Contribution of RC LACE into development of HY and NH dynamical core

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Outline

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1. Flexible iterative scheme
2. Implementation of alternative vertical velocity variables
3. VFE

Flexible iterative scheme

Main idea

Detect some global norm during model integration and choose either SI time step or PC scheme according actual value of that norm.

Norm

some kind of CFL, some measure of non-hydrostaticity, some measure of NL residual change

Choice of norm

We implemented norm

NL residual

$\mu = 1 - \frac{|R^t - R^{t-\delta t}|}{|R^t| + |R^{t-\delta t}|}$, if $\mu \approx 1$ then SI scheme, else PC scheme

with R being nonlinear residual of some prognostic quantity.

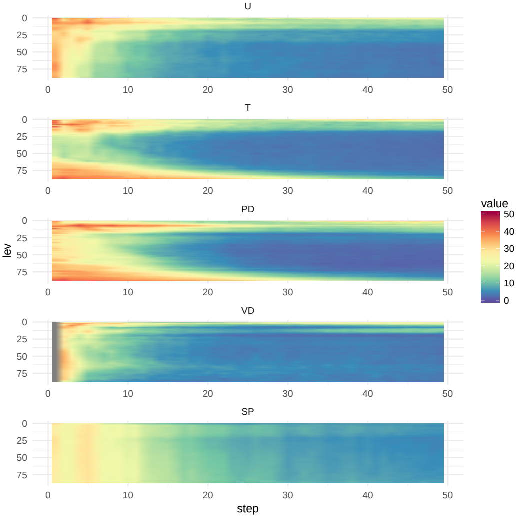


Figure : percentage of points for each vertical level where PC scheme is required because $\mu < 1$. Computed for small domain with $dx = 1km$. Horizontal axis is model time step, vertical axis is model level. Computed from NL residual of U wind component, T, PD, VD and 3D part of continuity equation.

Condition

If $\mu \approx 1$ in more that $> 90\%$ of grid points (upper atmosphere excluded) for $\hat{q}$, we switch to SETTLS scheme with NSITER=0, otherwise PC NESC with NSITER=1.

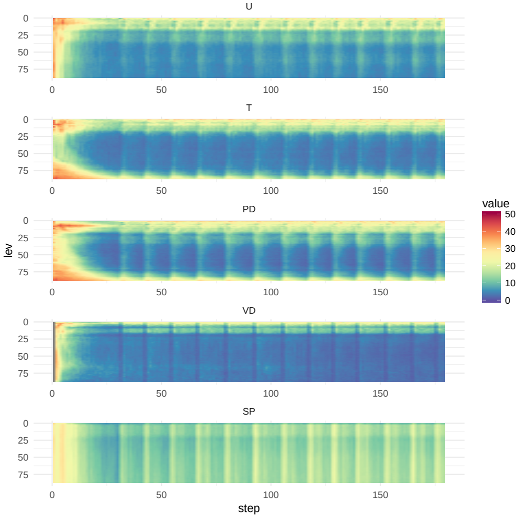


Figure : The percentage of points where $\mu < 1$. When global average $< 10\%$ we turn on SI SETTLS scheme. 3h integration is shown, computed with $\delta t = 60s$ for domain with $dx = 1km$. In total 26 time steps were SI SETTLS in this simulation and approx. $5 - 10\%$ of CPU was saved.

Unification of NESC and SETTLS

Idea

After one time step of SI SETTLS, balances were canceled and we have to stabilize next time steps with PC NESC scheme.
IDEA :keep extrapolation of NL residual as consistent as possible in SI and PC scheme, but try to profit from better stability of NESC approach.

Combined extrapolation scheme

$$R_M^{t+\frac{\delta t}{2}} \approx \frac{1}{2}\left(R_A^t + R_D^t\right) + \frac{\beta}{2}\left(R_D^t - R_D^{t-\delta t}\right) \tag{1}$$

β may change arbitrarily from 0 to 1. NESC scheme for $\beta = 0$ and SETTLS scheme for $\beta = 1$.

Unification of NESC and SETTLS

We analyzed stability of equation

$$\frac{df(t,x)}{dt} = R(t,x) = (\lambda + i\omega) f(t,x)$$

discretized in the form

$$\frac{f_A^{t+\delta t} - f_D^t}{\delta t} = \frac{1}{2}\left(R_A^t + R_D^t\right) + \frac{\beta}{2}\left(R_D^t - R_D^{t-\delta t}\right)$$

.

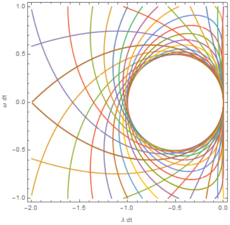
We analyze single Fourier component advected with constant wind U

$$f(n\delta t, jdx) = A^n e^{ijUk\delta t}$$

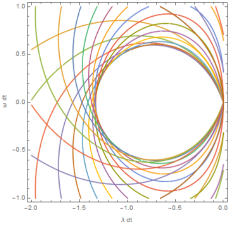
.

Stability reached when $|A| \leqslant 1$. Figures, x-axes $\lambda\delta t$, y-axes $\omega\delta t$.

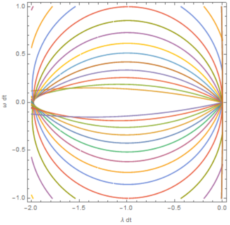
$\beta = 1(SETTLS)$



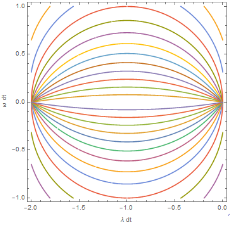
$\beta = 1/2$



$\beta = 1/100$

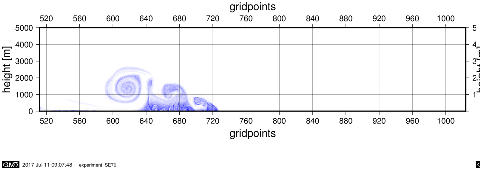


$\beta = 0(NESC)$

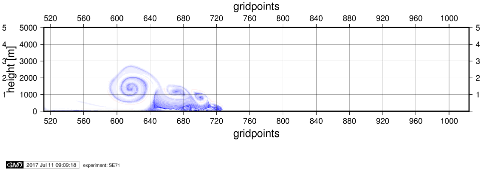


Combined PC CHEAP scheme evaluation

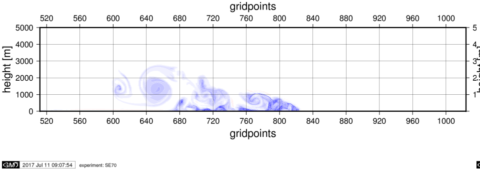
$t = 600s$ and NESC



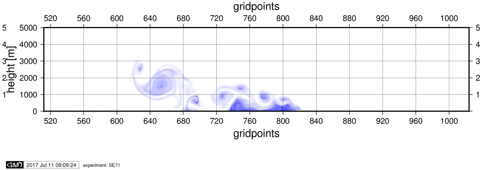
$t = 600s$ and SETTLS



$t = 900s$ and NESC



$t = 900s$ and SETTLS



Outline

Prognostic variables

Work based on proposal of Fabrice to ensure consistency of BBCs in LI and NL model.

LI model	NL model

BBC

LI model	NL model
$gw_s = 0$	$gw_s = \vec{v}_s \vec{\nabla} \phi_s$
$\frac{dgw_s}{dt} = 0$	$\frac{dgw_s}{dt} = \frac{d\vec{v}_s}{dt} \vec{\nabla} \phi_s + \vec{v}_s \vec{\nabla} (\vec{v}_s \dot{\phi}_s)$

new proposal, choose such gW that

BBC

LI model	NL model
$gW_s = 0$	$gW_s = 0$
$\frac{dgW_s}{dt} = 0$	$\frac{dgW_s}{dt} = 0$

no residual remains related to BBC condition in vertical momentum equation. Positive result expected.

We implemented three possible definitions of gW

Definition

$gW_n = gw + Y_n$ with

$$Y_5 = -\vec{v}\vec{\nabla}\phi$$

$$Y_6 = -\vec{v}\vec{\nabla}\phi_s$$

$$Y_7 = -\vec{v}_s\vec{\nabla}\phi_s$$

Spectral prognostic quantity d is modified as

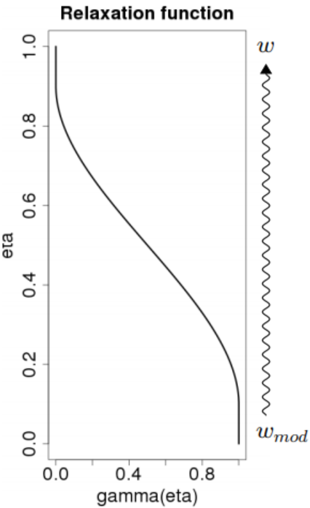
Modification of d

$$d_4 = -\frac{p}{mRT}\frac{\partial gw}{\partial \eta} + X_4$$

$d_n = -\frac{p}{mRT}\frac{\partial gW_n}{\partial \eta} + X_n$ with

$$X_n = \frac{p}{mRT}\frac{\partial Y_n}{\partial \eta} + X_4$$

Prognostic variables



However such definition just shifts problems from surface to model top ⇒ relaxation function $\gamma(\eta)$ in vertical is used to avoid "jet streams of gW close to model top.

$$gW_n = gw + \gamma(\eta) Y_n$$

Prognostic equation for gW_n is

$$\frac{dgW_n}{dt} = \frac{dgw}{dt} + \frac{d\gamma Y_n}{dt}$$

and for d_n variable

$$\frac{dd_n}{dt} = \frac{dd_4}{dt} + \frac{dX_n}{dt}$$

⇒ modification of gw inluences only evolution of X-term. The LI model remained unchanged for all gW_n.

Prognostic variables

The evolution of Y_n can be treated explicitly or approximative way

Explicit treatment | **Aprox. SL treatnent**

Evolution of Y_n

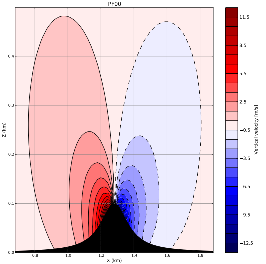
$$-$$

$$\frac{d(\gamma Y)}{dt} \approx \frac{(\gamma Y)_A^{t+\delta t} - (\gamma Y)_D^t}{\delta t}$$

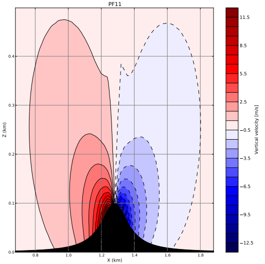
$$\frac{\gamma d Y_6}{dt} = -\gamma \left(\frac{d\vec{v}}{dt} \vec{\nabla} \phi_s - J_s \right) + \dot{\eta} \vec{v} \vec{\nabla} \phi_s \frac{\partial \gamma}{\partial \eta}$$

$$\frac{\gamma d Y_7}{dt} = -\gamma \left(\frac{d\vec{v}_s}{dt} \vec{\nabla} \phi_s - J_{ss} \right) + \dot{\eta} \vec{v}_s \vec{\nabla} \phi_s \frac{\partial \gamma}{\partial \eta}$$

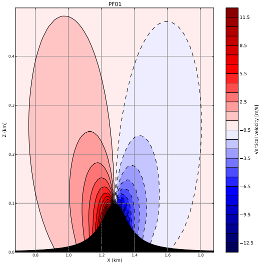
gw



gW_6 explicit treatment

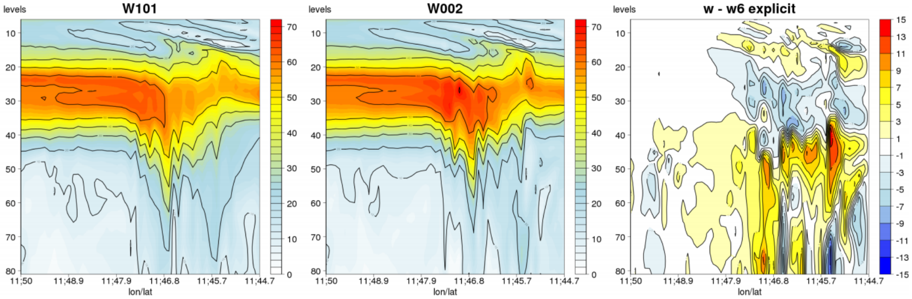


gW_6 approx. SL treatment



Real case - *gW*

Real simulations - 4.1.2017, strong wind across Alps.
Vertical cross section through the hor. wind speed field:



w $\qquad$ w_{mod} **explicit** $\qquad$ $w - w_{mod}$

We have to find a testbed to study the stability properties of the proposed alternatives.

Outline

Hybrid mass based coordinate definition

"π" represents the mass of column per m^2 in HY and NH model as well. In HY model $\pi = p$.

Finite differences model

Finite elements model

Definition

$\pi(\eta_{\tilde{l}}) = A(\eta_{\tilde{l}}) + B(\eta_{\tilde{l}})\pi_s(x, y)$

$m_l = \frac{\partial \pi(\eta_l)}{\partial \eta} = \frac{\partial A(\eta_l)}{\partial \eta} + \frac{\partial B(\eta_l)}{\partial \eta}\pi_s(x, y)$

Layer depth

$\delta\pi_l = \pi_{\tilde{l}} - \pi_{\tilde{l-1}}$

$m_l = \frac{\delta A_l}{\delta \eta_l} + \frac{\delta B_l}{\delta \eta_l}\pi_s$

$\delta\eta_l = \eta_{\tilde{l}} - \eta_{\tilde{l-1}}$

Integral operator

$\int_0^1 \frac{\partial \pi}{\partial \eta}\psi d\eta \approx \sum_{l=1}^{L} \psi_l \delta\pi_l$

η values remain implicit (semi-Lagrangian advection requires them)

$\int_0^1 \frac{\partial \pi}{\partial \eta}\psi d\eta \approx (\mathbf{K}m\psi)_L$

η requires explicit definition, operator $(\mathbf{K})$ is full level matrix and $(\mathbf{K})_l$ represent value of integral from top to level l

Hybrid mass based coordinate definition

Finite differences model

Finite elements model

Mass conservation

Finite differences model:

$$\sum_{l=1}^{L} \delta\pi_l = \sum_{l=1}^{L} \delta A_l + \sum_{l=1}^{L} \delta B_l \pi_s$$

$$\pi_s = (A_{\tilde{L}} - A_{\tilde{1}}) + (B_{\tilde{L}} - B_{\tilde{1}})\pi_s$$

Finite elements model:

$$(\mathbf{K} m)_L = \left(\mathbf{K} \frac{\delta A}{\delta \eta}\right)_L + \pi_s \left(\mathbf{K} \frac{\delta B}{\delta \eta}\right)_L$$

$$\left(\mathbf{K} \frac{\delta A}{\delta \eta}\right)_L = 0 \qquad \left(\mathbf{K} \frac{\delta B}{\delta \eta}\right)_L = 1$$

Adjustment

BCs of A and B satisfy mass conservation property

$$\tilde{\delta A}_l = A_{\tilde{l}} - A_{\widetilde{l-1}} \qquad \tilde{\delta B}_l = B_{\tilde{l}} - B_{\widetilde{l-1}}$$

$$\delta B_l = \frac{1}{\alpha} \tilde{\delta B}_l \qquad \alpha = \left(\mathbf{K} \frac{\tilde{\delta B}}{\delta \eta}\right)_L$$

$$\delta A_l = \frac{1}{\beta} \left(\tilde{\delta A}_l + \delta B_l \pi_r\right) - \delta B_l \pi_r$$

$$\beta = \left(\mathbf{K} \left(\frac{1}{\pi_r} \frac{\tilde{\delta A}_l}{\delta \eta} + \frac{\delta B_l}{\delta \eta}\right)\right)_L$$

Explicit definition of η

In finite difference model η is implicit, in finite elements model it must be defined explicitly.

LREGETA generalisation

$$\eta_k = \frac{\sum_{i=1}^{k} d\pi_k^{\alpha}}{\sum_{i=1}^{L} d\pi_k^{\alpha}}$$

$\alpha = 0 \Rightarrow LREGETA$,
$\alpha = 1 \Rightarrow .NOT.LREGETA$

Above definition is unstable for high order VFE schemes with spline order 5 and higher when tested with 137 levels. Stabilisation require higher density of levels close to BCs in η space

Denser levels close to BCs - LVFE_CENTRI

$$\eta_k = (1-\beta)\frac{k}{L} + \beta\left[\frac{1}{2} - \frac{1}{2}cos(\pi\frac{k}{L})\right]$$

$\beta = 0 \Rightarrow LREGETA$, $\beta \approx 1$ very dense close to BCs

Experimentally found that $\beta \approx 0.5$ is stable for high order operators and the eigenvalue of linear model $\mathbf{L}^*$ are pure imaginary one as required (susi, sunhsi). I implemented also implicit definition of η that allows to control density close to top BC and close to surface BC independently.

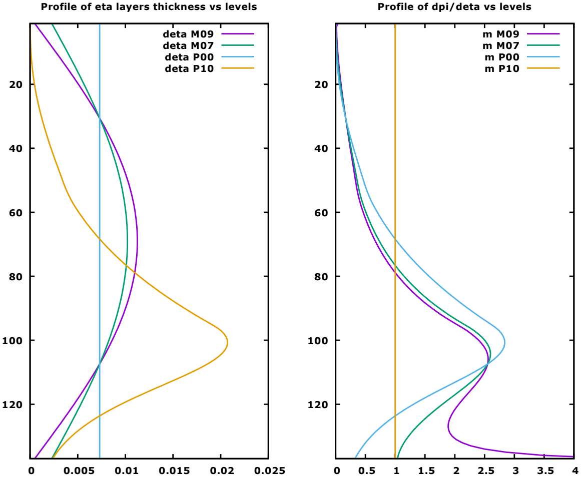
Denser levels close to BCs - LVFE_CHEB (testing to be done)

$$\frac{\partial s}{\partial \eta} = a\eta^2 + b\eta + c$$

$$\int_0^1 \frac{\partial s}{\partial \eta} = 1,\ \left(\frac{\partial s}{\partial \eta}\right)_0 = \alpha,\ \left(\frac{\partial s}{\partial \eta}\right)_1 = \beta$$

s represents LREGETA coordinate.

Explicit definition of η



Profiles of layer thickness $\delta\eta$ and $\frac{\partial\pi}{\partial\eta}$ vs levels index for various definitions of η coordinate (P00 - LREGETA=.T., P10 - LREGETA=.F., M07 - CETRI with $\beta = 0.7$, M09 - CENTRI with $\beta = 0.9$)

Analysis of stability (LVFE_STABILITY)

I implemented into SUSI and SUNHSI analysis of stability exactly as in Simmons,1978 article. We assume that atmosphere is described by $\overline{\mathbf{L}}(\overline{T},\overline{\pi_s})$, that is the same as $\mathcal{L}$ except linearisation is done around $(\overline{T},\overline{\pi_s})$ instead $(T^*,{\pi_s}^*)$

$$\frac{X^{t+\delta t}-X^t}{\delta t} = \frac{\overline{\mathbf{L}}X^{t+\delta t}+\overline{\mathbf{L}}X^t}{2}$$

part described by SI $\mathcal{L}$ model is implicit and residual part $RX = \overline{\mathbf{L}}X - \mathcal{L}X$ explicit using and iterative procedure

$$\begin{aligned}
\frac{X^{t+\delta t(0)}-X^t}{\delta t} &= RX^t + \frac{\mathcal{L}.X^{t+\delta t(0)}+\mathcal{L}X^t}{2} \\
\frac{X^{t+\delta t(n)}-X^t}{\delta t} &= \frac{RX^{t+\delta t(n-1)}+RX^t}{2} + \frac{\mathcal{L}.X^{t+\delta t(n)}+\mathcal{L}.X^t}{2}
\end{aligned}$$

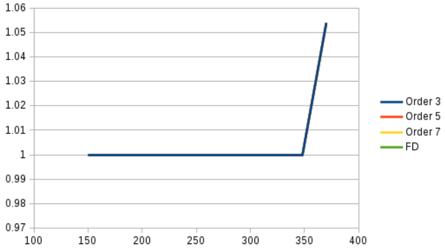
Largest eigenvalue of matrix M_c is showed

$$\begin{aligned}
M_p &= (I-\tau\mathcal{L})^{-1}\,(2\tau R + I + \tau\mathcal{L}) \\
M_c &= (I-\tau\mathcal{L})^{-1}\,(\tau R M_p + \tau R + I + \tau\mathcal{L})
\end{aligned}$$

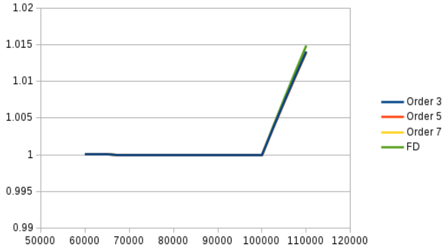
Analysis of stability (LVFE_STABILITY)

we examine stability of time stepping for single wave $\triangle = -\frac{n(n+1)}{a^2}$ for $n = 1000$ and $dt = 600s$ for FD and VFE scheme with various order of splines with 137 levels and $T^* = 350K$, $T_a{}^* = 50K, \pi_s{}^* = 100000Pa$.
Results are plotted for CENTRI definition with $\beta = 0.5$.

temperature dependence



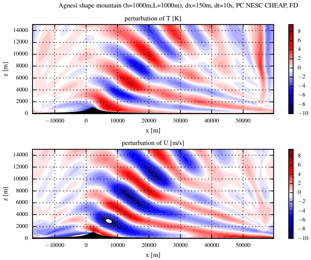
pressure dependence



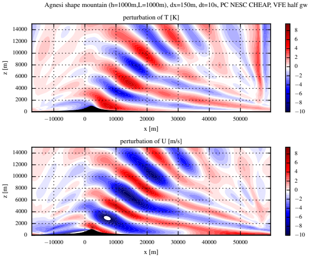
NLNH experiment small planet

Experiment with elliptic Agnesi mountain (h=1000m, L=1000m) with resolution $dx = 150m, dt = 10s, N = 0.02s^{-1}, U = 10ms^{-1}$.

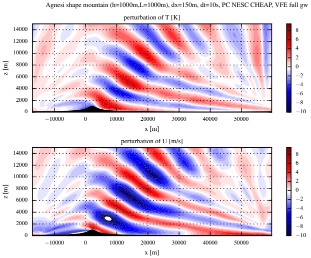
FD



VFE half gw



VFE full gw



I implemented full level option of *gw* in IFS using operators $\mathbf{P}$ and $\mathbf{D_1}$. But $\mathbf{P}.\mathbf{D_1} \neq I_d$. As you see on right figure this seems not to be obstacle. To be studied.

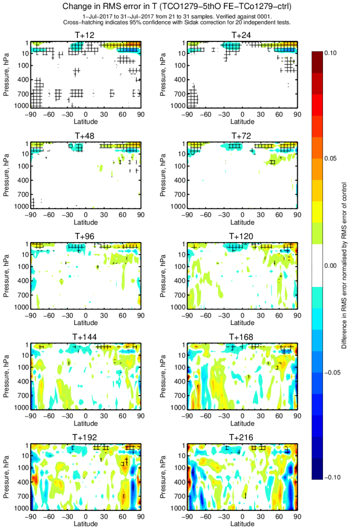
Real experiments with IFS - HY model

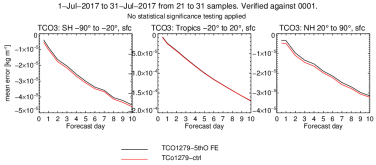
Comparison of ref 3rd order ECMWF VFE scheme against new VFE scheme using HY dynamics. TCO1279 and 137 levels.

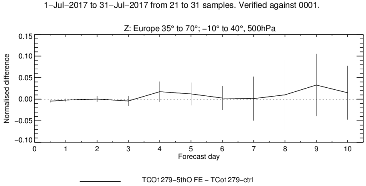
Ingredients

no ECMWF operators	LVFE_ECMWF=F
5th order spline basis	NVFE_TYPE=5
new definition of η (denser levels close to boundaries)	LVFE_CENTRI=T, RVFE_ALPHA=0.0,RVFE_BETA=0.5
different BCs of input quantity in integral operator	NVFE_INTBC=3

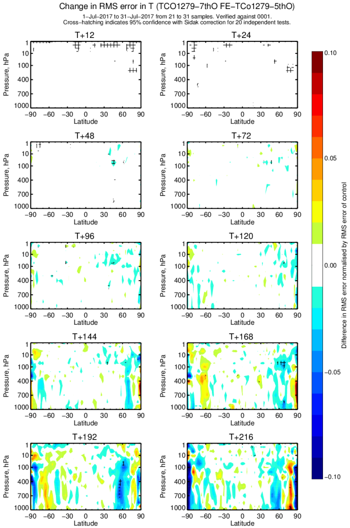
Real experiments with IFS - HY model





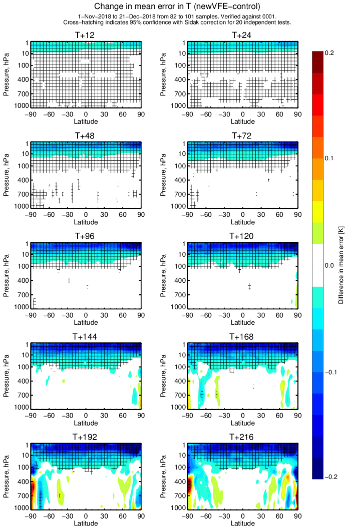


Real experiments with IFS - HY model - 7th vs. 5th order



Neutral results with potential :-)

Real experiments with IFS - lowres longterm validation



Ingredients

LVFE_ECMWF=F

NVFE_TYPE=3

LVFE_CENTRI=T,
RVFE_ALPHA=0.0,RVFE_BETA=0.5

NVFE_INTBC=3

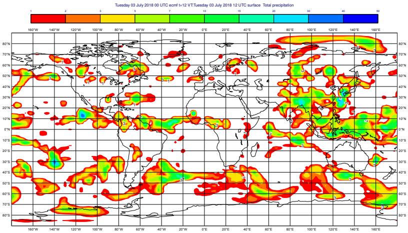
Cooling at the right place.

Ingredients

LVFE_ECMWF=F, NVFE_TYPE=3, LVFE_CENTRI=T, RVFE_ALPHA=0.0,RVFE_BETA=0.5

NVFE_INTBC=3

FD reference



VFE exp