AROME Training Course: Advection

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Introduction

- the topic advection is not very specific for NH modeling
- i will try to review the basic ideas behind the advection treatment implemented in the model ALADIN

The content of session - advection

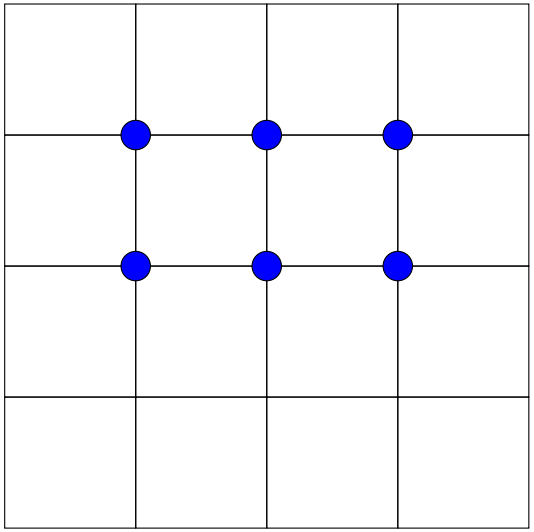
We will speak about:

1. Eulerian approach

2. Lagrangian - semi-Lagrangian approach
   (a) three time level and two time level schemes
   (b) research of trajectories
   (c) NH - specific item - LGWADV scheme
   (d) orographic resonance

Eulerian advection treatment

It is turn on by NAMCT0 - LSLAG=.F. namelist variable.



Lets consider the pure advection problem

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + u\frac{\partial\psi}{\partial x} = 0$$

- Evolution of ψ in time is evaluated in time fixed points as

$$\frac{\partial\psi}{\partial t} = -u\frac{\partial\psi}{\partial x}$$

Eulerian approach - stability criterion

The Eulerian advection is in ALADIN implemented only with three time level (leap-frog) time discretization NAMCT0-LTWOTL=.F.

Leap-frog time discretization gives $\frac{\psi^+ - \psi^-}{2\Delta t} = -u\frac{\partial \psi^t}{\partial x}$

if we consider the propagating wave $\psi = \hat{\psi} e^{ikx}$ and $\hat{\psi}^+ = \lambda \hat{\psi}^t$

then **spectral CFL criterion** of stability is obtained

$$|uk\Delta t| < 1$$

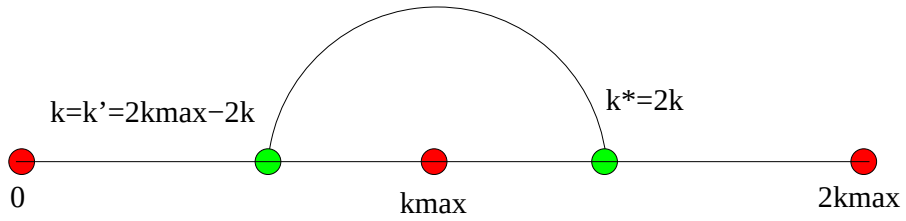
- the most unstable wave signal is the one with the greatest wavenumber k (the shortest wavelength)

- on the quadratic grid with J points and mesh size Δx it is the wave $k_{max} = \frac{2\pi(J-1)}{3J\Delta x} \approx \frac{2\pi}{3}$

$$|u\frac{2\pi}{3\Delta x}\Delta t| < 1 \Rightarrow |\frac{u\Delta t}{\Delta x}| < \frac{3}{2\pi}$$

The shortest wave can not travel more than a half of mesh size.

Eulerian approach - non-linear aliasing



The term $u(x,t)\frac{\partial\psi(x,t)}{\partial x}$ is non-linear and the both signals can be represented as waves:

$$u = \hat{u}e^{ik_1x} \qquad \psi = \hat{\psi}e^{ik_2x} \quad \Rightarrow \quad e^{ik_1x}e^{ik_2}x = e^{i(k_1+k_2)x}$$

The waves with $k^* = (k_1 + k_2) > k_{max} = \frac{2\pi}{2\Delta x}$ are produced by this process. They cannot be recognized on the collocation grid and there are falsely represented as waves $k' = 2k_{max} - k^*$.

No aliasing occurs if all waves are shorter than a wave that is aliased on itself ($k = 2k_{max} - 2k$). This gives $k < \frac{2}{3}k_{max}$ and the condition for quadratic truncation

$$N_{trunc} = \frac{J-1}{3}$$

N_{trunc} in ALADIN are represented by namelist variables NSMAX and NMSMAX.

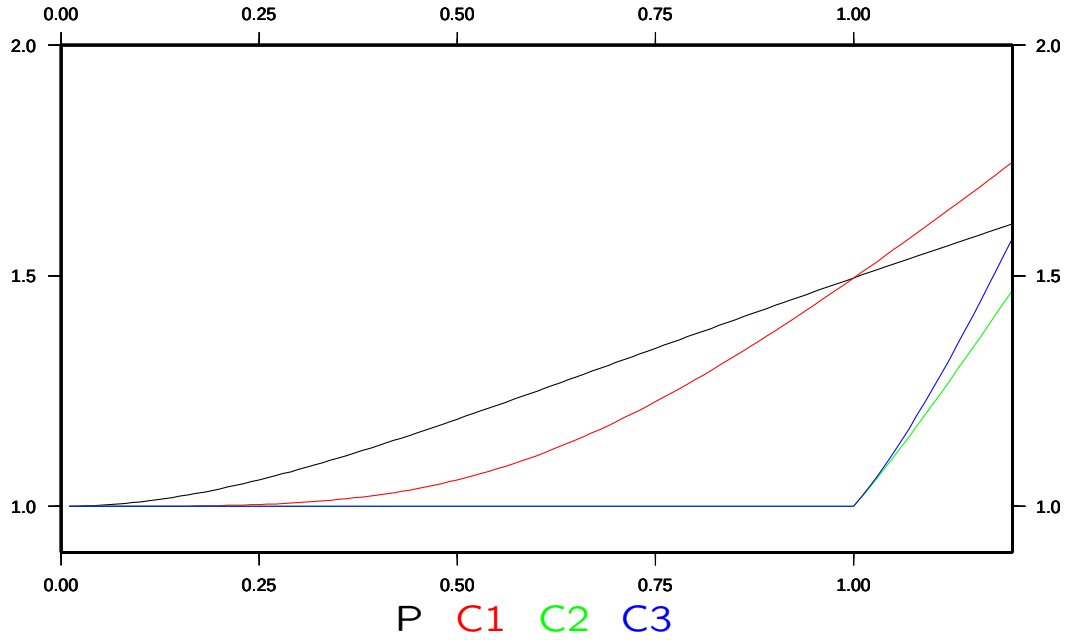
Eulerian approach - the two-time level treatment

The two time level Eulerian schemes are unstable. Here is the example of possible iterative treatment.

P: $\frac{X^{+(0)}-X^t}{2\Delta t} = \bar{U}\frac{\partial X^t}{\partial x}$

C: $\frac{X^{+(n+1)}-X^t}{2\Delta t} = \bar{U}\frac{\partial}{\partial x}\frac{X^{+(n)}+X^t}{2}$

Two iteration necessary to gain stability.



Semi-Lagrangian approach - introduction

Semi-Lagrangian advection is turn on by NAMCT0-LSLAG=.T..

The evolution is now fixed to the moving particle. For our pure advection problem it gives

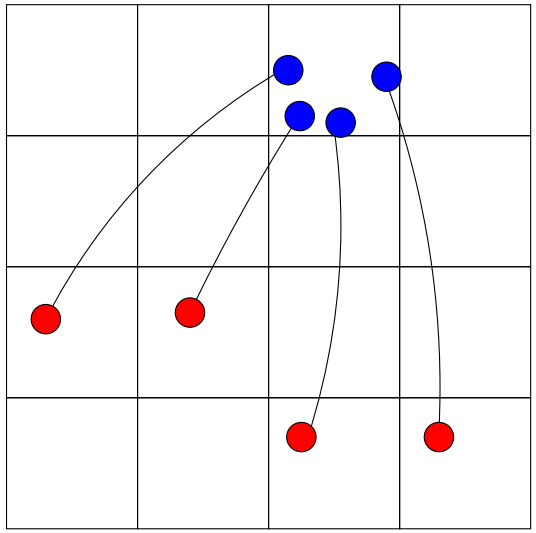
$$\frac{d\psi}{dt} = 0 => \frac{\psi_F^+ - \psi_0^-}{2\Delta t} = 0$$

- Lagrangian approach - evolution of fixed set of particles along their trajectories. It can not be used in spectral model with regular FFT, because the particles are highly irregular.

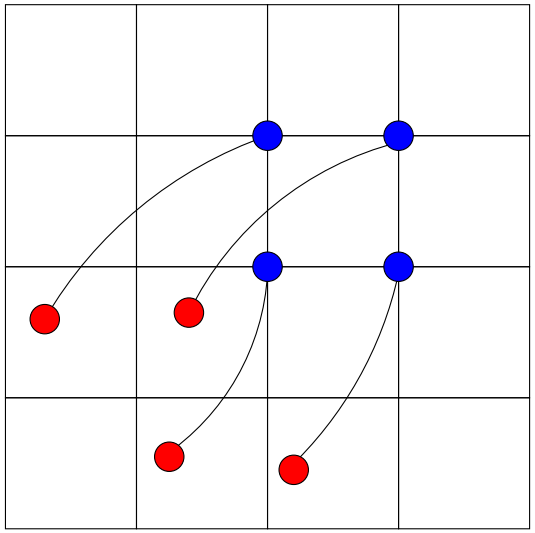
- semi-Lagrangian approach - different set of the particles each time step. Each particle arrives at the points of regular mesh each time step. This allow to use efficient regular FFT.

- semi-Lagrangian approach is unconditionally stable and non-dispersive.

Lagrangian



Semi-Lagrangian



Semi-Lagrangian approach - algorithm

Now consider more general example with non-zero RHS

$$\frac{\psi_F^+ - \psi_0^-}{2\Delta t} = A_O + B_F$$

The semi-Lagrangian scheme is implemented in a following way

1. computation of A and B on regular grid
2. research of trajectories (computation of O point position)
3. interpolation of quantities ψ^- and A into O point
4. evaluation of new state ψ^+ on regular grid as

$$\psi_F^+ = \psi_0^- + 2\Delta t\,(A_O + B_F)$$

Semi-Lagrangian approach - stability

Lipschitz criteria - the trajectories from adjacent points can not intersect

$$|\frac{\partial u}{\partial x}|\Delta t < 1$$

The scheme could run with approx. 3 times longer time steps and it is approx. 2 time more efficient than Eulerian scheme.

No aliasing problem because the non-linear terms are replaced by the interpolations along trajectory. The linear truncation can be used (NSMAX and NMSMAX)

$$N_{trunc} = \frac{J-1}{2}$$

The three time level scheme in the full model

The full non-linear model 3tl time stepping is

$$\frac{X_F^+ - X_0^-}{2\Delta t} = \mathcal{R}(X)_M^t + \mathcal{L}^* \left(\frac{X_F^+ + X_0^-}{2} \right) + P_O^-$$

- the interpolations into middle point M and the origin O point are required. Therefore the space averaging along the trajectory is implemented

$$\mathcal{R}(X)_M^t = \frac{\mathcal{R}(X)_O^t + \mathcal{R}(X)_F^t}{2}$$

- putting implicit terms on the LHS and grouping the origin point terms gives

$$(I - \Delta t \mathcal{L}^*) X_F^+ = \left((I + \Delta t \mathcal{L}^*) X^t + P^- + \Delta t \mathcal{R}(X)^t \right)_O + \Delta t \mathcal{R}(X)_F^t$$

Now only origin point interpolations are required. But ...

The three time level scheme in the full model

Origin point interpolations for each equation could be done in a two different ways according NAMDYN/N[W,T,V,SPD,SVD]LAG parameter.

1. NXLAG=2 - all origin point terms are interpolated together using high order interpolations

$$(I - \Delta t \mathcal{L}^*) X_F^+ = \left((I + \Delta t \mathcal{L}^*) X^t + P^- + \Delta t \mathcal{R}(X)^t \right)_O + \Delta t \mathcal{R}(X)_F^t$$

2. NXLAG=3 - first term is interpolated using high order interpolations and the second one with linear interpolations

$$(I - \Delta t \mathcal{L}^*) X_F^+ = \left(X^t + P^- \right)_O + \left(\Delta t \mathcal{L}^* X^t \Delta t + \mathcal{R}(X)^t \right)_O + \Delta t \mathcal{R}(X)_F^t$$

Key	Variable
NWLAG	horizontal wind
NTLAG	temperature equation
NVLAG	continuity equation
NSPDLAG	pressure departure
NSVDLAG	vertical divergence

The two time level schemes

The two-time level (2TL) scheme is switched on by NAMCT0/LTWOTL=.T..

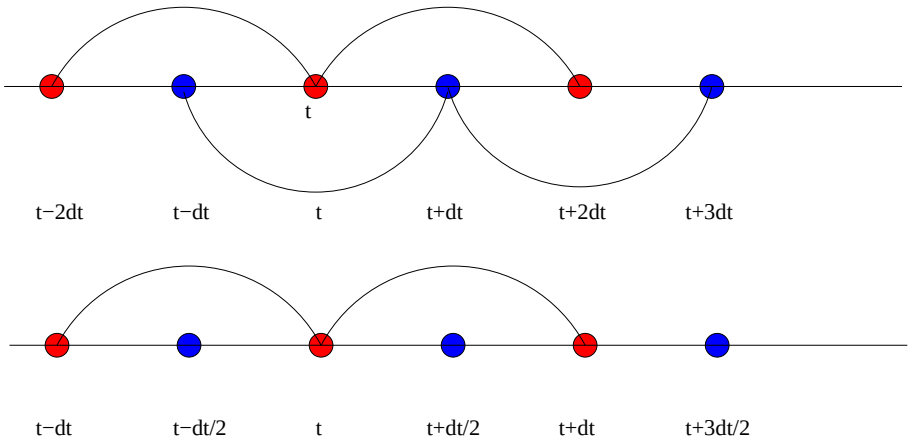
- From point of view of advection terms the three time levels odd and even steps are not decoupled

$$\psi_F^{t+\Delta t} = \psi_O^{t-\Delta t} \qquad \psi^t = \psi^{t-2\Delta t}$$

- it is reasonable to skip one of this "lines" and to use only one with the two times longer time step

$$\Delta t_{2TL} = 2\Delta t_{3TL} \Rightarrow \psi_F^{t+\Delta t} = \psi_O^t$$

- 2tl scheme runs with the two times longer time step and is therefore twice efficient than 3tl scheme



The two time level scheme in the full model

The full non-linear model 2tl time stepping is

$$\frac{X_F^+ - X_0^t}{\Delta t} = \mathcal{R}(X)_M^{t+\frac{\Delta t}{2}} + \mathcal{L}^* \left(\frac{X_F^+ + X_0^t}{2} \right) + P_O^0$$

How to implement the two time levels scheme into non-linear model where the non-linear terms have to be centered to maintain the $O(\Delta t^2)$ accuracy of time discretization ?

1. to extrapolate those terms using at least $O(\Delta t^2)$ method

$$\mathcal{R}(X)_M^{t+\frac{\Delta t}{2}} = \left(\frac{3}{2} \mathcal{R}(X)^t - \frac{1}{2} \mathcal{R}(X)^{t-\Delta t} \right)_M$$

2. (LSETTLS=.T.) to extrapolate the terms with second order method in time and space with uniform acceleration

$$\mathcal{R}(X)_M^{t+\frac{\Delta t}{2}} = \frac{1}{2} \left(2\mathcal{R}(X)^t - \mathcal{R}(X)^{t-\Delta t} \right)_O + \frac{1}{2} \left(\mathcal{R}(X)_F^t \right)$$

3. (LPC_NESC=.T.) to use iterative centered approach (the first guess can be only $O(\Delta t)$. This can be combined only with iterative schemes where $O(\Delta t^2)$ is retained during iterations.

$$P : \mathcal{R}(X)_M^{t+\frac{\Delta t}{2}} = \mathcal{R}(X)_M^t \quad | \quad C : \mathcal{R}(X)_M^{t+\frac{\Delta t}{2}} = \left(\frac{\mathcal{R}(X)_F^{t+\Delta t(*)} + \mathcal{R}(X)_O^t}{2} \right)$$

The two time level scheme in the full model

It was found that the extrapolations of Coriolis terms caused model instability. This instability can be pacified by one of the following methods

1. NAMDYN/LIMPF=.T. The Coriolis force is added to the $\mathcal{L}^*$ operator and is therefore not present in a nonlinear residual $\mathcal{R}$ (not yet implemented in ALADIN).

2. NAMDYN/LADVF=.T. The Coriolis force is expressed as (example for u component)

$$\frac{du}{dt} = \ldots - fv \ldots = \ldots - f\frac{dy}{dt} \ldots$$

the variability of f term in horizontal direction is neglected. Then

$$\frac{du}{dt} + \frac{fy_F^+ - fy_O^t}{\Delta t} = \ldots$$

Trajectory research

- The accuracy of trajectories computation is crucial to the accuracy of SL scheme
- the trajectories shall be at least $O(\Delta t^2)$ accurate
- the research of trajectories is an iterative procedure (NITMP number of iteration during trajectory research)

When searching for trajectory we solve iteratively following equation.

$$\frac{dr}{dt} = \vec{v}$$

in 3TL and in x direction it gives

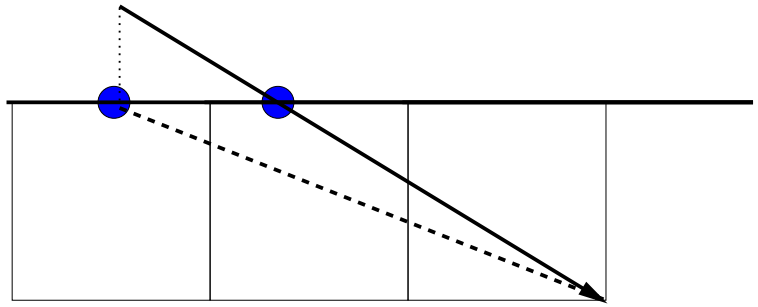
$$OF_s = 2\Delta t u_M^t$$

The computation of trajectories for the 2TL schemes is based on formula

$$OF_s = 2\Delta t u_M^{t+\frac{\Delta t}{2}}$$

and the treatment of $u^{t+\frac{\Delta t}{2}}$ quantity is consistent with the keys $LSETTLS$ and LPC_NESC.

Trajectory research - boundary conditions



The value is evaluated at the projected position of origin point on boundary and the new trajectory is obtained.

The two iterations of trajectory calculations are enough to get reasonably accurate estimate.

Interpolations - the explicit definition of η coordinate

The SL interpolations are performed in (x,y,η) space. The coordinate η is defined implicitly by the relation $\pi = A(\eta) + B(\eta)\pi_s$, and for the interpolation process has to be defined explicitly. The two methods are available

1. LREGETA=.T. - regular definition of η on $l-th$ half-level (L is number of vertical levels)

$$\eta(l) = \frac{l}{L}$$

2. LREGETA=.F. - regular definition of η on $l-th$ level (L is number of vertical levels)

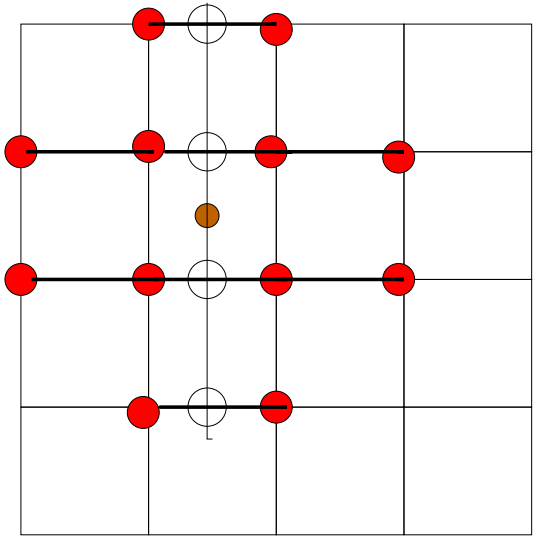
$$\eta(l) = \frac{A_l}{p_{00}} + B_l$$

with p_{00} being chosen constant.

Interpolations

- have a important impact on the acurracy and efficiency
- linear interpolation are very cheap (13 operations) but too diffusive
- Lagrange cubic interpolations (polynomials of 3-th order) are enough precise but more expensive (37 operations)

Interpolations - efficient algorithm



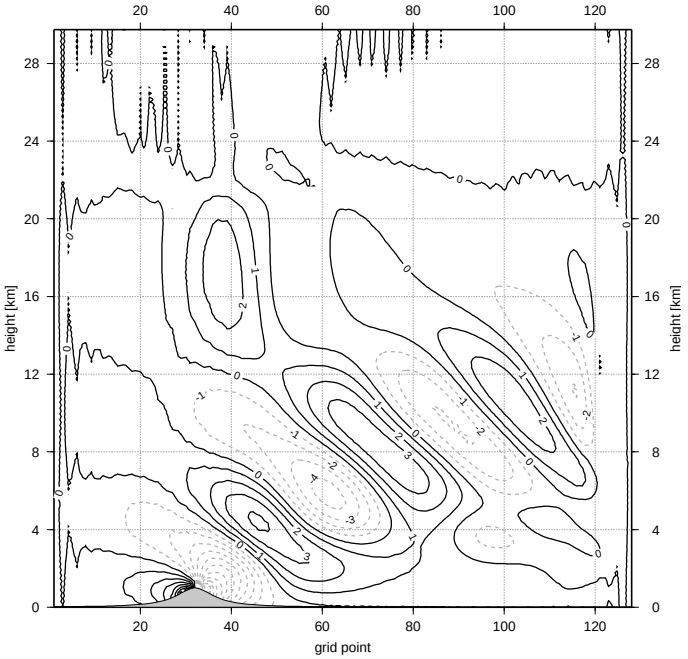
The trick is to combine the linear and cubic interpolations.

- in 2D:16 points involved and 5 cubic interpolations $\Rightarrow$ 12 points involved and 3 cubic and 2 linear interpolations
- in 3D:64 points involved and 21 cubic interpolations $\Rightarrow$ 32 points involved and 7 cubic and 10 linear interpolations
- shape preserving version of cubic interpolation is availaible (LQMW, LQMT, LQMSPD, LQMSVD ...)

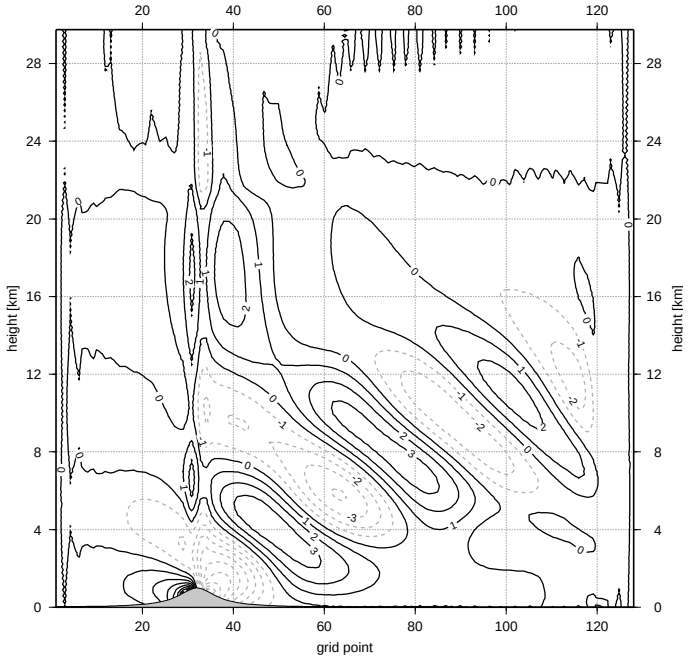
NH specificities - advection of w

The scheme was proposed by Smith and Brozkova to cure the problems of artificial "chimneys" over orography.

$(LGWADV = T)$



$(LGWADV = F)$



NH specificities - advection of w

The main idea of LGWADV scheme is that the grid point calculations are performed with the prognostic variable w and the spectral computations are performed with prognostic variable d_4.

We can not use the prognostic variable w inside spectral calculation because such model would be unstable.

The transformations are performed between:

- the states at time instant t at the beginning of grid-point calculations

$$d_4{}^t \Rightarrow w^t$$

- then the explicit evolution of the w is computed (Y is vector of variables with w)

$$\tilde{Y}_F^{t+\Delta t(0)} = Y_O^t + \Delta t \mathcal{M}_w(Y)_M^{t+\frac{\Delta t}{2}}$$

- the explicit guess of $w^{t+\Delta t(0)}$ is then transformed back

$$w^{t+\Delta t(0)} \Rightarrow d_4{}^{t+\Delta t(0)}$$

and then relevant terms are added to the explicit guess to complete the RHS which is then transformed into spectral space.

NH specificities - advection of w

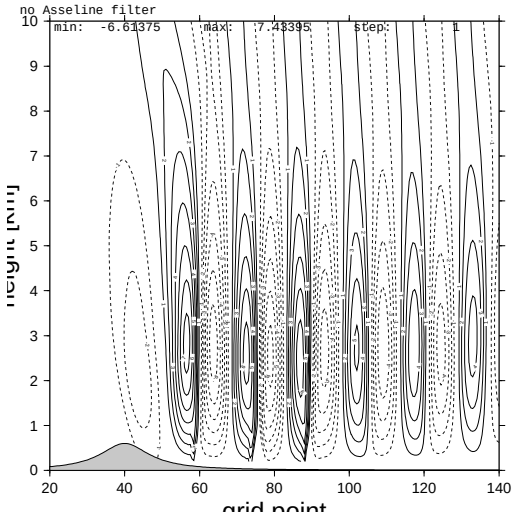
The LGWADV scheme has following consequences on SL scheme

- it can be implemented only as 2TL ICI scheme with non-extrapolating approach (LPC_NESC=T). In other cases additional interpolations of separated terms would be required.

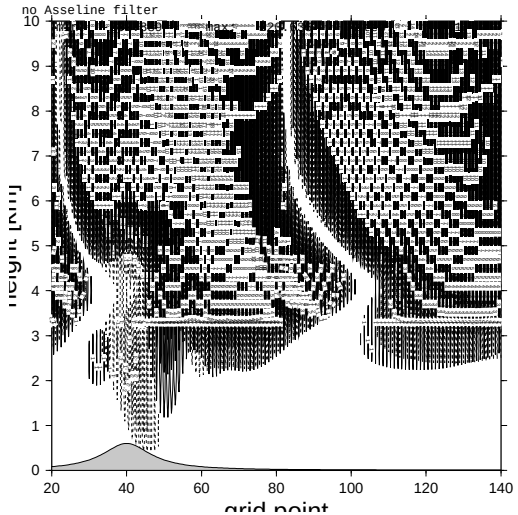
- the w is half level prognostic variable and the new set of trajectories must be computed (all other prog. variables are on full levels)

LGWADV scheme has been found to be more stable in the idealized simulations than the traditional scheme at very high resolutions.

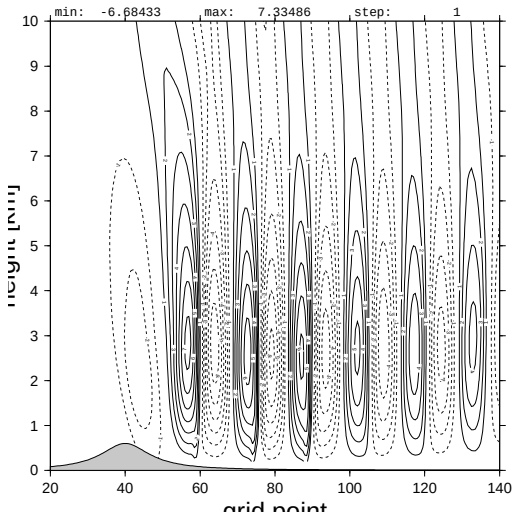
$(LGWADV = T)$



$(LGWADV = F)$



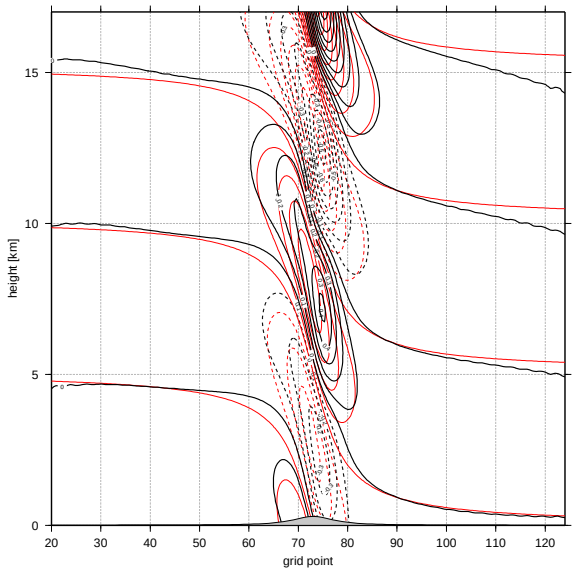
$(LGWADV = F + HD)$



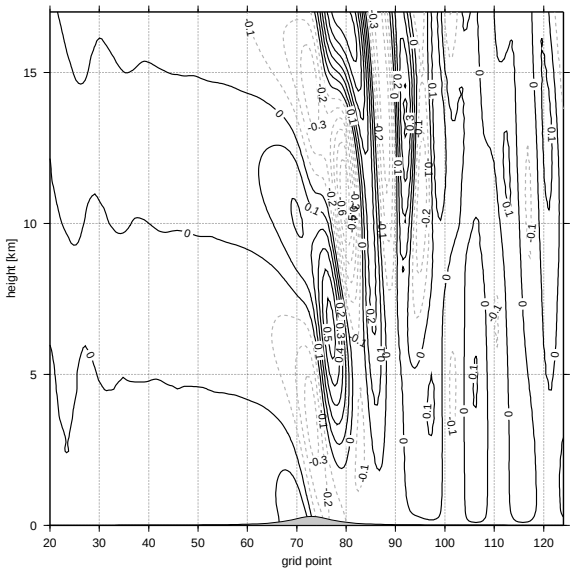
Orographic resonance

When the time step in SL scheme is increased beyond some critical value , the flow over orography gives the wrong stationary solution.

($CFL = 0.3$)



($CFL = 3$)



There are two cures available in model ALADIN of this problem:

1. Tanguay-Ritchie correction ($RCMSLP = 1.0$)

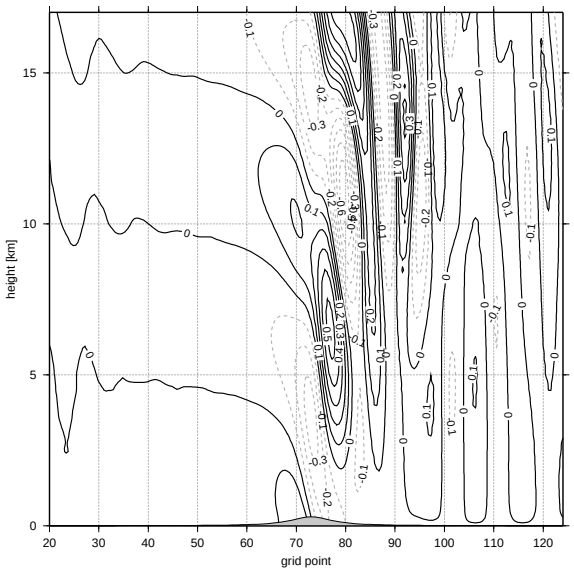
2. first-order decentering ($VESL > 0$)

Orographic resonance - Tanguay-Ritchie correction

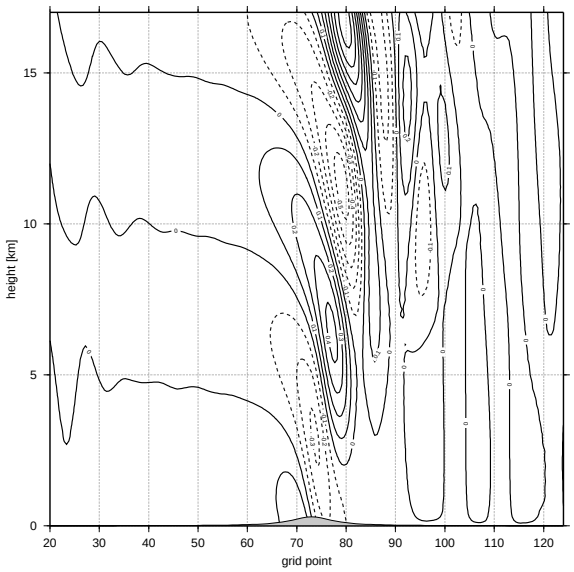
When RCMSLP=1.0 (δ_{TR}) is set, then the new prognostic variables are used that is less sensitive to the orography. The second term in the prognostic equation is evaluated in Eulerian way and averaged along trajectory.

$$\bar{q_s} = log(\pi_s) + \delta_{TR}\frac{\phi_s}{R_d T_{st}} \qquad \bar{T} = T + \delta_{TR}\frac{\phi_s}{R_d T_{st}}$$

$(RCMSLP = 0)$



$(RCMSLP = 1)$



Orographic resonance - first-order decentering

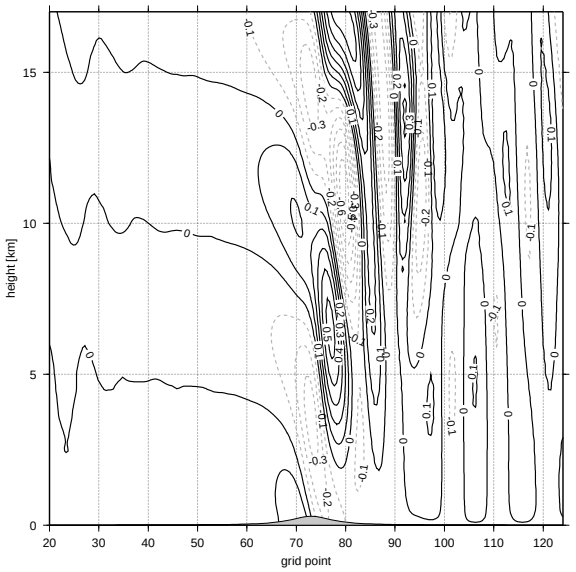
Each term is SL discretization which is averaged between the final and origin point is adjusted as

$$\frac{X_F + X_O}{2} \Rightarrow \frac{(1+\alpha)X_F + (1-\alpha)X_O}{2}$$

Decentering makes the scheme more implicit and

- makes the scheme only $O(\Delta t)$
- damps the waves in the whole spectra, not only those responsible for resonance

$(VESL = 0)$



$(VESL = 0.1)$