New SL(HD) interpolators

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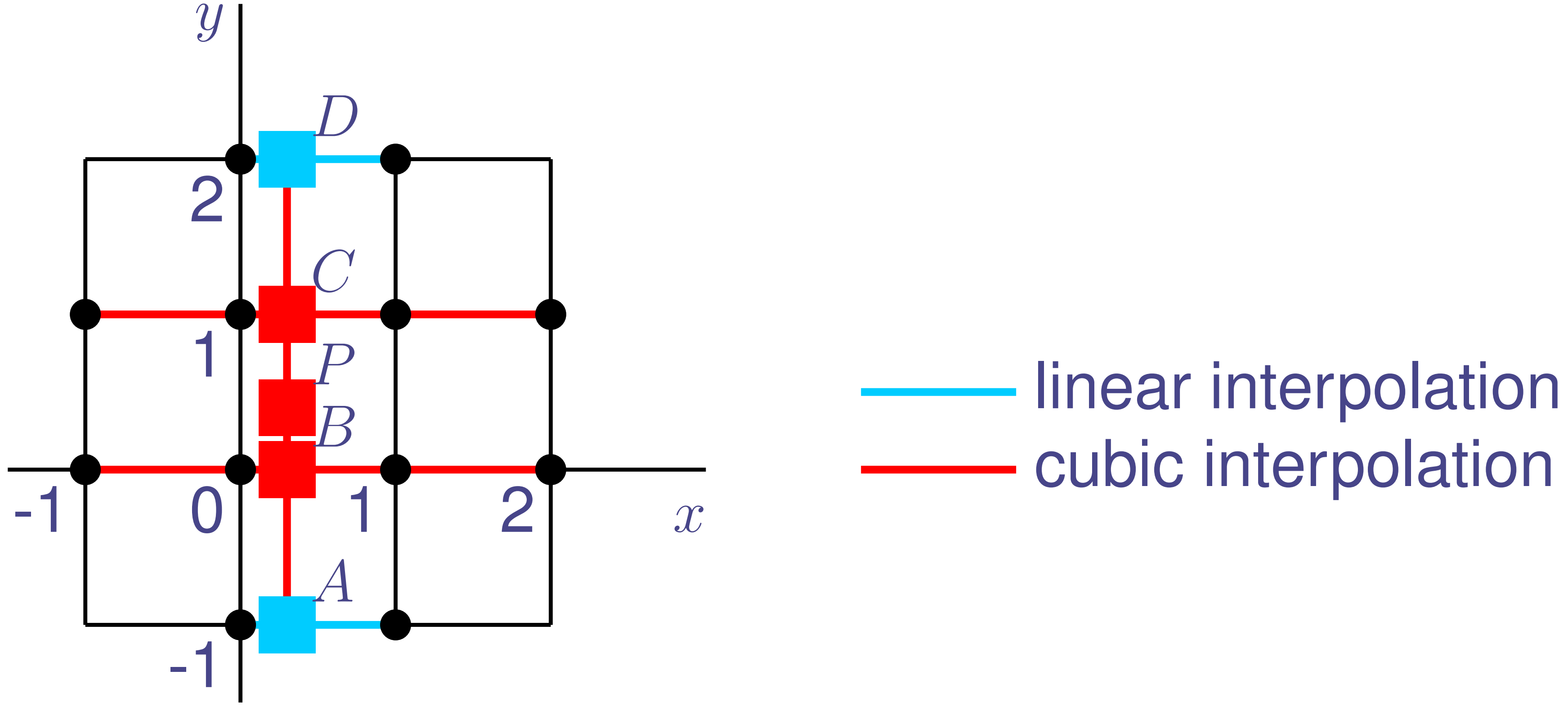
ONPP / ČHMÚ - LACE

High order SL interpolation before CY35T1

- High order interpolation exclusively performed by Lagrangian cubic interpolator. (With the only exception for ozone where vertical B-splines or Hermite cubic interpolation are allowed along vertical.)



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- SLHD (Semi-Lagrangian Horizontal Diffusion) interpolator selectively combining Lagrangian cubic interpolation with a product of linear interpolation and smoother.
- Both previous exist also in QM and QMH alternatives.

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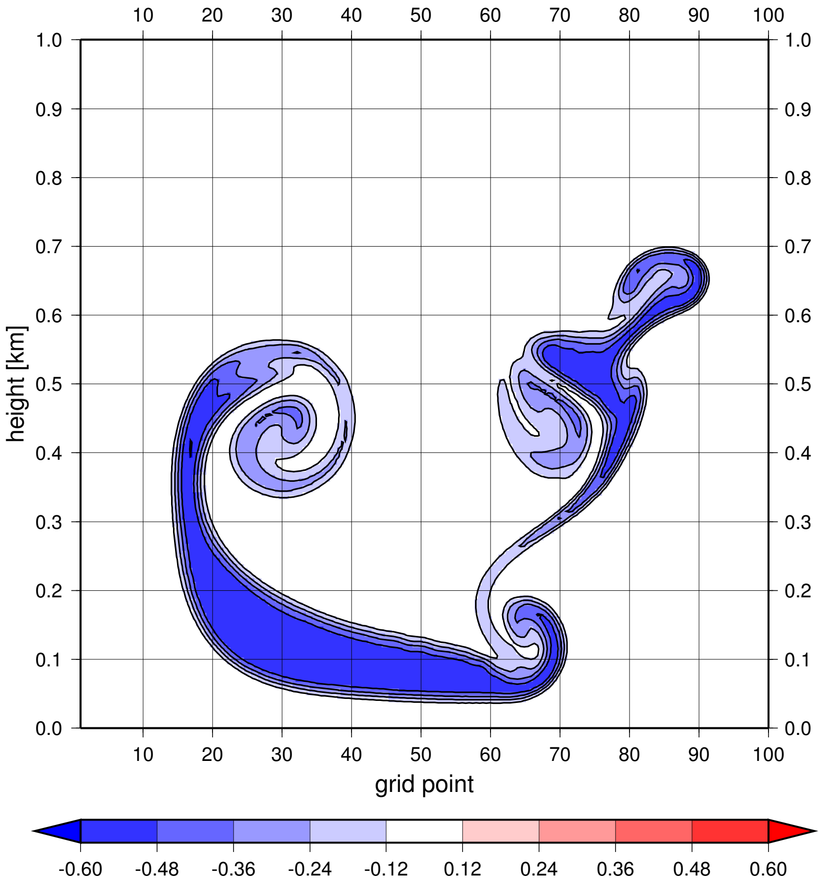
- Cubic Lagrange polynomial was replaced with more accurate natural cubic spline (on 4 point stencil).
- It indeed reduced positive MSL pressure bias, but detrimental effect on other fields was observed.
- In order to understand what is going on, detailed examination of semi-Lagrangian interpolators followed.

Problem with natural cubic spline

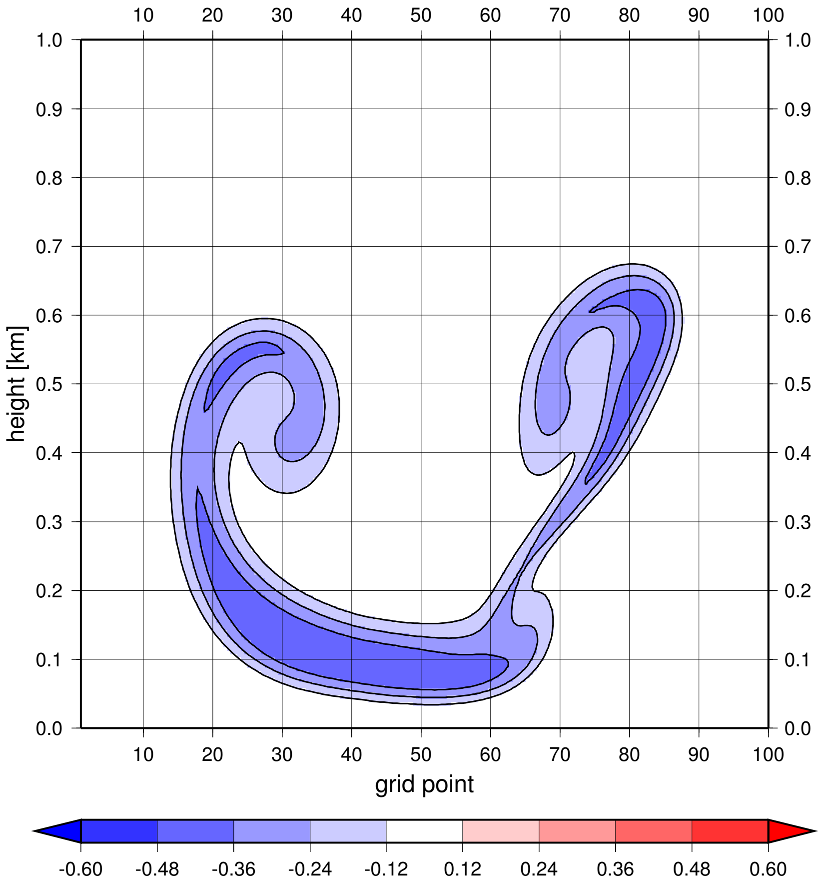
cubic Lagrange polynomial

(reliable reference)



linear interpolator

(details smoothed out)



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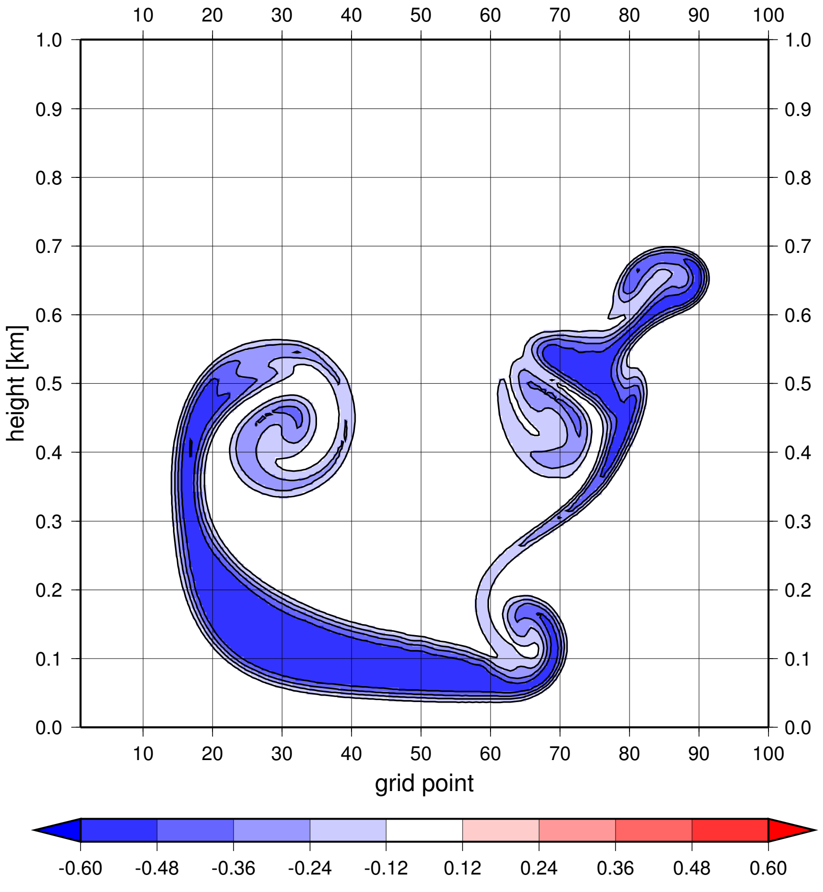
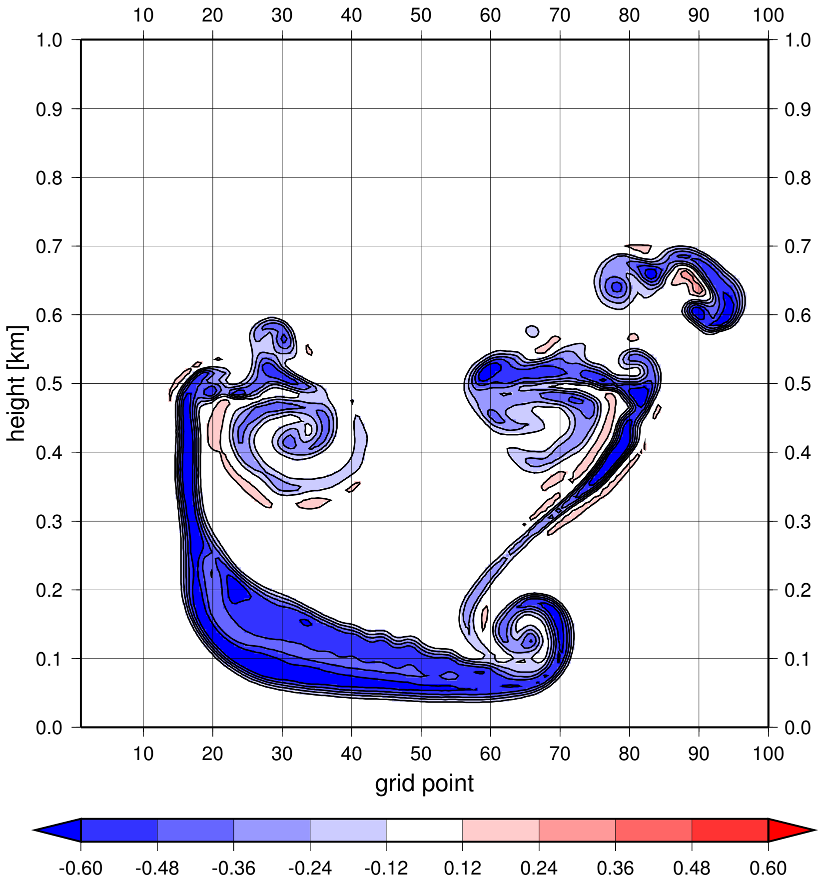


natural cubic spline

(distortion and overshoots)



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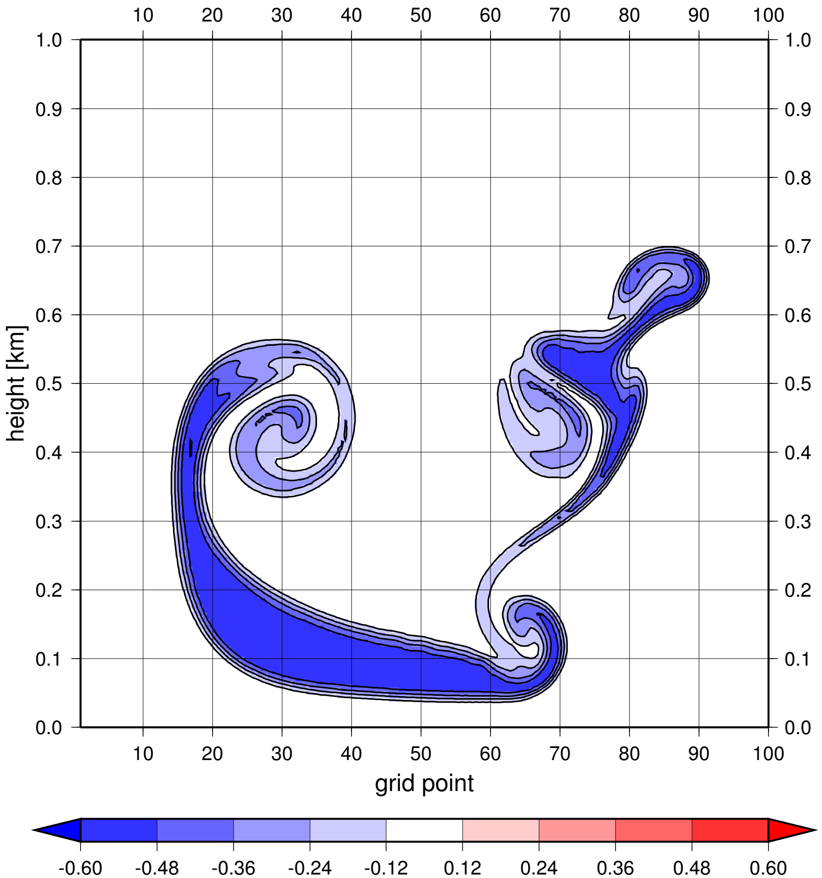
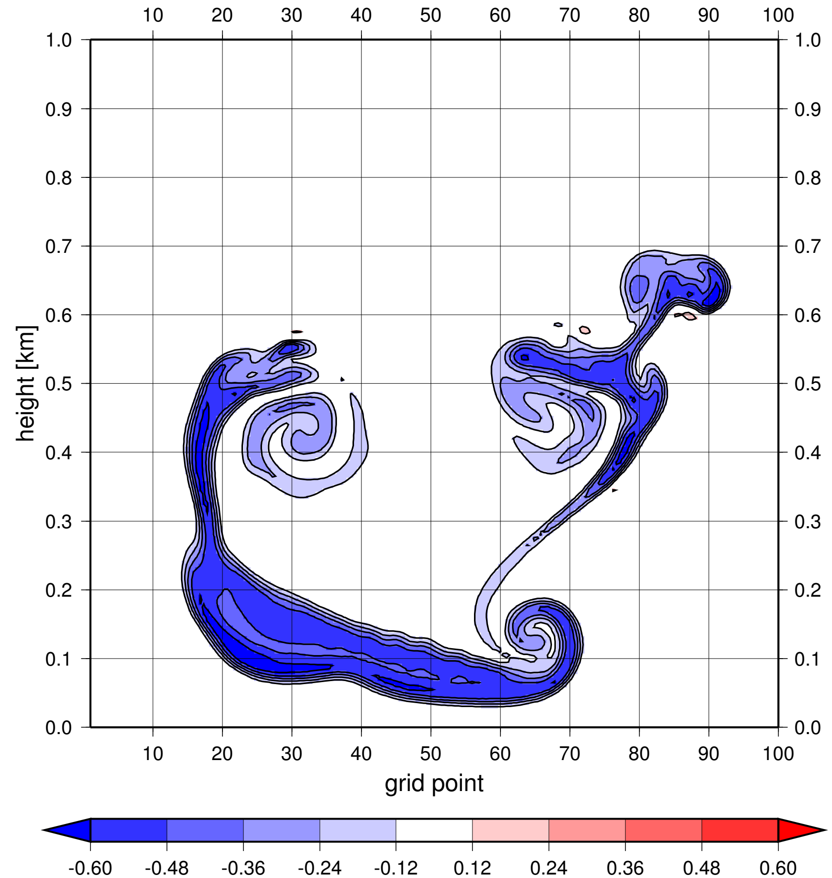


natural cubic spline, QM version

(distortion, overshoots cut off)



Decent 4-point interpolator requirements

- linearity with respect to $\mathbf{y} = (y_{-1}, y_0, y_1, y_2)$:
$$F(x, \mathbf{y}) = w_{-1}(x)y_{-1} + w_0(x)y_0 + w_1(x)y_1 + w_2(x)y_2$$
- invariance with respect to horizontal mirroring:
$$F(1-x, y_2, y_1, y_0, y_{-1}) = F(x, y_{-1}, y_0, y_1, y_2)$$
- invariance with respect to vertical shift:
$$F(x, \mathbf{y} + c) = F(x, \mathbf{y}) + c$$
- reproducing of values y_0, y_1:
$$F(0, \mathbf{y}) = y_0$$
$$F(1, \mathbf{y}) = y_1$$
- reproducing of linear function $y = x$:
$$F(x, -1, 0, 1, 2) = x$$

Family of cubic 4-point interpolators

- when weights w_{-1}, w_0, w_1, w_2 are constrained to polynomials of degree at most 3, an interpolator F is restricted to the form:

$$F(x, \mathbf{y}) = u(x)y_{-1} + v(x)y_0 + v(1-x)y_1 + u(1-x)y_2$$

$$u(x) = a_1 x + a_2 x^2 - (a_1 + a_2)x^3$$

$$v(x) = 1 + (a_2 - 1)x - (3a_1 + 4a_2)x^2 + 3(a_1 + a_2)x^3$$

$$a_1, a_2 \in \mathbb{R}$$

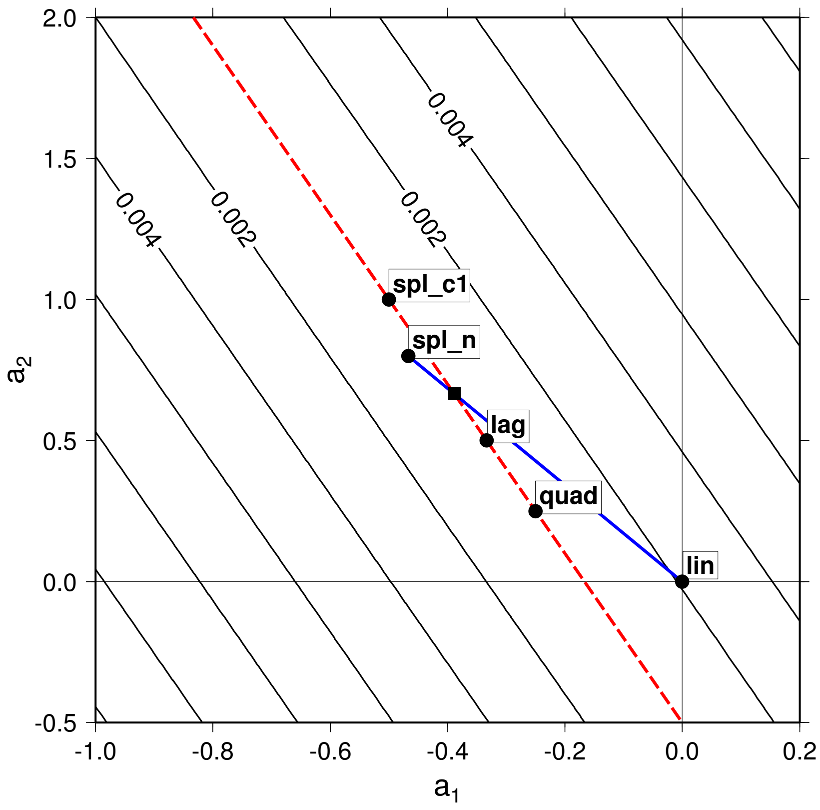
- every decent 4-point cubic interpolator can be represented by a point in the (a_1, a_2) plane
- requirement that F reproduces also quadratic function $y = x^2$ (which implies second order accuracy) defines the straight line:

$$6a_1 + 2a_2 = -1$$

Accuracy maps

accuracy measured by weighted MAE

weight function $\exp(-25m/M)$



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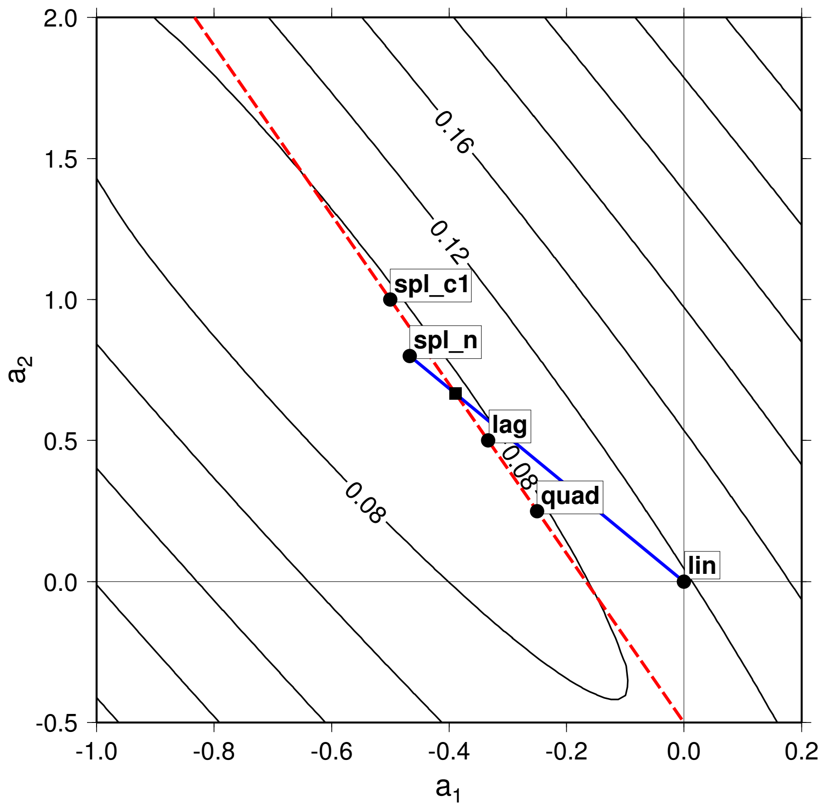
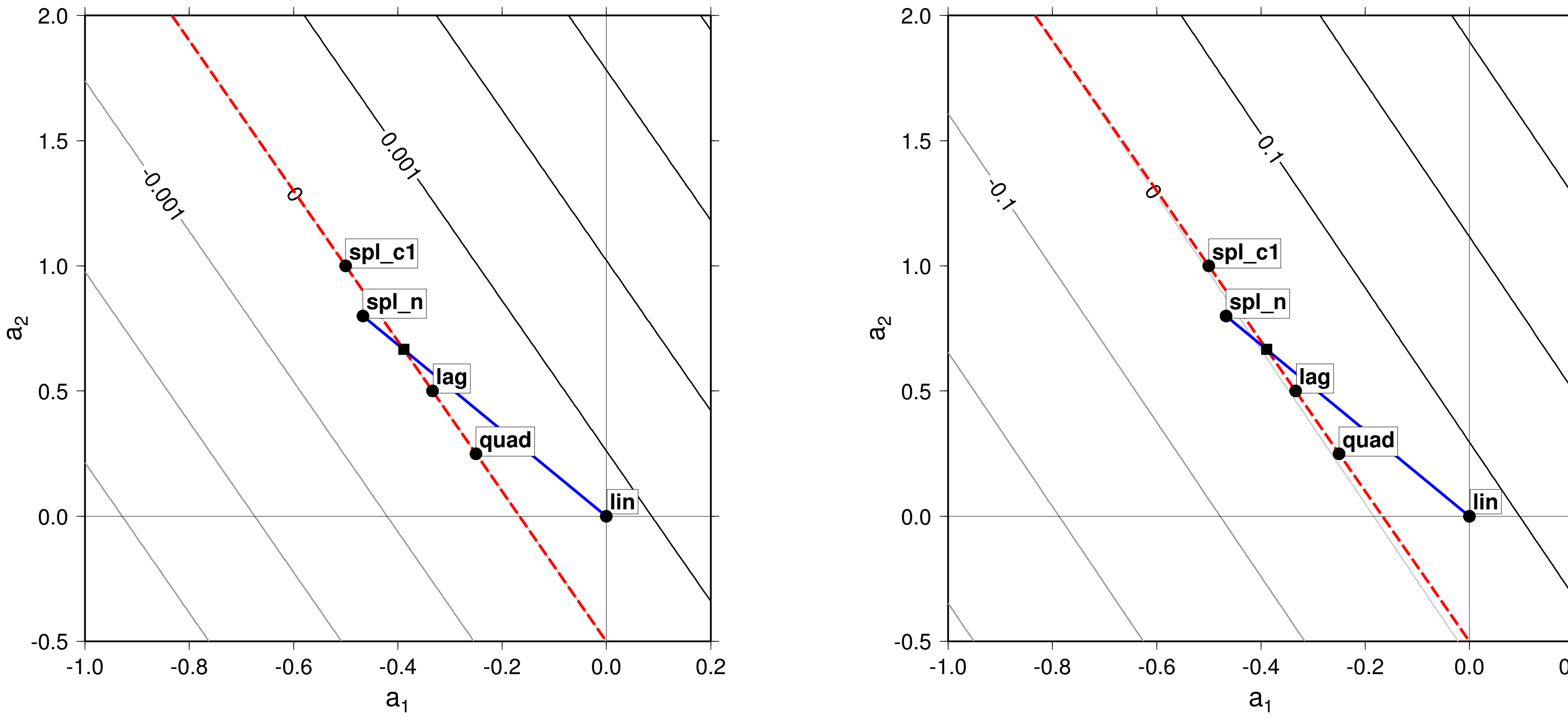


Diffusivity maps

dimensionless damping rate

$\lambda = 100\Delta x$ $\qquad\qquad$ $\lambda = 10\Delta x$



Diffusivity maps

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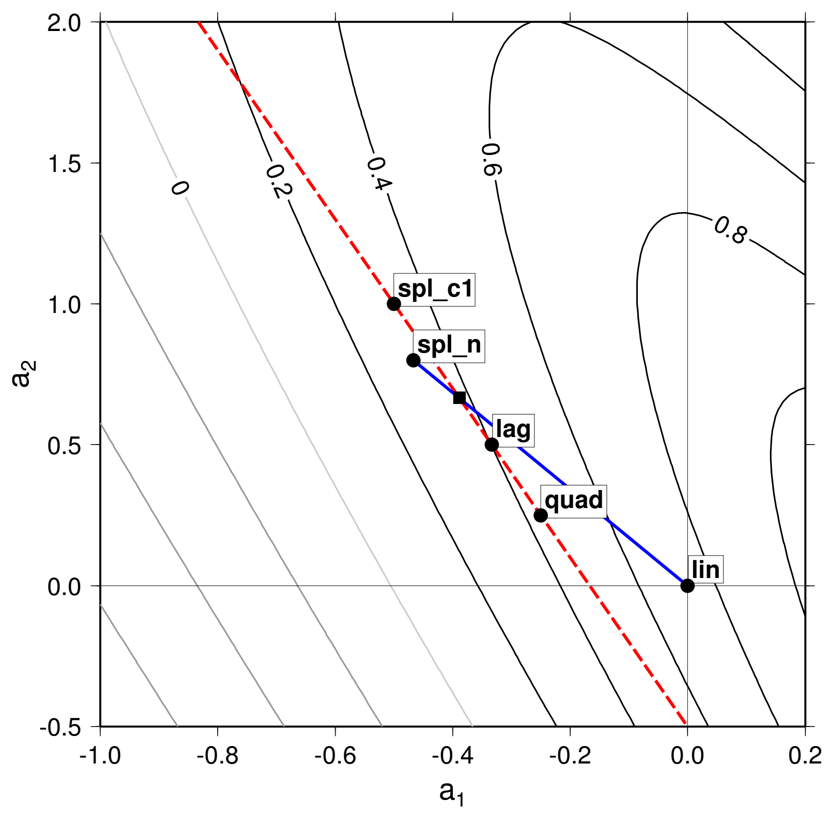
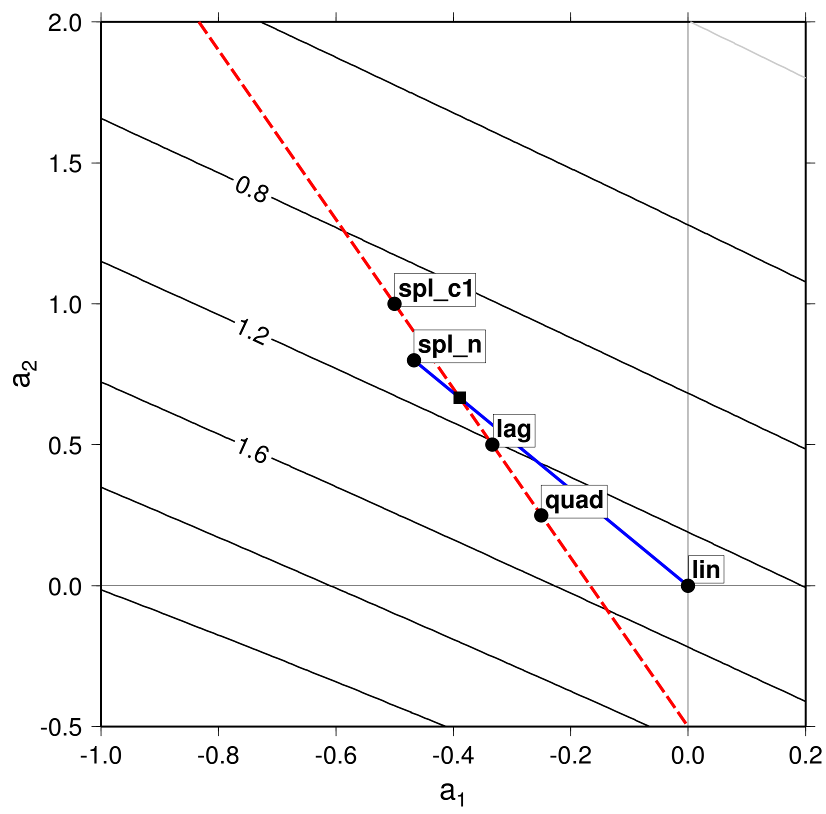
$\lambda = 3.0\Delta x$



$\lambda = 2.0\Delta x$



Model implementation

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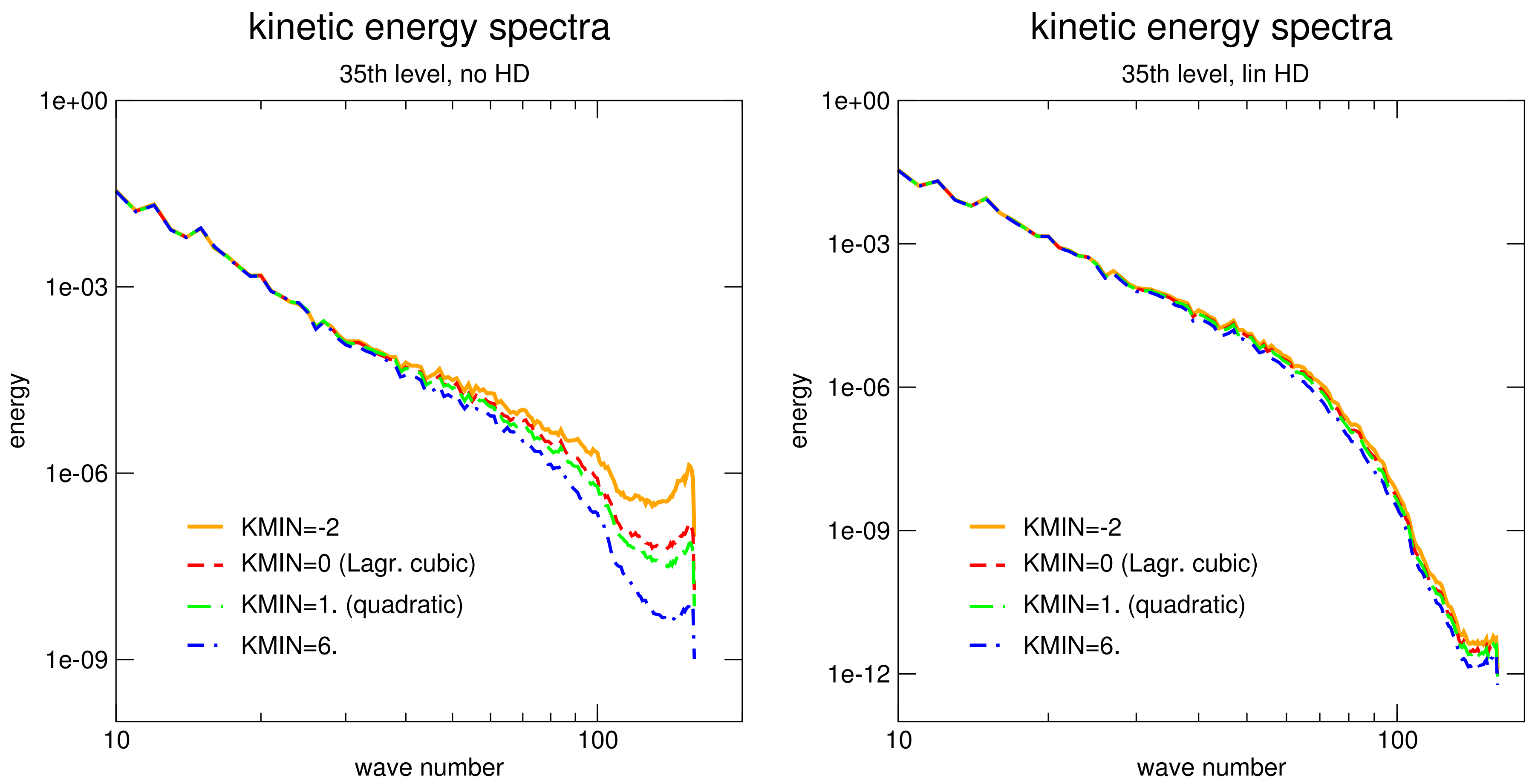
- Identity of some important inhabitants of (a_1, a_2) plane:

a_1	a_2	name	order of accuracy	parameter κ
0	0	linear interpolator	1	—
$-\frac{1}{4}$	$\frac{1}{4}$	quadratic interpolator	2	1
$-\frac{1}{3}$	$\frac{1}{2}$	cubic Lagrange polynomial	3	0
$-\frac{1}{2}$	1	quasi-cubic spline	2	-2
$-\frac{7}{15}$	$\frac{4}{5}$	natural cubic spline	1	—

Impact to KE spectra

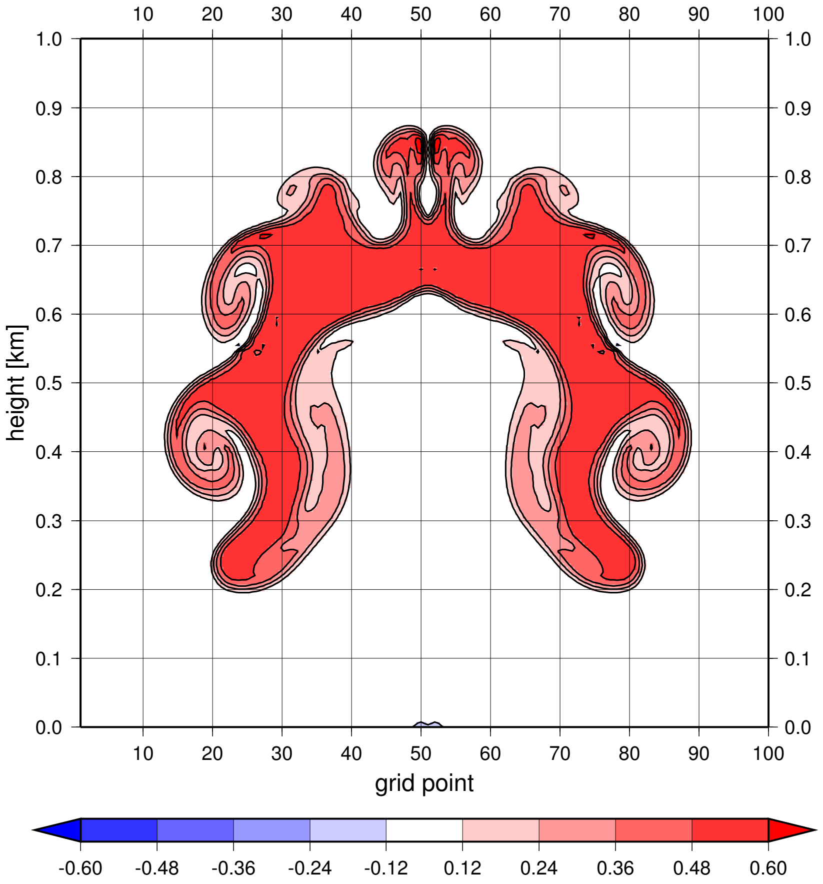
3h adiabatic forecast



Damping properties of new interpolators (1)

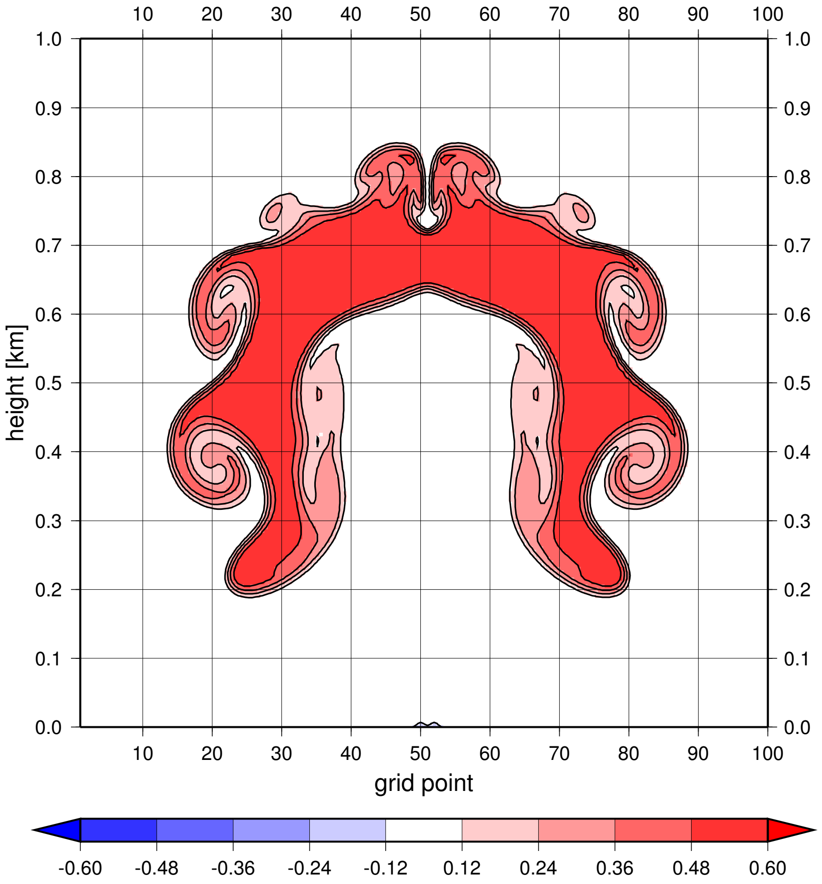
quasi-cubic spline

$(\kappa = -2)$



cubic Lagrange polynomial

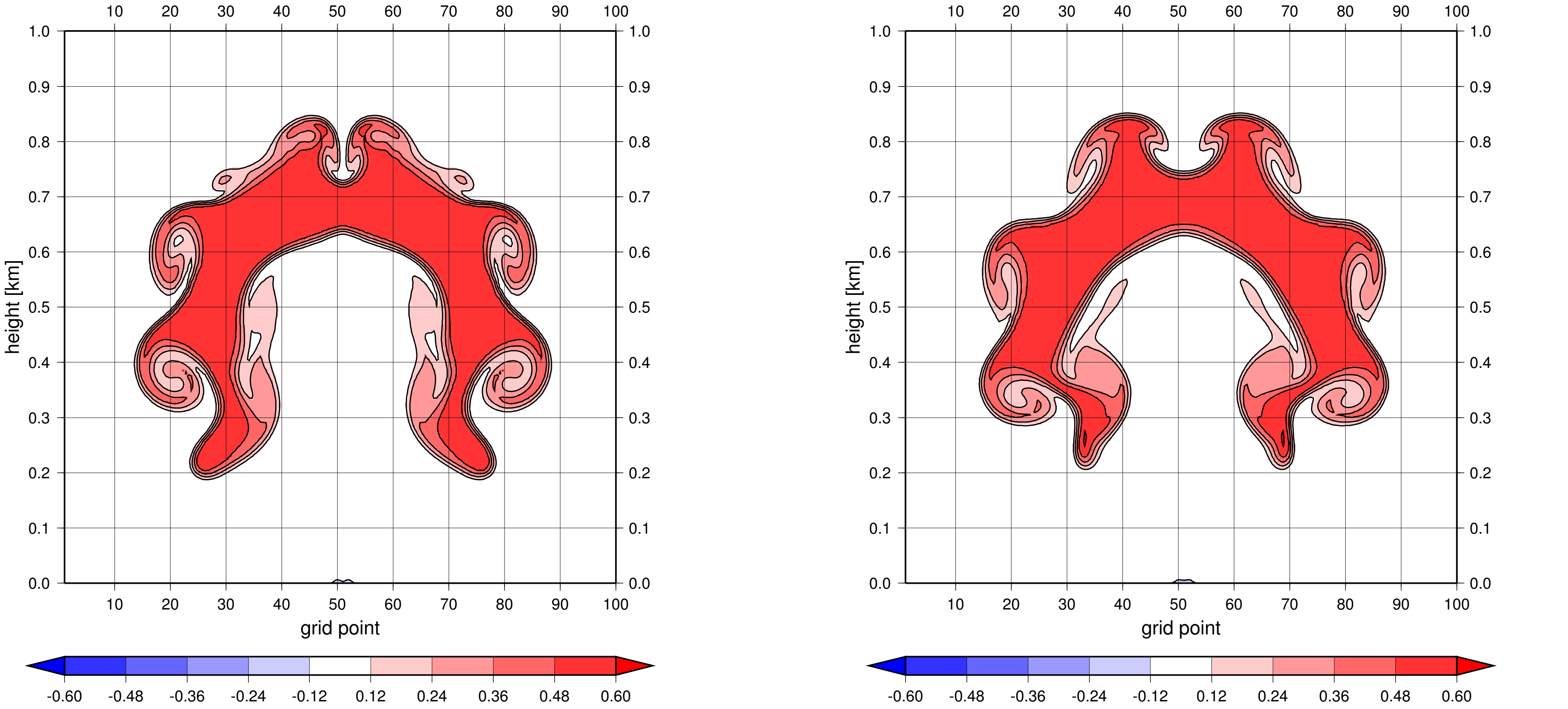
$(\kappa = 0)$



Damping properties of new interpolators (1)

quadratic interpolator ($\kappa = 1$)

strongly diffusive second order accurate interpolator ($\kappa = 6$)



SLHD bonus – Laplacian smoother

- Laplacian smoother S can be applied before semi-Lagrangian interpolations:

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- Finite difference formula for second order derivative at inner nodes and prescribing it to be zero at outer nodes gives:

$$S(\mathbf{y}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \varepsilon & 1-2\varepsilon & \varepsilon & 0 \\ 0 & \varepsilon & 1-2\varepsilon & \varepsilon \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_{-1} \\ y_0 \\ y_1 \\ y_2 \end{pmatrix}$$

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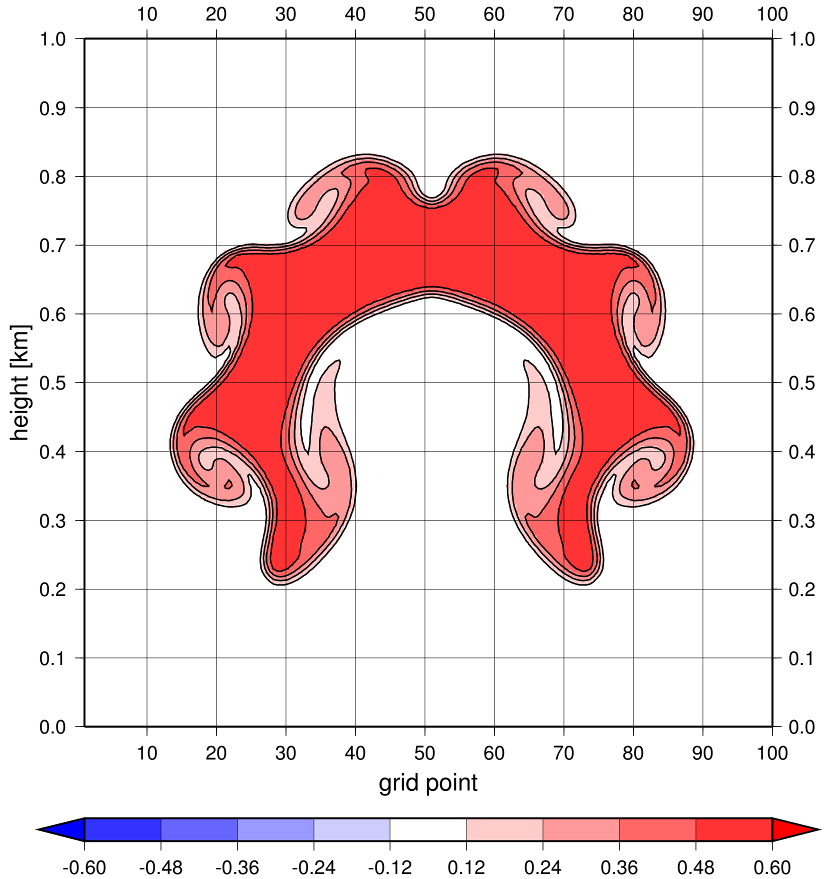
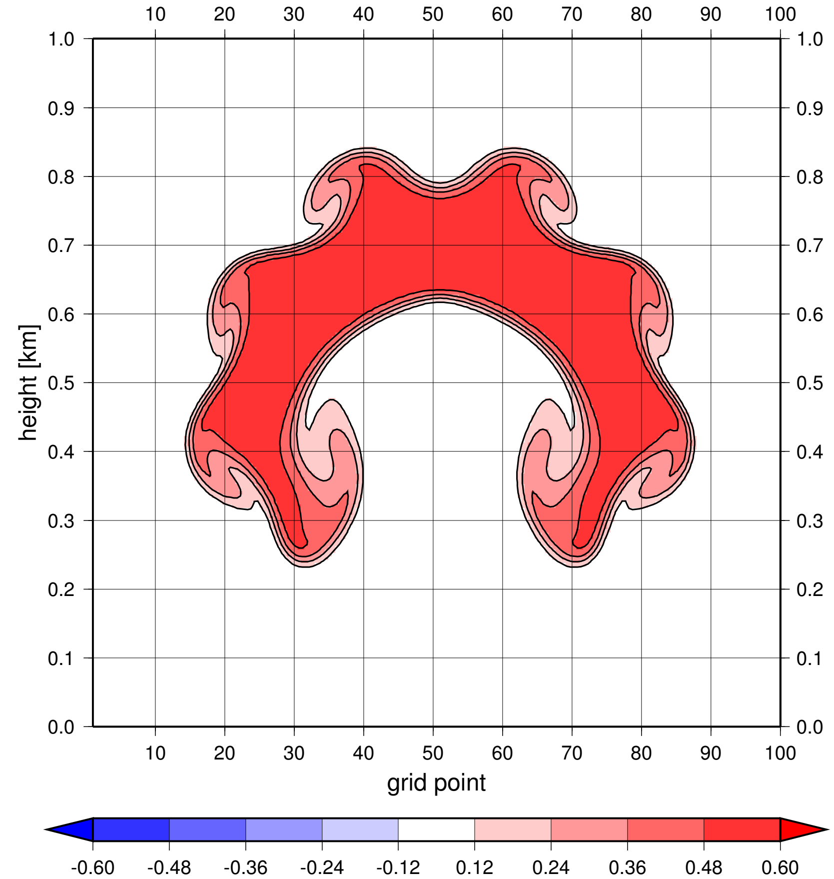
- Explicitly treated diffusion $\Rightarrow$ safety limit on smoother strength ε. Model implementation distinguishes between ε_H and ε_V.

Damping properties of new interpolators (2)

cubic Lagr. polynomial with smoother, ($\kappa = 0$, $\varepsilon = 0.01$)



cubic Lagr. polynomial with smoother, ($\kappa = 0$, $\varepsilon = 0.02$)



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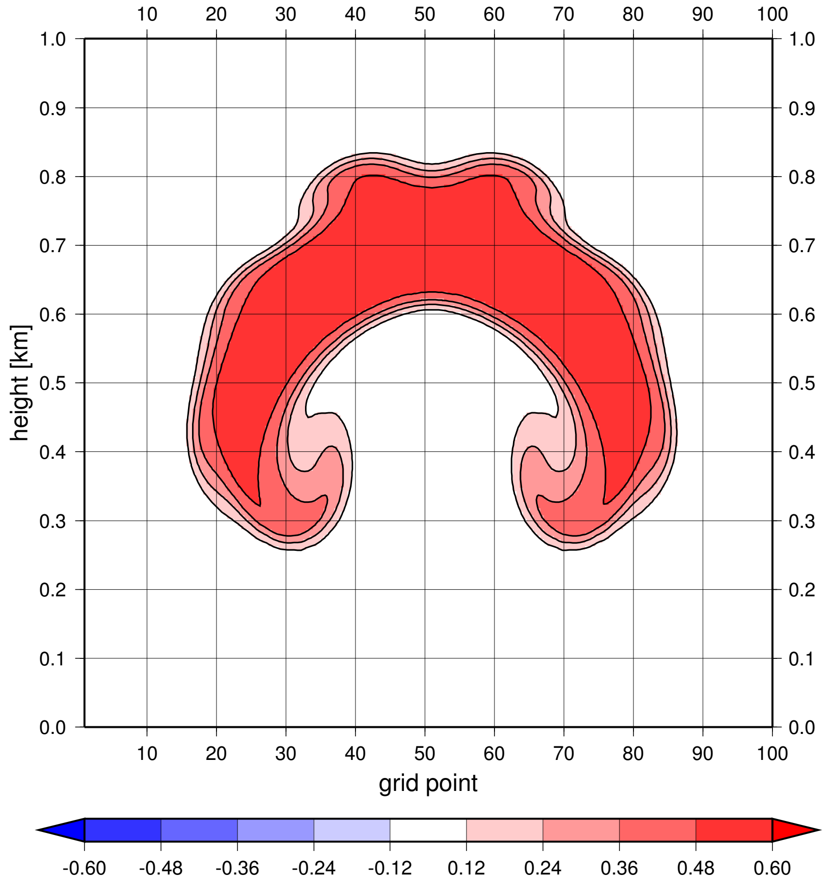
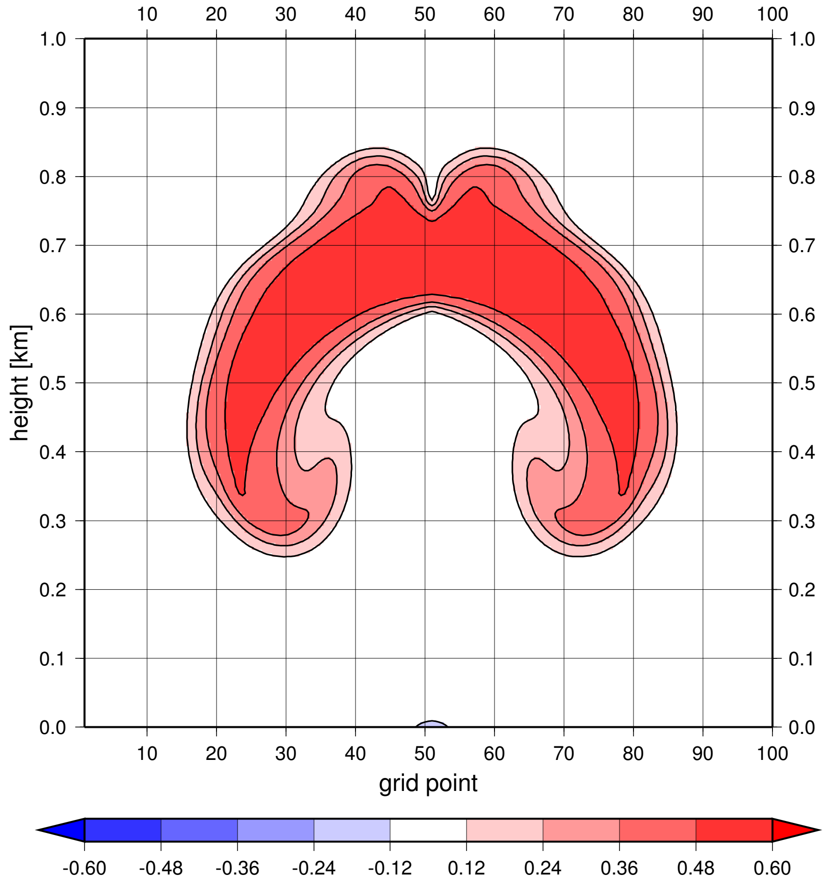
cubic Lagr. polynomial with smoother, ($\kappa = 0$, $\varepsilon = 0.04$)



linear interpolator



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- Lagrangian cubic (default, `SLHDKMIN=0.`)
- General second order accurate interpolation (`SLHDKMIN` $\in \langle -2., 6. \rangle$)
- Two interpolator sets, controlled by `SLHDKMIN` and `SLHDKMAX` (`SLHDKMIN` $\leq$ `SLHDKMAX`),
  - `LSLHD_STATIC=`.true.
  - Prognostic quantities belonging to the "`SLHDKMAX`" group are selected by activating appropriate LSLHD key.
  - Additionally, for the second group the Laplacian smoother can be activated. (Specific case: `SLHDKMIN` $=$ `SLHDKMAX`!)

SLHD interpolation since CY35T1

- Emulation of the "old" SLHD within the new data-flow. (LSLHD_OLD=.true., SLHDKMIN, SLHDEPSH, SLHDEPSV; SLHDKMAX has no meaning here)

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- New SLHD with all interpolations being 2nd order accurate (SLHDKMIN $\leq$ SLHDKMAX, SLHDEPSH, SLHDEPSV)
- Laplacian smoother only as the specific case when SLHDKMIN $=$ SLHDKMAX. (Beware of the stability for SLHDEPSV!)

Namelist parameters

Keys

`LSLHD_[x]`	-	Keys to activate SLHD for GMV [x]
`%LSLHD`	-	GFL attribute to activate SLHD
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Tuning parameters

`SLHDKMIN`	-	property of the basic interpolator (Optimum seems to be around 0.!)
`SLHDKMAX`	-	property of the maximum diffusive limit (Optimum between 6.-10.)
`SLHDEPSH`	-	horizontal Laplacian smoother
`SLHDEPSV`	-	vertical Laplacian smoother

Experience from the real atmosphere

Based on ALADIN/CE (LACE domain, Δx=9 km) simulations

- Decreasing of `SLHDKMIN` leads to:
  - general improvement of wind speed and MSL pressure (conservation of mass)
  - other dynamics variables are improved in upper troposphere (above 500 hPa) and above 100 hPa
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  - detrimental effects in PBL and lower stratosphere
- Decreased `SLHDKMIN` needs to be compensated by increased horizontal and vertical diffusion!
- `SLHDEPSH` significantly influences KE spectra, weak effect to model scores; `SLHDEPSV` nearly no effect to KE spectra, strong impact to scores and mass conservation

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- Natural entry point to real 3D turbulence.