What can we expect from grid point AROME ?

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Problem

Two issues :

- Scalability
- Steep slopes

A common solution ?

- Grid Point approach

AROME 2D - presentation

- ICI constant coefficient, SL, A-grid, mass-based coordinate, etc
- No physics, idealised test cases
- Spectral or grid point versions

Grid point version :

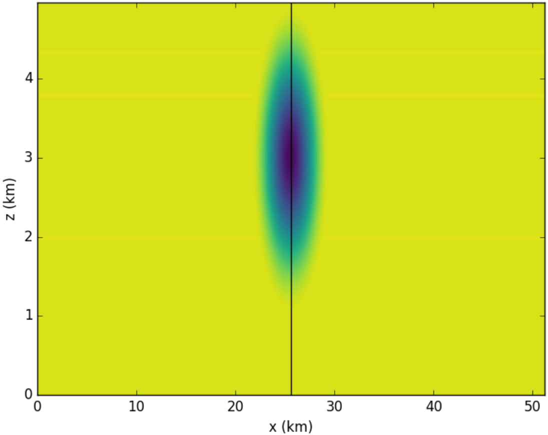
- Explicit diffusion
- Krylov solver

Stop criterion :

$$\varepsilon = \frac{\sqrt{(Ax-b)^T(Ax-b)}}{\sqrt{bb^T}}$$

Density current test case

Spectral vs Grid Point 4^{th} order, $N_{iter} \approx 10$



Hypothesis

Parallelisation :

- No MPI, no Open-MP

Geometry :

- 500 x 500 pts (vs 1536 x 1440 pts in operational AROME)

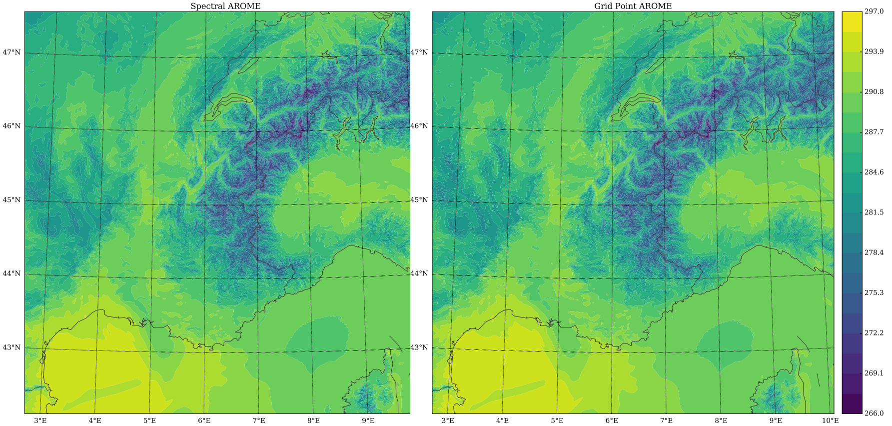
Spectral computations :

- From $D^{t+\Delta t}$ to $U^{t+\Delta t}$ and $V^{t+\Delta t}$
- Implicit diffusion
- RHS

Grid point computations :

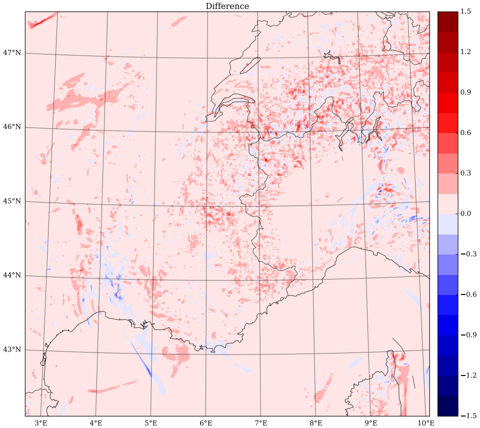
- Derivatives in the linear operator
- Krylov solver

Spectral vs Grid Point



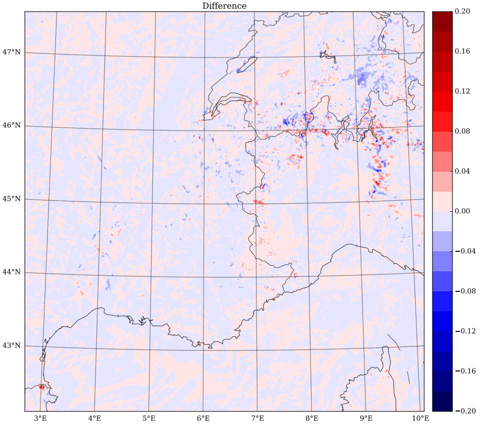
T80, $\delta t = 50\ s$, $T = 2\ h$, $\Delta x = 1.3\ km$, $N_{iter} \approx 13$, 8^{th} order

Spectral vs Grid Point



$\delta(T80)$, $\delta t = 50\ s$, $T = 2\ h$, $\Delta x = 1.3\ km$, $N_{iter} \approx 13$, 8^{th} order

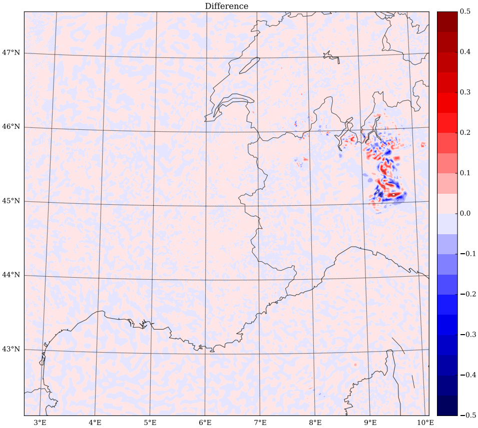
Sensitivity test case



$\delta(T80)$, $\delta t = 50\ s$, $T = 2\ h$, $\Delta x = 1.3\ km$

One more iteration in the ICI

Sensitivity test case



$\delta(T80)$, $\delta t = 50\ s$, $T = 2\ h$, $\Delta x = 1.3\ km$

Random noise at the bottom of the atmosphere ($\sigma = 0.01\ K$) at $t = 0$

Comparisons

- Comparisons between AROME and some observations : nearly identical scores after 4 hours (not shown)

Perspectives :

- To extend study to $8 * 24$ hours forecast

Solver for constant coefficient SI

$$\left[1-\delta t^2 \mathbf{B}\Delta\right] D^{t+\delta t} = D^{\bullet\bullet}$$

B non-symmetric matrix (boundary conditions) : GMRES method

Solver for constant coefficient SI

By projecting in the eigenspace of **B** :

$$\left[1 - \delta t^2 b_m \Delta\right] QD^{t+\delta t} = QD^{\bullet\bullet}$$

where $b_m \in [10^{-2}, 10^{5}]\ m^2 s^{-2}$
($\sqrt{b_m} \in [0.1, 320]\ ms^{-1}$)

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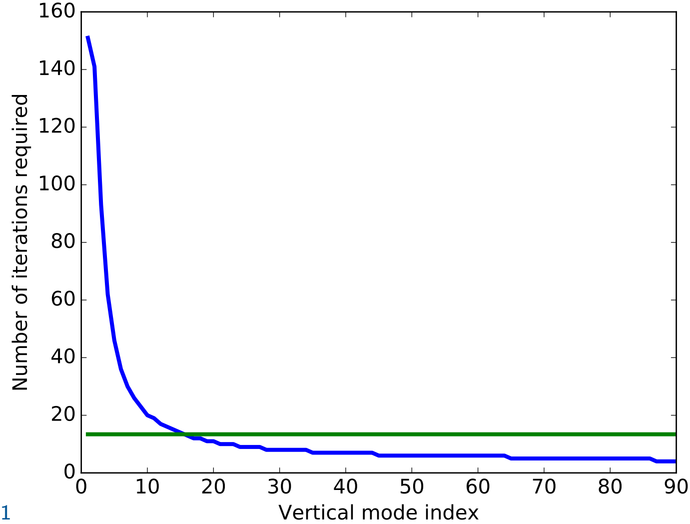
$$cond \approx \frac{1+\Delta t^2 b_m \pi^2/\Delta x^2}{1+\Delta t^2 b_m 4\pi^2/L^2} \approx 1+\delta t^2 b_m \frac{\pi^2}{\Delta x^2} = 1 + C_*^2$$

where C_* is the CFL number

Convergence behaviour

$\varepsilon = 10^{-8}$



Numerical cost

AROME operational configuration (2019)

- 170 nodes on Bull SX supercomputer
- output bandwidth from a node : 7 Go/s
- network latency : 0.864 ms

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Without projection on vertical modes (150 iterations required) :
"Total" cost : $27.5 + 0.1 \approx 27.6$ s

With projection on vertical modes (13 iterations required) :
"Total" cost : $2.4 + 1 \approx 3.4$ s

Comparison with an HEVI model

Cost of 1 iteration in the solver ≈ Cost of 1 acoustic time step

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$\delta t = 50\ \mathtt{s}$, $\Delta x = 1300\ \mathtt{m}$, $c = 350\ \mathtt{m/s}$
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$$\Delta t \approx 4\ \mathtt{s}$$

Comparison with an HEVI model

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SI grid point model :

$$\Delta \tau = \frac{\delta t}{2N_{iter}} = \frac{50}{2*13} \approx 2\ \mathtt{s}$$

Conclusion

Results

- Non exact derivatives –> Order ≥ 6
- Convergence –> $N_{iter} \approx 13$
- Technical viability
- Simulated computational cost seems low

Linear equations without orographic terms

$$\frac{\partial U'}{\partial t} = -RT^* \frac{\partial q'}{\partial x} - \frac{RT^*}{\pi_S^*}\frac{\partial \pi_S'}{\partial x} - R\int_\eta^1 \frac{m^*}{\pi^*}\frac{\partial T'}{\partial x} d\eta' + RT^* \int_\eta^1 \frac{m^*}{\pi^*}\frac{\partial q'}{\partial x} d\eta'$$

$$\frac{\partial d'}{\partial t} = -\frac{g}{rH^*}\partial^*(\partial^* + 1)q'$$

$$\frac{\partial q'}{\partial t} = -\frac{C_p}{C_v}\left(\frac{\partial U'}{\partial x} + d'\right) + \frac{1}{\pi^*}\int_0^\eta m^* \frac{\partial U'}{\partial x} d\eta'$$

$$\frac{\partial T'}{\partial t} = -\frac{RT^*}{C_v}\left(\frac{\partial U'}{\partial x} + d'\right)$$

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Linear equations σ-coor with orographic terms

$$\frac{\partial U'}{\partial t} = \ldots + \frac{RT^*}{\pi_S^{*2}}\frac{\partial \pi_S^*}{\partial x}\pi'_S + \frac{1}{T^*}\frac{\partial \phi^*}{\partial x}T' - \frac{\partial \phi^*}{\partial x}q' - \frac{\pi^*}{m^*}\frac{\partial \phi^*}{\partial x}\frac{\partial q'}{\partial \eta}$$

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Linear equations η-coor with orographic terms

$$\frac{\partial U'}{\partial t} = \ldots + R_d T^* \int_{\eta}^{1} \frac{\partial}{\partial x}\left(\frac{m^*}{\pi^*}\right) q d\eta' - R_d \int_{\eta}^{1} \frac{\partial}{\partial x}\left(\frac{m^*}{\pi^*}\right) T' d\eta'$$

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$$\frac{\partial \pi'_S}{\partial t} = \ldots$$

Linear equations η-coor with orographic terms

In general :

$$\frac{\partial X}{\partial t} = L(X)$$

With a 2-TL discretisation :

$$\left[I - \frac{\delta t}{2} L\right] X^{+} = X^{\bullet}$$

SI scheme

$$\begin{pmatrix} I & A \\ B & C \end{pmatrix} \begin{pmatrix} U^{+} \\ d^{+} \\ q^{+} \\ T^{+} \\ \pi_S^{+} \end{pmatrix} = \begin{pmatrix} U^{\bullet} \\ d^{\bullet} \\ q^{\bullet} \\ T^{\bullet} \\ \pi_S^{\bullet} \end{pmatrix}$$

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$$U^{+} + A\Phi^{+} = U^{\bullet}$$

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We can reduce the problem to only one equation :

$$\left[I - AC^{-1}B\right] U^{+} = U^{\bullet} - AC^{-1}\Phi^{\bullet}$$

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- Orographic idealised test cases
- Identify the instability contribution of each orographic term

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Conclusion & perspectives

Scalability :

- Grid point approach seems viable in AROME
- Grid point approach seems competitive

Perspectives

- To test some preconditioners ?
- To remove completely spectral computations in AROME ?

Steep slopes :

Perspectives

- To test it in AROME 2D
- To measure the interest/potential

End

Thank you for your attention !

Do you have some questions ?

Linearisation

Constant coefficient approach :

$$\pi(x,\eta) = \pi^*(\eta) + \pi'(x,\eta)$$

where :

$$\pi^*(\eta) = A(\eta) + B(\eta)\pi_S^*$$

Linearisation around a state which contains orography :

$$\pi(x,\eta) = \pi^*(x,\eta) + \pi'(x,\eta)$$

where :

$$\pi^*(x,\eta) = A(\eta) + B(\eta)\pi_S^*(x)$$

In σ-coordinate : $A(\eta) = 0$, hence :

$$\frac{m^*}{\pi^*} = \frac{\partial_\eta B(\eta)\pi_S^*(x)}{B(\eta)\pi_S^*(x)} = \frac{\partial_\eta B(\eta)}{B(\eta)}$$

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C is a block diagonal matrix that contains $N_X N_Y \approx 10^6$ blocks of matrix of size $N_L = 90$
Solution ? Some blocks are so similar that we can suppose they are identical, then we have to invert only few (1000 ?) blocks

Accélération de la convergence

$$Ax = b$$

est équivalent à :

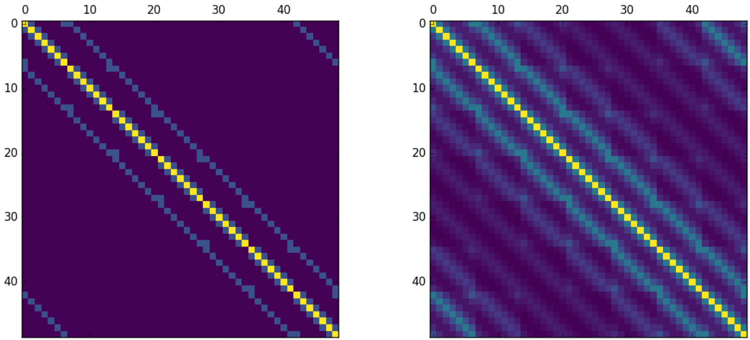
$$PAx = Pb$$

P est un bon préconditionneur si $P \approx A^{-1}$ ie :

- $\text{cond}(PA) << \text{cond}(A)$
- coût CPU faible du produit PA
- coût communication faible du produit PA

Méthode point de grille

$\mathrm{cond}(A) = 262$



A A^{-1}

Préconditionneur

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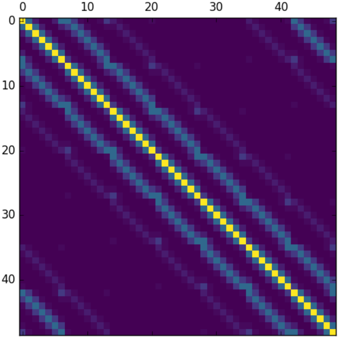

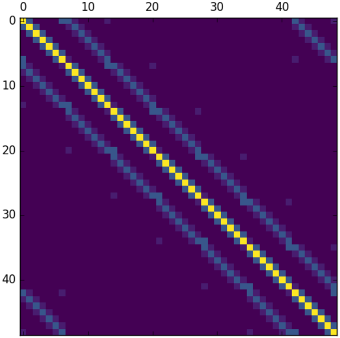

$\text{cond}(PA) = 31,\ \alpha = 47\%$

$\text{cond}(PA) = 55,\ \alpha = 7\%$