Laboratoire d'accueil: CNRM/CEN (Grenoble) ou Laboratoire Paul Painlevé (Lille)
Titre du stage: Using the second principle of thermodynamics as a numerical stability criterion in snow models
Responsables de stage: Kévin Fourteau (CNRM/CEN), Clément Cancès (Paul Painlevé), Marie Dumont (CNRM/CEN)
Coordonnées des responsables: kevin.fourteau@meteo.fr, clement.cances@inria.fr, marie.dumont@meteo.fr

Sujet du Stage:

Snow is a crucial component of the Earth system, with a significant impact on global, energy and mass budgets[1]. Therefore, accurate and robust modeling of the snow cover is of primary importance for our understanding of past and future climates. The equations used to model the snow cover are coupled mass and energy conservation laws, involving multiple concurrent processes such as heat conduction, phase changes, or liquid water transport. However, the numerical treatment of these coupled equations still poses problems, notably of numerical stability[2].

Specific care is currently given in snow models to ensure that they respect the conservation of energy, i.e. the first principle of thermodynamics. However, little attention was given to the second principle, which states that macroscopic physical systems' evolution in time can only result in the creation of entropy. In parallel, the last decade has seen the development of entropy-based methods in the field of applied mathematics[3]. They are based on the idea that a large family of partial differential equations exhibits a so-called entropy-structure: there exist quantities (analogous to the entropy in physical thermodynamics) that can only increase over time. This entropy-structure ensures the stability of the equations and should therefore be preserved by numerical methods.

The goal of this internship is to use Classical Irreversible Thermodynamics (CIT) to favor physically-consistent and numerically-robust snow models. The internship revolves around two main stages. First, within the framework of CIT, we will check the consistency of the physical equations used in snow models with the second principle of thermodynamics. Then, we will determine which discrete numerical methods (spatial discretization, operator splitting, time-stepping) preserve these entropy-structures and under which conditions.

The candidate is expected to be familiar with PDEs and their numerical discretization (FVM, FEM, time-stepping, etc.) and with equilibrium thermodynamics. If needed, an introduction to CIT will be provided.

The candidate will either be hosted at Centre d'Étude de la Neige (Grenoble, France) or Laboratoire Paul Painlevé (Lille, France). In both cases, regular visits to the other laboratory are planned.

[1] - IPCC: IPCC Special Report on the Ocean and the Cryosphere in a Changing Climate, https://www.ipcc.ch/srocc/, 2019

[2] - Brondex, J., Fourteau, K., Dumont, M., Hagenmuller, P., Calonne, N., Tuzet, F., and Löwe, H.: A finite-element framework to explore the numerical solution of the coupled problem of heat conduction, water vapor diffusion and settlement in dry snow (IvoriFEM v0.1.0), Geosci. Model Dev. Discuss. [preprint], https://doi.org/10.5194/gmd-2023-97, in review, 2023.

[3] - Ahmed Ait Hammou Oulhaj, Clément Cancès, Claire Chainais-Hillairet. Numerical analysis of a nonlinearly stable and positive Control Volume Finite Element scheme for Richards equation with anisotropy. ESAIM: Mathematical Modelling and Numerical Analysis, 2018, 52 (4), pp.1532-1567, 10.1051/m2an/2017012, hal-01372954