Research on VFE in HIRLAM

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on behalf of
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with thanks to P. Bénard, Météo-France

Sets of NH equations including geopotential

S1: $V, w, \phi, T, q \equiv \ln \pi_s$

S2: $V, w, \phi, p, m \equiv \partial \pi / \partial \eta$

S3: $V, w, \phi, \ \hat{q} \equiv \ln\left(p / \pi\right), q \equiv \ln \pi_s$

The results presented here are for set S1, for which P. Bénard has shown to be SHB stable

Linearized system using $D, w, T, \varphi, q = \ln \pi_s$ is SHB stable

A slab x-σ model has been coded to ease testing of different options
The equation set for the linearized version of this model is:

$$\frac{\partial D}{\partial t} + R\Delta T' + RT^* \nabla q' - (1 + \tilde{\partial})\Delta \phi' = 0$$

$$\frac{\partial w}{\partial t} - \frac{g}{R_d T^*}\left(R\left(1 + \tilde{\partial}\right) T' + \left(\left[\tilde{\partial}^2\right] + \tilde{\partial}\right)\phi'\right) = 0$$

$$\frac{\partial T'}{\partial t} = -\frac{RT^*}{c_v}\left(D - \frac{g}{R_d T^*}\tilde{\partial} w\right)$$

$$\frac{\partial \phi'}{\partial t} - gw - RT^*\left(\tilde{N} - \tilde{S}\right)D = 0$$

$$\frac{\partial q'}{\partial t} + \tilde{N}D = 0$$

here:

$$\tilde{\partial} \equiv \sigma \frac{\partial}{\partial \sigma}; \quad \tilde{S}\left(f(\sigma)\right) \equiv \frac{1}{\sigma}\int_0^{\sigma} f(\sigma')d\sigma'; \quad \tilde{N}\left(f(\sigma)\right) \equiv \int_0^1 f(\sigma')d\sigma'$$

A sufficient set of conditions to allow elimination to arrive at a single equation in w is:

C1: $$\tilde{\partial}\ \tilde{N} = 0$$

C2: $$\left(1+\tilde{\partial}\right)\tilde{S} = 1$$

C3: $$\left[\tilde{\partial}^2\right] \equiv \tilde{\partial}\ \tilde{\partial}$$

Then we obtain:

$$\left(1-\beta^2 c_*^2\left(\Delta+\frac{1}{H_*^2}\left(1+\tilde{\partial}\right)\tilde{\partial}\right)-\beta^4 N_*^2 c_*^2 \Delta\right)w^+ = \hat{R}_w$$

Eigenvalues of the "vertical laplacian" operator : $\tilde{L} \equiv \left(1+\tilde{\partial}\right)\tilde{\partial}$

should be real and negative for stability, which it is not easy to fulfill

Boundary conditions can modify eigenvalues and this is the case here

If we set the lowest model level at the surface

$$\phi_N^{+.} = \phi_N^E \quad \equiv \quad \phi_s$$

the structure equation becomes:

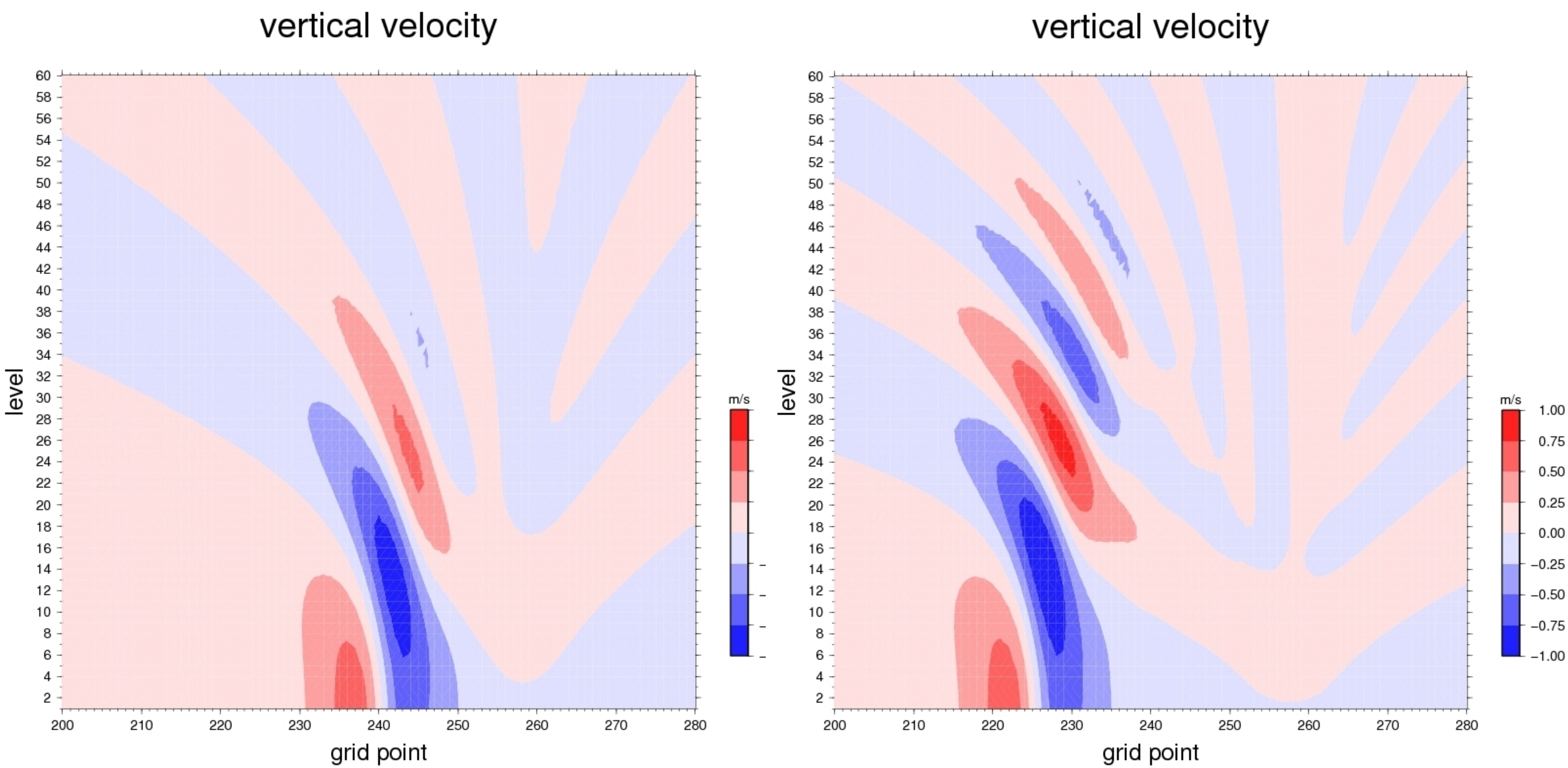
$$\left(1-\beta^2 c_*^2\left(\Delta+\mathbf{P}\frac{1}{H_*^2}\left(1+\tilde{\partial}\right)\tilde{\partial}\right)-\mathbf{P}\beta^4 N_*^2 c_*^2 \Delta\right)w^+ = \tilde{R}_w^*$$

where

$$\mathbf{P} = diag\left(\mathbf{1},\ldots,\mathbf{1},\mathbf{0}\right)$$

Matrix $\mathbf{P}(1+\tilde{\partial})\,\tilde{\partial}$ has real and negative eigenvalues

Preliminary tests: Linear model (SHB style) at rest with a hill moving to the left



Future work

- Non-linear model with orography is unstable
  - Investigate the influence in 2-time-levels of the reference temperature on the stability