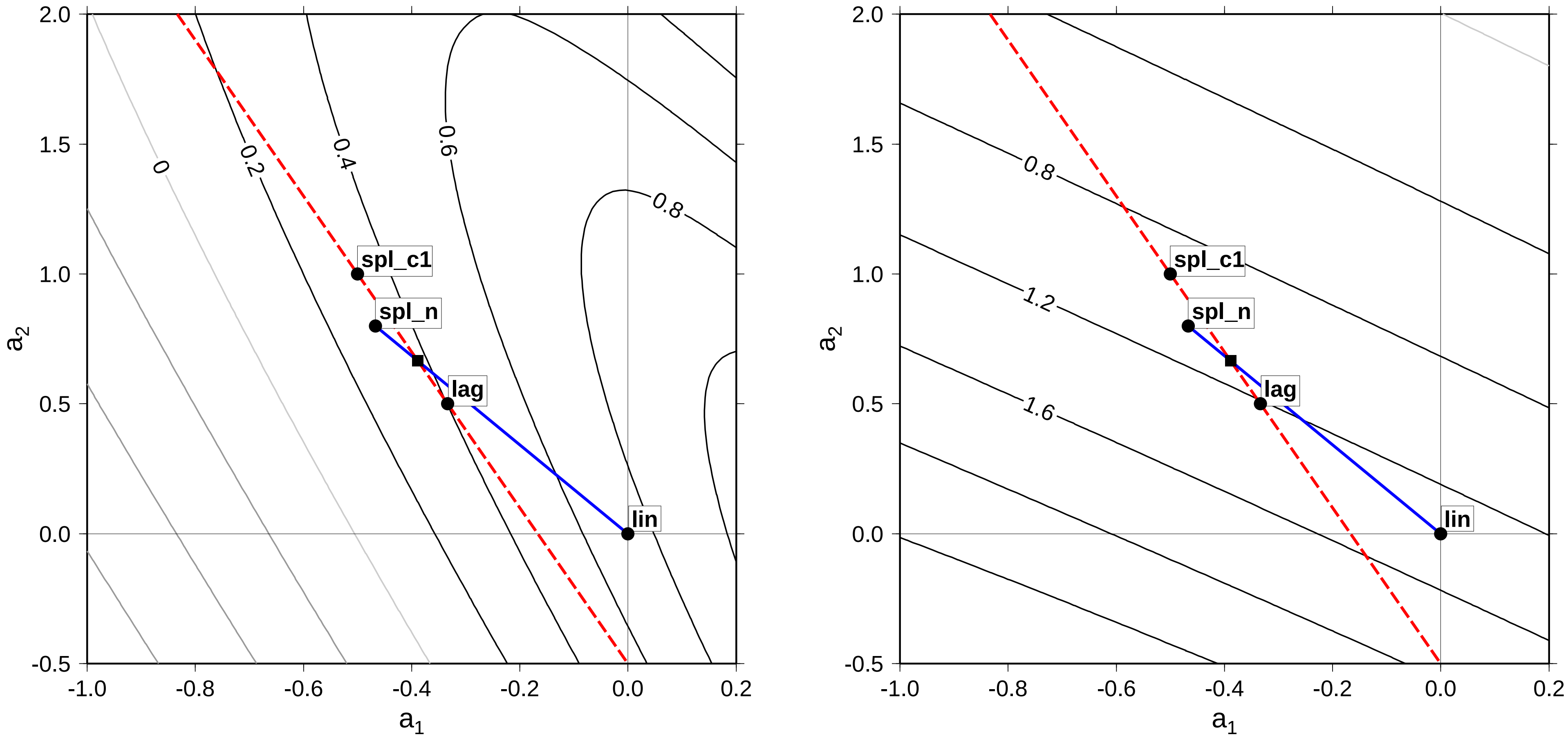




Diffusivity maps (2)

dimensionless damping rate

$\lambda = 3.0\Delta x$ $\lambda = 2.0\Delta x$



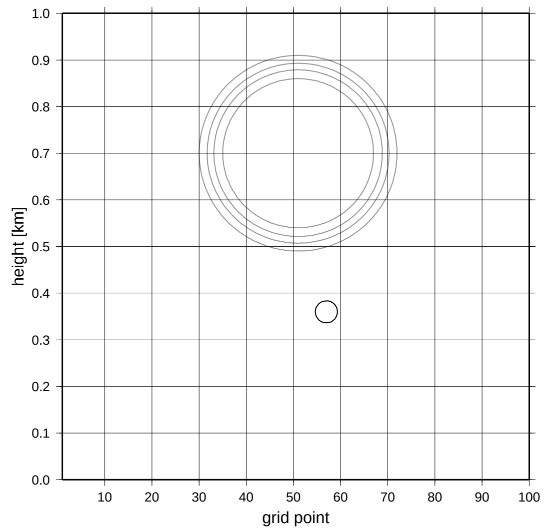
Part II – 2D experiments

Cold and warm bubble test

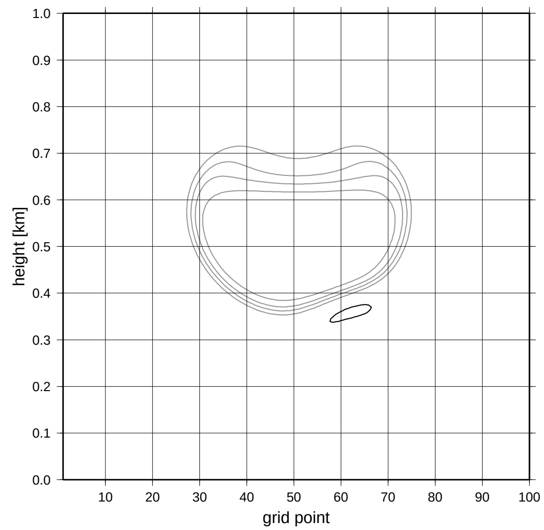
neutrally stratified
resting background state
in domain $1 \times 1\,\mathrm{km}$,
$\Delta x = \Delta z = 10\,\mathrm{m}$

SL2TL ICI scheme with
advection of w, $\Delta t = 5\,\mathrm{s}$

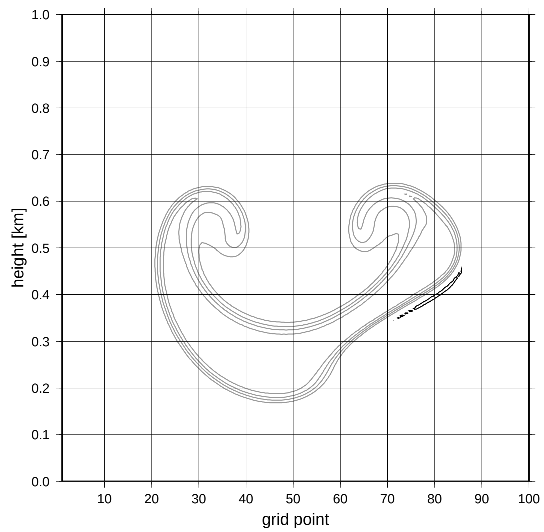
cubic Lagrange SL
interpolator, no other
source of damping



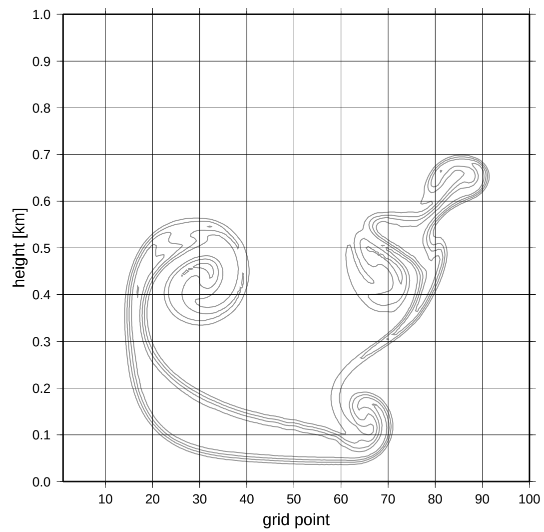
$t = 0\,\mathrm{min}$



$t = 4\,\mathrm{min}$



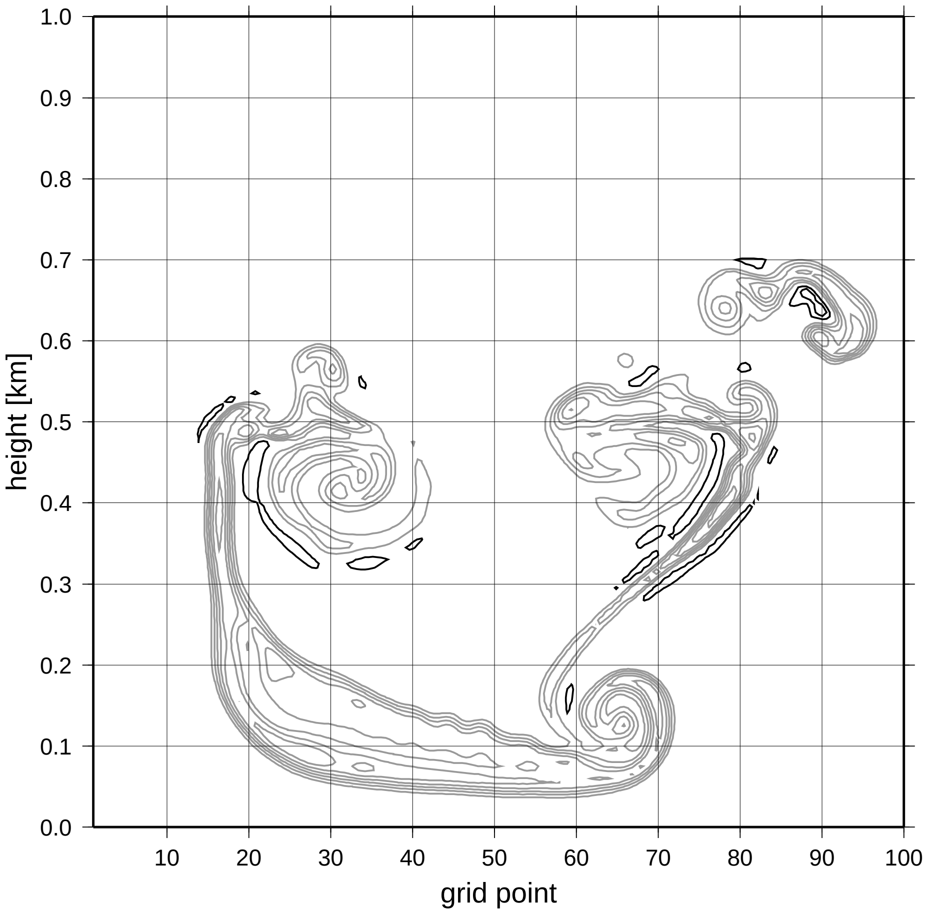
$t = 7\,\mathrm{min}$



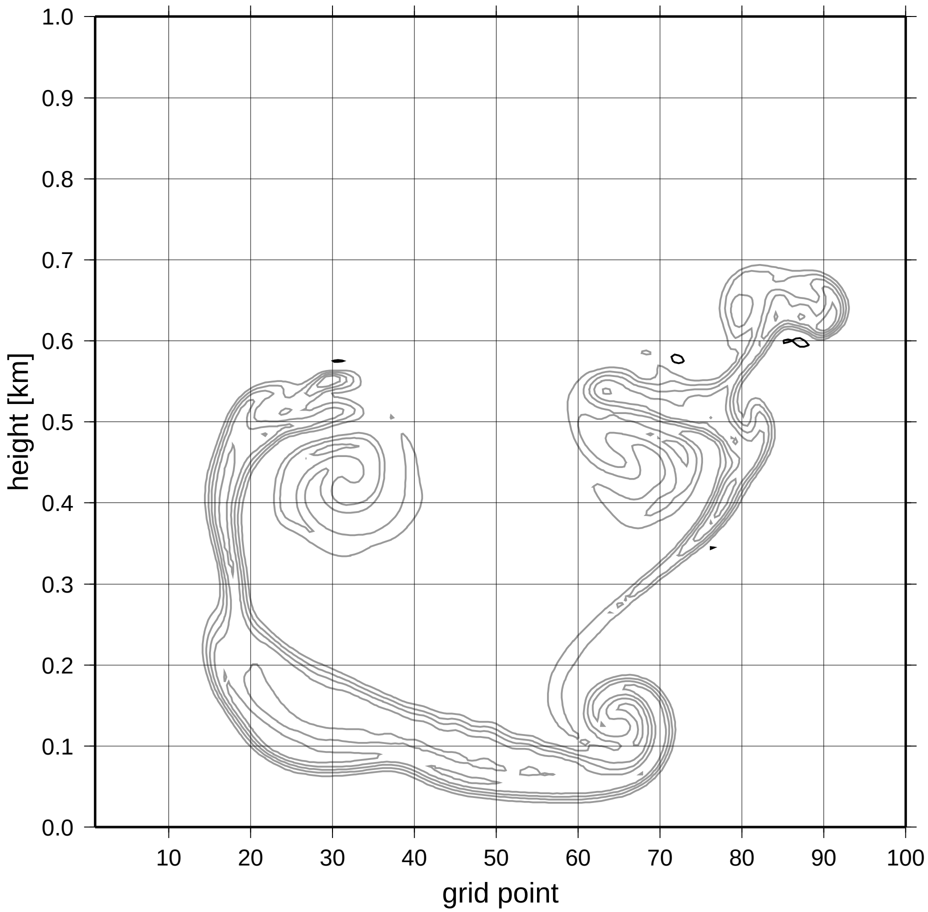
$t = 10\,\mathrm{min}$

Performance of various SL interpolators (1)

natural cubic spline $\left(-\frac{7}{15}, \frac{4}{5}\right)$
(long waves amplified)

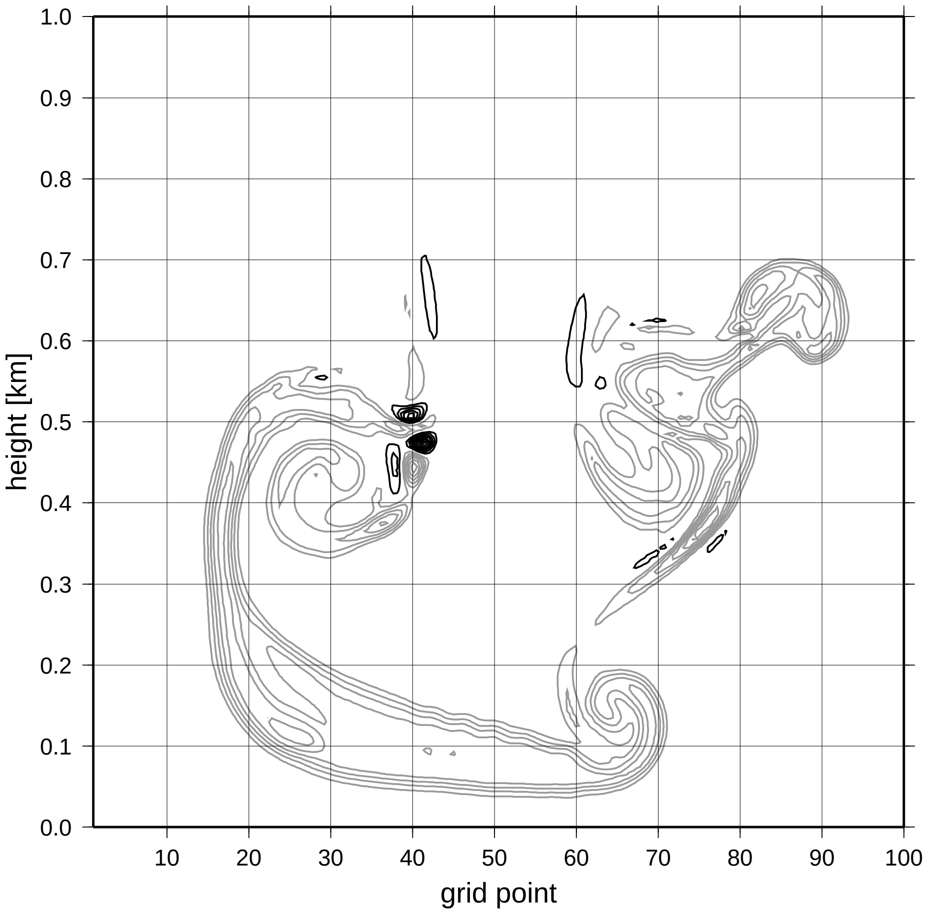


natural cubic spline, QM version
(no overshoots)

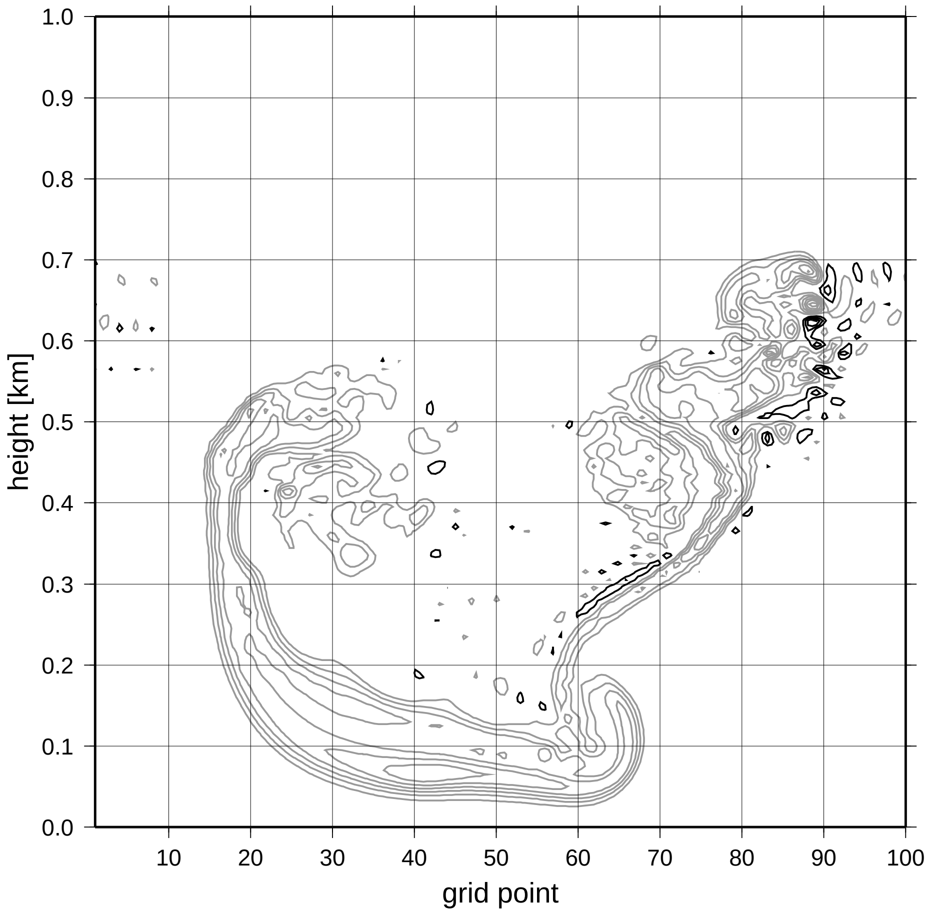


Performance of various SL interpolators (2)

interpolator $\left(-\frac{2}{3}, \frac{3}{2}\right)$
(weak 4th order diffusion)

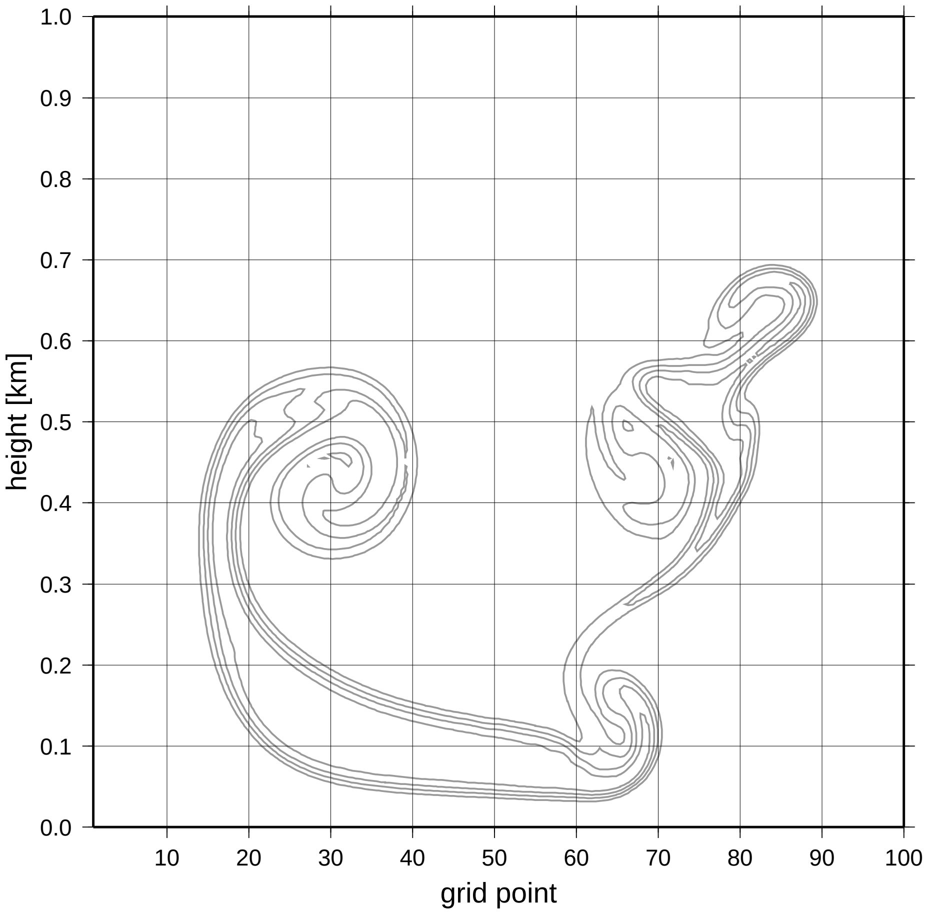


3TL Eulerian scheme, $\Delta t = 1\,\mathrm{s}$
(no diffusion, but Asselin filter)

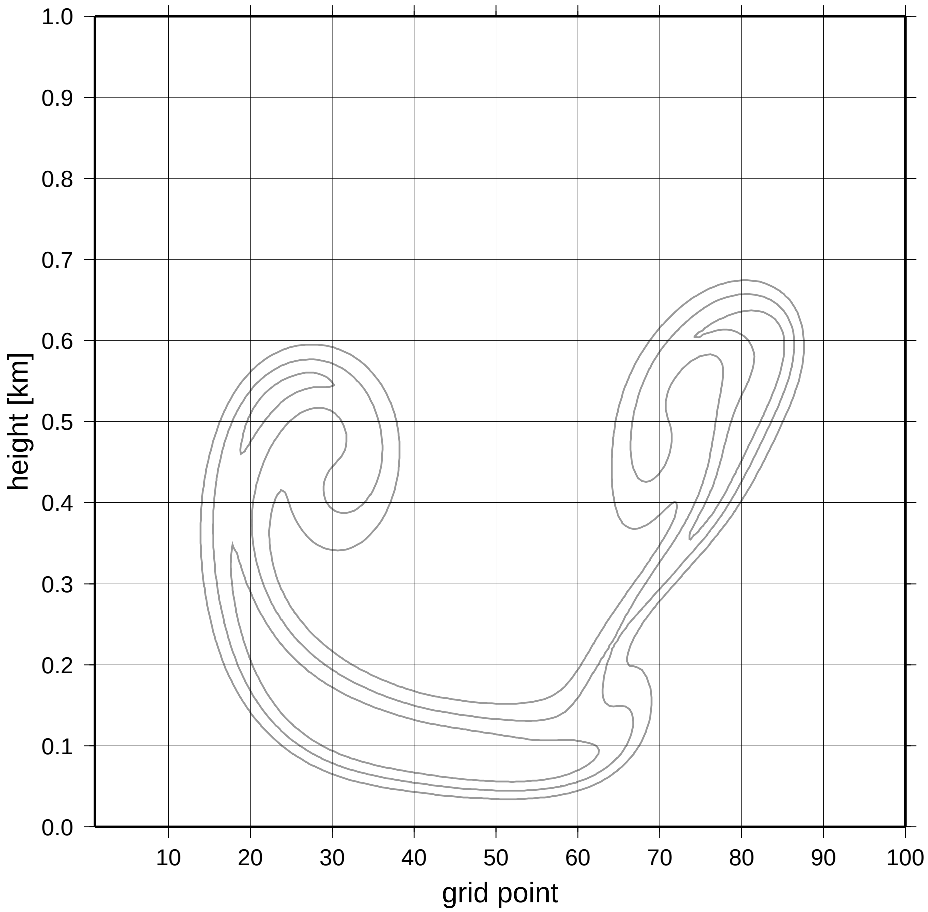


Performance of various SL interpolators (3)

interpolator $\left(0, -\frac{1}{2}\right)$
(strong 4th order diffusion)



linear interpolator $(0, 0)$
(strong 2nd order diffusion)



Conclusions

- there exists 1-parametric class of second order accurate 4-point cubic interpolators with diffusivity tunable in wide range

- 2D bubble tests showed clear superiority of this class when compared to current solution used in SLHD scheme, which is only first order accurate

- slight instability of natural cubic splines might further contribute to the problem in regions where SLHD is not active

- implementation in ALADIN (general case with irregular nodes) and 3D real case tests should follow

- if successful, recoding of semi-Lagrangian TL/AD will be necessary

Additional info

More details can be found in stay report on RC LACE web page:

www.rclace.eu
$\longrightarrow$ Research areas
  $\longrightarrow$ Dynamics and Coupling
    $\longrightarrow$ Reports
      $\longrightarrow$ 2006