Part VI
Deep convection

1 Generalities

The model *large scale equations* yield the evolution of the *large scale model variables*, which are values at discrete grid points, representing an average value over 1 grid mesh. Subgrid effects are hidden into the so-called "turbulent stress". All parameterizations are intended to represent the effects of subgrid phenomena on the large scale model variables.

The occurrence of *deep convection* (i.e. the convection producing precipitation), affects mainly the vertical component of the turbulent flux.
Some models distinguish a *deep* convection associated to large scale moisture convergence, a *shallow* convection associated to local evaporation alone, and a *mid-level* convection, which originates above the boundary layer; in ARPÈGE-ALADIN's frame, the shallow convection is treated as a correction in turbulent fluxes parameterization, while the "deep" convection needs large scale moisture convergence, and produces subgrid precipitation; it also includes the mid-level convection.

We must emphasize that what is handled by the parameterization is actually the subgrid effect of convective processes; the vertical movements induced by *fronts* have scales big enough to be resolved by the model grid, and the associated precipitation is handled by the large-scale precipitation scheme. But also in convective situations, part of the vertical movements are perceived at the large scale, and handled by the large scale precipitation scheme. So the so-called "deep convection" parameterisation scheme is only concerned with what the large scale cannot distinguish.
As stated by KUO [1965, 1974], wherever the conditions for deep convection are fulfilled, the large scale equations cannot describe correctly the evolution of heating and moisture, as the release of latent heat is accomplished by the vertical motion associated with the convective clouds (much smaller than the grid box), and not by the vertical component of the large scale wind. Cumulus clouds serve collectively as a heat source for the mean flow field, in addition to their function as agents of diffusion. The deep convection scheme has to estimate the part of the large scale heating Q_1 and of the moisture sink $-Q_2$ associated to the subgrid motions; the convective precipitation corresponds to the vertical integral of the moisture sink. Besides, the convective mass flux induces a vertical redistribution of heat, moisture, and momentum.

Cloudiness is treated separately from the convective scheme: "deep convection cloudiness" will be derived from the convective precipitation flux, while shallow convection cloudiness is diagnosed at the same time as large scale cloudiness.
Figure 5 presents the main characteristics of a single cumulus cloud. Main active elements are the updraught and the downdraught. Air is entrained from the environment, and cloudy air is also detrained to the environment. The updraught activity is fed by the large scale convergence and also by the local evaporation. The upper part of the cloud is made of ice, the lower part of liquid water. Evaporation of falling precipitation cools the environment, which can spawn a downdraught motion. The cooler air induces a "local high" (pressure) near the ground, which causes a wake of cool air and a gust front.

These clouds are often organized in cloud systems, but our parameterization will use a bulk formulation, replacing the subgrid turbulent structure by a single updraught and a single downdraught.

- The updraught is computed first, as resulting from the occurrence of both moisture convergence and positive buoyancy.
- Saturation spawns a cloud condensate, which converts into precipitation when the saturated layer reaches a critical thickness.
- The closure of the subgrid mass budget may require a local compensating subsidence, but we'll see another vertical advection term abusively called "compensating subsidence", appearing in the equations following the choice of the referential. This term implies anyway a vertical reorganisation of the (variables describing the) immediate (subgrid) environment of the updraught.
- This affects the moisture budget and can result in a vertical convergence of the precipitation flux, representing the evaporation of falling precipitation.

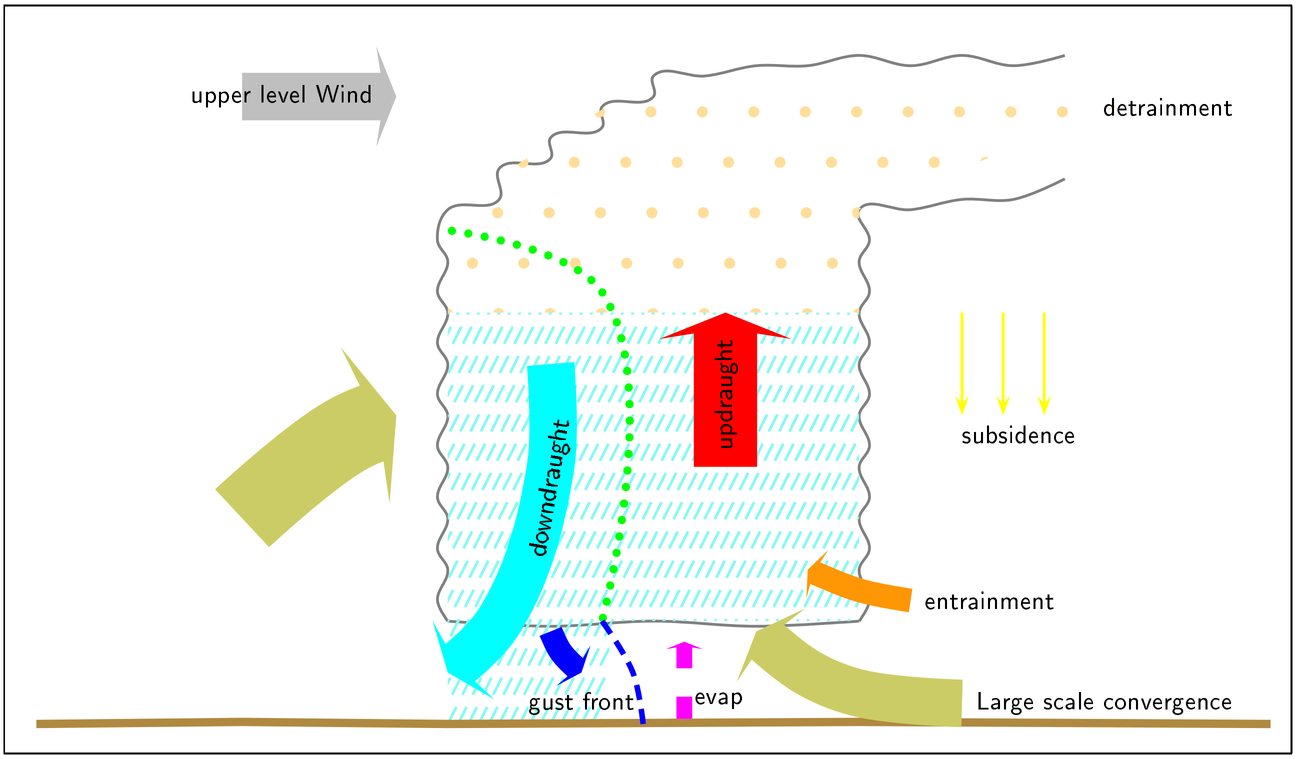


Figure 5: Schematic representation of a single cumulus cloud

- The downdraught scheme is computed separately, as resulting from the occurrence of both vertical convergence of the precipitation flux and negative buoyancy. The latter results actually of precipitation evaporation cooling the mid-level layers.

Separation of phenomena drives us to express a partial tendency for the convective effect. We write conventionally the large scale equations (i.e. prognostic equations for the large scale model variables, and dropping the viscous stress terms) using the hydrostatic pressure as vertical coordinate. For this paragraph only, we note the large scale values with a bar, reminding they represent a mean value over 1 grid mesh, as:

$$\frac{d\overline{\psi}}{dt} = (\text{Source}) - (\text{Turbulent Stress})$$

or (using the notation Q_1, Q_2 introduced by Yanai et al. [1973]):

- Large scale apparent heating Q_1 (K/day):

$$c_p \cdot Q_1 = \frac{\partial \overline{s}}{\partial t} + \overline{\mathbf{V}} \cdot \nabla \overline{s} + \overline{\omega} \frac{\partial \overline{s}}{\partial p} = L \cdot (\mathcal{C} - \mathcal{E}) + c_p \cdot Q_R - \frac{\partial \overline{\omega' s'}}{\partial p} - \left(\frac{\partial \overline{u' s'}}{\partial x} + \frac{\partial \overline{v' s'}}{\partial y} \right) \tag{1}$$

- Large scale apparent moisture sink $-Q_2$ (K/day) (i.e. cooling / heating brought by moisture variations):

$$\frac{-c_p \cdot Q_2}{L} = \frac{\partial \overline{q}}{\partial t} + \overline{\mathbf{V}} \cdot \nabla \overline{q} + \overline{\omega} \frac{\partial \overline{q}}{\partial p} = -(\mathcal{C} - \mathcal{E}) - \frac{\partial \overline{\omega' q'}}{\partial p} - \left(\frac{\partial \overline{u' q'}}{\partial x} + \frac{\partial \overline{v' q'}}{\partial y} \right) \tag{2}$$

  In addition, we may consider the

- Horizontal momentum equation

$$\frac{\partial \overline{\mathbf{V}}}{\partial t} + \overline{\mathbf{V}} \cdot \nabla \overline{\mathbf{V}} + \overline{\omega} \frac{\partial \overline{\mathbf{V}}}{\partial p} - f\, \mathbf{k} \wedge \overline{\mathbf{V}} = -\nabla \overline{\phi} - \frac{\partial \overline{\omega' \mathbf{V}'}}{\partial p} - \left(\frac{\partial \overline{u' \mathbf{V}'}}{\partial x} + \frac{\partial \overline{v' \mathbf{V}'}}{\partial y} \right) \tag{3}$$

$\mathcal{C} - \mathcal{E}$ is the net condensation-evaporation rate, Q_R is the radiative local heating (K/day): these source terms can be separated between the part occurring at large scale, and the part occurring at subgrid scale.
So we express that the heating of a (3D) mesh-scale material parcel is due to

- large-scale and subgrid-scale phase transitions,
- large-scale and subgrid radiative heating and
- subgrid reorganization effects (divergence of the eddy transport of sensible heat).

The apparent moisture sink measured by Q_2 is due to the net condensation and the divergence of the eddy transport of moisture. A complete resolution of the parameterization problem would be to derive expressions for the subgrid part of $\mathcal{C}-\mathcal{E}$ and Q_R, and the turbulent fluxes: as this is hardly thinkable, we opt instead for a bulk description of the resulting large-scale effects, seeking an expression for $Q_1 - Q_R$ and $-Q_2$, accounting for the effects of both deep convection and turbulent transports; and we want to determine Q_1 and $-Q_2$ from the large-scale environment.
Outside the regions of deep convection the convective part of Q_1 and $-Q_2$ is zero, while the turbulent processes still give an input.

The set of large scale equations may be rewritten in the form:

$$\frac{\partial\overline{\psi}}{\partial t} = -\overline{\mathbf{V}}\nabla\overline{\psi} - \overline{\omega}\frac{\partial\overline{\psi}}{\partial p} - \frac{\partial\overline{\omega'\psi'}}{\partial p} + S_\psi \tag{4}$$

where ψ stands for s, q, or $h = s + Lq$ and $\mathbf{V}$. The vertical flux $\overline{\omega'\psi'}$ represents all subgrid scale transports (linked to either micro-scale, cloud-scale, or meso-scale circulations) and S_ψ the source/sink terms (radiation contribution for thermodynamic variables, geopotential gradient for the momentum equation). We drop here the horizontal turbulent stresses and the Coriolis term as they do not affect directly the convective process.

Equation (4) can be considered as:

$$\frac{\partial\overline{\psi}}{\partial t} = \left(\frac{\partial\overline{\psi}}{\partial t}\right)_{LS} + S'_\psi + \left(\frac{\partial\overline{\psi}}{\partial t}\right)_{\mathrm{subgrid}} \tag{5}$$

The subscript $_{LS}$ denotes the contribution to the time derivative by the large-scale advective process and $_{\mathrm{subgrid}}$ for those having a space scale smaller than the grid size. S'_ψ represents the source terms treated outside the convection parameterisation scheme (the large scale parts). The subgrid tendency becomes then:

$$\left(\frac{\partial\overline{\psi}}{\partial t}\right)_{\mathrm{subgrid}} = \mathrm{source} - \frac{\partial\overline{\omega'\psi'}}{\partial p} = \left(\frac{\partial\overline{\psi}}{\partial t}\right)_{\mathrm{conv}} + \left(\frac{\partial\overline{\psi}}{\partial t}\right)_{\mathrm{vert\_diff}} \tag{6}$$

1.1 Mass flux approach

The approach performs an average over the "cloudy" (i.e. convectively active) and "no-cloudy" (inactive) area of the grid point and may allow some approximations as the active cloud covers a fractional area much smaller than one.
The *mass flux approach* partitions the flux between contributions from convective updraughts (subscript $_u$), convective downdraughts (subscript $_d$), and the environment (subscript $_e$):

$$\overline{\psi'\omega'} = \sigma_u\ \overline{\psi'\omega'}\big|_u + \sigma_d\ \overline{\psi'\omega'}\big|_d + \sigma_e\ \overline{\psi'\omega'}\big|_e \tag{7}$$

where ψ stands for $\mathbf{V}$, s, or q and the σ represent the fractional areas covered by convective up- and downdraughts, and the environment.

$$\psi = \overline{\psi} + \psi' \tag{8}$$

where $\overline{\psi}$ represents the average value over the grid box, and ψ' the fluctuations from it. We have:

$$\begin{aligned}\overline{\psi} &= \sigma_u\cdot\psi_u + \sigma_d\cdot\psi_d + (1-\sigma_u-\sigma_d)\cdot\psi_e \\ \overline{\omega} &= \sigma_u\cdot\omega_u + \sigma_d\cdot\omega_d + (1-\sigma_u-\sigma_d)\cdot\omega_e\end{aligned} \tag{9}$$

hence

$$\begin{aligned}\overline{\omega'\psi'} &= \overline{\omega\cdot\psi} - \overline{\omega}\cdot\overline{\psi} \\ &= \sigma_u\cdot\omega_u\psi_u + \sigma_d\cdot\omega_d\psi_d + (1-\sigma_u-\sigma_d)\cdot\omega_e\psi_e - (\sigma_u\cdot\omega_u + \sigma_d\cdot\omega_d + (1-\sigma_u-\sigma_d)\cdot\omega_e)\cdot\overline{\psi} \\ &= \sigma_u\cdot\omega_u\cdot\left(\psi_u - \overline{\psi}\right) + \sigma_d\cdot\omega_d\cdot\left(\psi_d - \overline{\psi}\right)\end{aligned}$$

$$
\begin{aligned}
& +\omega_e \left\{-\sigma_u \left(\psi_u - \psi_e\right) - \sigma_d \left(\psi_d - \psi_e\right) - \sigma_u \left(\psi_e - \overline{\psi}\right) - \sigma_d \left(\psi_e - \overline{\psi}\right)\right\} \\
= \quad & \sigma_u \cdot (\omega_u - \omega_e) \cdot \left(\psi_u - \overline{\psi}\right) + \sigma_d \cdot (\omega_d - \omega_e) \cdot \left(\psi_d - \overline{\psi}\right) && (10) \\
= \quad & \sigma_u \cdot (\omega_u - \overline{\omega}) \cdot (\psi_u - \psi_e) + \sigma_d \cdot (\omega_d - \overline{\omega}) \cdot (\psi_d - \psi_e) && (11)
\end{aligned}
$$

The last equality results from the symmetry between ψ and ω in the calculation.
We define

$$\overset{\wedge}{\omega^*} \equiv \sigma_u \cdot (\omega_u - \omega_e) \qquad , \qquad \overset{\vee}{\omega^*} \equiv \sigma_d \cdot (\omega_d - \omega_e) \tag{12}$$

the relative up- and downdraught mass fluxes with respect to the environment.
The absolute draught mass fluxes (by the absolute cloud velocity),

$$
\begin{aligned}
M_u \equiv -\sigma_u \cdot \omega_u = -(\overset{\wedge}{\omega^*} + \sigma_u \omega_e) \quad & \Longleftrightarrow \quad \overset{\wedge}{\omega^*} = -(M_u + \sigma_u \omega_e) \\
M_d \equiv \sigma_d \cdot \omega_d = (\overset{\vee}{\omega^*} + \sigma_d \omega_e) \quad & \Longleftrightarrow \quad \overset{\vee}{\omega^*} = (M_d - \sigma_d \omega_e)
\end{aligned} \tag{13}
$$

Following assumption is generally applied

$$-\overset{\wedge}{\omega^*} \sim M_u \equiv -\sigma_u \cdot \omega_u \qquad \text{and} \qquad \overset{\vee}{\omega^*} \sim M_d \equiv \sigma_d \cdot \omega_d \tag{14}$$

This will be realized if

- $\sigma_u \ll 1$: remember that σ_u represents only the fraction of the grid box occupied by the updraught. We have then

  $$\sigma_u \ll 1 \Longrightarrow \omega_u \gg \omega_e$$

  Various articles, like Asai and Kasahara [1967] propose to estimate an affordable section for the updraught by considering complete Bénard-like cells, considering that space is required between the updraughts for compensating subsidence. For such a situation, they obtain that the updraught should not pass over 15% of the section of the cloud. Such a view is rather academic: in our case, most of the compensating subsidence could occur outside the grid box containing the cloud system we try to simulate, because if it was within the box, we should not observe the large scale convergence that we put as a necessary condition for convective activity. Practically, with large grid boxes, there could be significant compensating subsidence within the box, but this part would then be hidden in the parameterization; on the other hand, with grid boxes of a few kilometers, cloud systems could extend over several grid boxes, and the active mesh fraction would not need to be small.

- $\omega_e \sim 0$ could be less restrictive. The large scale vertical velocity is related to the large scale convergence, which is supposed here to feed the convective process: so the average large scale upward motion should come essentially from a larger updraught velocity over the small updraught mesh fraction σ_u. Note that actually, the *local surface fluxes* also feed the updraught (see §1.2), so that this balance could not be perfect.
  Outside the updraught, only the compensating subsidence could produce a non-zero subgrid environmental vertical velocity. In the case both an updraught and a downdraught are present, the downdraught's compensating ascent acts opposite to the compensating subsidence, reducing the net effect, and the so-called compensating vertical fluxes could be seen as mere tools to close the subgrid budgets, with no net impact on the large scale.

Bougeault [1985]'s experiments with a single updraught showed already similar profiles for his variables $\omega^* = \sigma_u(\omega_u - \omega_e)$ (representing in his terminology the "net vertical ascent occurring at subgrid scale") and the large scale vertical velocity $\overline{\omega} = \omega^* + \omega_e$, the first being nevertheless slightly bigger, suggesting the need of some correction by environment compensating subsidence (but his scheme did not include a downdraught). Anyway it is important to maintain a clear distinction between the fluxes in the reasoning.
The final expression is thus

$$\overline{\psi'\omega'} = \overset{\wedge}{\omega^*} \left(\psi_u - \overline{\psi}\right) + \overset{\vee}{\omega^*} \left(\psi_d - \overline{\psi}\right) \tag{15}$$

In the general equations we must express the turbulent constraints as

$$-\frac{\partial \overline{\psi'\omega'}}{\partial p} = -\frac{\partial \overset{\wedge}{\omega^*}(\psi_u - \overline{\psi})}{\partial p} - \frac{\partial \overset{\vee}{\omega^*}(\psi_d - \overline{\psi})}{\partial p} \tag{16}$$

1.2 Formulation

BOUGEAULT [1985] uses the following guidelines:

- Most of the vertical mass ascent needed to satisfy the large-scale mass budget is not visible at the large scale but only at the sub-grid scale, where
  - the ascent cooling effect is widely balanced by the condensation heat release
  - the moistening is widely balanced by the rain fallout

  Similarly, we'll suppose that the downdraught moisture and energy budgets are closed without mixing with the environment: the evaporation of precipitation is the only source of q and s in the downdraught and is totally devoted to it.

- When deep convection occurs, the large scale values of s and q are also modified by mixing of cloudy air with the environment, which is called **detrainment**. For this, Kuo's scheme uses a relaxation of the large-scale variables towards a single cloud profile (s_c, q_c) with a time constant independent of the altitude (practical implementation will refine this scheme, introducing enhanced detrainment at the cloud top).

- As soon as deep convection occurs, the validity of a separate treatment of vertical turbulent diffusion becomes doubtful: a simple way to compensate for the contribution of the turbulent diffusion scheme added in a subsequent computation step, is to subtract the turbulent tendencies in the expression of the convective tendencies. When closing the moisture budget, this requires to subtract the vertical divergence of turbulent moisture flux (and similarly for the associated dry static energy) from the total convective tendencies. In addition (see §6.1.1), it has been proposed to add the effect of the turbulent moisture flux to the large scale moisture convergence feeding the convective process, which reflects the fact that *large scale moisture advection alone is not always the main moisture provider to the updraught: local surface evaporation – expressed in this turbulent vertical flux – sometimes brings a substantial contribution.*

Considering the mixing of the cloud air with the environment, through an entrainment rate $E \geq 0$ and a detrainment rate $D \geq 0$, we can write cloud-scale budgets, with the hypotheses that $\psi_e = \overline{\psi}$, $\sigma_u, \sigma_d \ll 1$, and $\frac{\partial \sigma_u \psi_u}{\partial t} \equiv 0$ as we neglect the contributions of the cloud variables tendencies (stationarity of cloud properties over the time step).
For the updraught :

$$\begin{aligned}
\frac{\partial M_u}{\partial p} &= D_u - E_u \\
\frac{\partial M_u s_u}{\partial p} &= D_u s_u - E_u \overline{s} - Lc \\
\frac{\partial M_u q_u}{\partial p} &= D_u q_u - E_u \overline{q} + c \\
\frac{\partial M_u \mathbf{V}_u}{\partial p} &= D_u \mathbf{V}_u - E_u \overline{\mathbf{V}}
\end{aligned} \tag{17}$$

where c is in-cloud condensation rate (there is no evaporation inside the cloud).

For the downdraught, we have, similarly:

$$\begin{aligned}
\frac{\partial M_d}{\partial p} &= E_d - D_d \\
\frac{\partial M_d s_d}{\partial p} &= E_d \overline{s} - D_d s_d - L\,e \\
\frac{\partial M_d q_d}{\partial p} &= E_d \overline{q} - D_d q_d + e \\
\frac{\partial M_d \mathbf{V}_d}{\partial p} &= E_d \overline{\mathbf{V}} - D_d \mathbf{V}_d
\end{aligned} \tag{18}$$

where this time the source term e is the evaporation (measured through the convergence of the precipitation flux), and there is no condensation as the downdraught no longer works when saturated.

With (17), (18) we can derive:

$$\begin{aligned}
\frac{\partial M_u (s_u - \overline{s})}{\partial p} &= D_u (s_u - \overline{s}) - M_u \frac{\partial \overline{s}}{\partial p} - Lc \\
\frac{\partial M_d (s_d - \overline{s})}{\partial p} &= -D_d (s_d - \overline{s}) - M_d \frac{\partial \overline{s}}{\partial p} - L\,e
\end{aligned}$$

and similar relations for q and $\mathbf{V}$, hence the general form

$$
\begin{aligned}
-\frac{\partial \overset{\wedge}{\omega^*}\left(\psi_u-\overline{\psi}\right)}{\partial p} &= D_u(\psi_u-\overline{\psi})+\overset{\wedge}{\omega^*}\frac{\partial\overline{\psi}}{\partial p}-\text{source}\\
\frac{\partial \overset{\vee}{\omega^*}\left(\psi_d-\overline{\psi}\right)}{\partial p} &= -D_d(\psi_d-\overline{\psi})-\overset{\vee}{\omega^*}\frac{\partial\overline{\psi}}{\partial p}-\text{source}
\end{aligned}
$$

Combining with (16) and (6), we obtain as final expression, applying to regions of deep convection:

$$
\left(\frac{\partial\overline{\psi}}{\partial t}\right)_{\text{conv}} = \underbrace{\overset{\wedge}{\omega^*}\frac{\partial\overline{\psi}}{\partial p}}_{\text{pseudo subs.}} + \underbrace{K_u(\psi_u-\overline{\psi})}_{\text{Detrainment}} + \underbrace{\overset{\vee}{\omega^*}\frac{\partial\overline{\psi}}{\partial p}}_{\text{pseudo asc.}} + \underbrace{K_d(\psi_d-\overline{\psi})}_{\text{Detrainment}} + \underbrace{g\frac{\partial J_\psi}{\partial p}}_{\text{turb. vert. diffusion}} \tag{19}
$$

where we noted the detrainment rates D as Kuo-type detrainment coefficients K_u and K_d.
The last term of the RHS is the subtraction of the separate vertical diffusion scheme contribution, following Bougeault's third guideline.
This equation expresses the part of the tendency of the large scale model variables due to convection, and the different terms can receive an external interpretation, as indicated: pseudo subsidence and ascent, and detrainment of cloudy air which modifies the environment by mixing to it.

We could summarize the development as follows:

- Outer side: We try to express here the environmental effects of the convection. Those are
  - A *pseudo downward advection* for the updraught (often abusively called *compensating subsidence*) and pseudo-upward advection for the downdraught (abusively called *compensating ascent*). Those two terms are linked to the fact that large scale variables have the large scale vertical velocity, while the draught actual environment has vertical velocity ω_e:

    $$\overline{\omega}=\sigma_u\omega_u+\sigma_d\omega_d+\sigma_e\omega_e \quad\Longrightarrow\quad \overset{\wedge}{\omega^*}+\overset{\vee}{\omega^*}=\overline{\omega}-\sigma_e\omega_e$$

    and we see these pseudo-advection terms by the two draughts fluxes combine in a relative advection of the large scale variable by the difference between large scale vertical velocity and the subgrid average environment velocity.
    No environmental vertical motion effect of this term is experienced at large scale.
  - When deep convection occurs, the large scale values are affected via the *detrainment* process.
  - We suppose that the convective scheme includes the vertical turbulent diffusion effects in the case of convection, so we have to subtract the contribution of the separate diffusion scheme that would be redundant and alter the results.
- Inner side: The subgrid behaviour will be replaced by a single updraught and a single downdraught. This is the part concerned by the parameterization, which must take into account:
  - The source of the convective activity: *moisture convergence and positive buoyancy* for the updraught, and *evaporation of the precipitation and negative buoyancy* for the downdraught.
  - The effects of *entrainment* of environmental air into the up/downdraughts.
  - The effects of the pressure gradient on momentum, due to the fact that the pressures into the updraught, the downdraught, and the environment may be different.

Let's rewrite equation (19) for the actual model variables, and compare with the case with no convection.
In the convective region, we have to add convection's contribution :

$$
\begin{aligned}
c_p\cdot Q_1^{\text{cu}}=\mathcal{C}\cdot L-\frac{\partial\overline{\omega' s'}}{\partial p} &= \overset{\wedge}{\omega^*}\frac{\partial\overline{s}}{\partial p}+K_u\left(s_u-\overline{s}\right)+\overset{\vee}{\omega^*}\frac{\partial\overline{s}}{\partial p}+K_d\left(s_d-\overline{s}\right)+g\frac{\partial J_s}{\partial p} && (20)\\
-\frac{c_p}{L}\cdot Q_2^{\text{cu}}=-\mathcal{C}-\frac{\partial\overline{\omega' q'}}{\partial p} &= \overset{\wedge}{\omega^*}\frac{\partial\overline{q}}{\partial p}+K_u\left(q_u-\overline{q}\right)+\overset{\vee}{\omega^*}\frac{\partial\overline{q}}{\partial p}+K_d\left(q_d-\overline{q}\right)+g\frac{\partial J_q}{\partial p} && (21)\\
Q_3=\left(\frac{\partial\overline{\mathbf{V}}}{\partial t}\right)^{\text{cu}} &= \overset{\wedge}{\omega^*}\frac{\partial\overline{\mathbf{V}}}{\partial p}+K_u\left(\mathbf{V}_u-\overline{\mathbf{V}}\right)+\overset{\vee}{\omega^*}\frac{\partial\overline{\mathbf{V}}}{\partial p}+K_d\left(\mathbf{V}_d-\overline{\mathbf{V}}\right) && (22)
\end{aligned}
$$

To the turbulent fluxes, found everywhere:

$$c_p \cdot Q_1^{\rm diff} = c_p \cdot (Q_1 - Q_R) = -g\frac{\partial J_s}{\partial p} \tag{23}$$

$$-\frac{c_p}{L} \cdot Q_2^{\rm diff} = -g\frac{\partial J_q}{\partial p} \tag{24}$$

$$Q_3^{\rm diff} = -g\frac{\partial J_{\mathbf{V}}}{\partial p} \tag{25}$$

In the case of momentum, it doesn't seem wise to introduce a turbulent vertical diffusion term (see §5.2): unlike s and q, momentum has no active contribution to convection itself, while Q_3 (22) acts as a pure redistribution of horizontal momentum. So we let turbulent diffusion of horizontal momentum act everywhere through the turbulent diffusion scheme.

To go farther, we need expressions for the parameters K_u and K_d, for the cloudy mass fluxes $\hat{\omega}^*$, $\check{\omega}^*$, and for the cloudy profiles ψ_u, ψ_d.

For all this, we have to look at the *inner side* of the parameterization.

In the following sections, we'll first derive the pseudo subsidence/ascent terms and the detrainment coefficients K_u and K_d, supposing the cloudy mass fluxes are known.

After, we will build the cloudy profiles, and finally examine the different closure hypotheses allowing to determine the mass fluxes.

2 Mesh size effects on parameterization

The aim of "deep convection" parameterization is primarily to address the unresolved phenomena inducing precipitation. Large scale precipitation is based widely on large scale vertical velocities, themselves representing averages over one grid mesh.

When increasing the resolution, more and more precipitation represented by the parameterization scheme will also be diagnosed by the large scale scheme, leading to a double count of precipitation.

Practically cumulo-mimbus clouds are totally resolved by a 1 km grid mesh, and ignored by a 100 km grid mesh.

Between those mesh sizes, you need to modulate the contribution of the deep convection scheme, and this is particularly sensible in ARPÈGE global model, where the mesh size varies over the domain.

The modulation is performed by affecting the large scale moisture convergence fed to the deep convection routine.

2.1 Classic method

Moisture convergence at a given level is expressed by

$$\mathsf{CVGQ} \equiv -\mathcal{R}\left[\mathbf{V}\cdot\nabla q + \omega\frac{\partial q}{\partial p}\right] - g\frac{\partial J_q}{\partial p} \tag{26}$$

where $\mathcal{R}$ is a modulation factor to take into account mesh size effects.

Note the addition of the divergence of the turbulent diffusion flux, which is an important characteristic of the scheme (see §6.1.1).

The ancient way (LSRCON=.FALSE.) defines this modulation factor as

$$\mathcal{R} = \frac{1}{(1 + \mathsf{PGM(JLON)} \cdot \frac{\triangle x_{\rm ref}}{\triangle x_{\rm equiv}})^{\Upsilon}} \tag{27}$$

where

- PGM is the local map factor,
- $\Upsilon \equiv$ GCOMOD is a tuning exponent,
- $\triangle x_{\rm equiv}$ is the model equivalent mesh size, defined as

  $\triangle x_{\rm equiv} \equiv \frac{2\pi r_a}{3\mathsf{NSMAX}}$ in the global model ARPÈGE, and

$\triangle x_{\mathrm{equiv}} \equiv \min(\mathsf{EDELX}, \mathsf{EDELY})$ in ALADIN

where r_a is the earth radius, NSMAX the model truncation, EDELX and EDELY the local area model mesh dimensions $[m]$.

- $\triangle x_{\mathrm{ref}}$ is a reference length $[m]$ suitable to the closure type of the convection scheme, corresponding to parameters REFLKUO (Kuo closure) of REFLCAPE (CAPE closure).

2.2 Smarter method

Activated by the key (LSRCON=.TRUE.), it consists to add, to the moisture convergence CVGQ, the vertical divergence of total the large scale precipitation flux (returned by CPFHPFS), before passing it to the deep convection routine.

$$\mathsf{CVGQ} \equiv -\mathcal{R}\left[\mathbf{V}\cdot\nabla q + \omega\frac{\partial q}{\partial p}\right] - g\frac{\partial J_q}{\partial p} - g\frac{\partial \mathcal{P}_{\mathrm{LS}}}{\partial p} \tag{28}$$

In this case, it is advisable to have $\mathcal{R} = 1$, by setting GCOMOD $= 0.$ or REFLKUO $= 0.$ since there is no need to perform the large scale moisture convergence modulation in two ways simultaneously.

3 Pseudo vertical advection effects

The original development can be found in the appendix of [GELEYN et al., 1982].
In a separate resolution, we consider here only the first and the third terms of the RHS of equation (19): so we have to solve

$$\begin{aligned}
\left(\frac{\partial\psi}{\partial t}\right)_{ps} &= \overset{\wedge}{\omega^*}\frac{\partial\psi}{\partial p} = \frac{\partial\overset{\wedge}{\omega^*}\psi}{\partial p} - \psi\frac{\partial\overset{\wedge}{\omega^*}}{\partial p} = -g\frac{\partial F_\psi^{ps}}{\partial p} \\
\left(\frac{\partial\psi}{\partial t}\right)_{pa} &= \overset{\vee}{\omega^*}\frac{\partial\psi}{\partial p} = \frac{\partial\overset{\vee}{\omega^*}\psi}{\partial p} - \psi\frac{\partial\overset{\vee}{\omega^*}}{\partial p} = -g\frac{\partial F_\psi^{pa}}{\partial p}
\end{aligned} \tag{29}$$

where subscript $_{ps}$ represents the *pseudo-subsidence* and $_{pa}$ the *pseudo ascent*. The resolution is identical for the updraught and the downdraught.

$$\left(\frac{\partial\psi}{\partial t}\right)_{ps} = \overset{\wedge}{\omega^*}\frac{\partial\psi}{\partial p} = \frac{\partial\overset{\wedge}{\omega^*}\psi}{\partial p} - \psi\frac{\partial\overset{\wedge}{\omega^*}}{\partial p} = -g\frac{\partial F_\psi^{ps}}{\partial p}$$

Let's define ZFORM $= c = -\overset{\wedge}{\omega^*}\Delta t \geq 0$ the updraught mass flux.
Putting $\delta^l \equiv p^{\overline{l}} - p^{\overline{l-1}}$ we get, using a split-implicit algorithm for the intermediate determination of ψ_{ps}:

$$\psi_{ps}^l - \psi^l = -\frac{1}{\delta^l}\left\{c_{\overline{l}}\,\frac{\psi_{ps}^{l+1}+\psi_{ps}^l}{2} - c_{\overline{l-1}}\,\frac{\psi_{ps}^l+\psi_{ps}^{l-1}}{2}\right\} + \psi_{ps}^l\,\frac{c_{\overline{l}}-c_{\overline{l-1}}}{\delta^l}$$

As this system is not diagonal dominant, we prefer to write:

$$\begin{aligned}
\psi_{ps}^l - \psi^l &= -\frac{1}{\delta^l}\left\{c_{\overline{l}}\,\frac{\psi^{l+1}+\left(\psi_{ps}^l-\psi^l\right)+\psi_{ps}^l}{2} - c_{\overline{l-1}}\,\frac{\psi^l+\left(\psi_{ps}^{l-1}-\psi^{l-1}\right)+\psi_{ps}^{l-1}}{2}\right\} + \psi_{ps}^l\,\frac{c_{\overline{l}}-c_{\overline{l-1}}}{\delta^l} \\
&= -\frac{c_{\overline{l}}}{\delta^l}\,\frac{\psi^{l+1}-\psi^l}{2} - \frac{c_{\overline{l-1}}}{\delta^l}\,\frac{\psi^l-\psi^{l-1}}{2} + \frac{c_{\overline{l-1}}}{\delta^l}(\psi_{ps}^{l-1}-\psi_{ps}^l)
\end{aligned}$$

$$\begin{aligned}
\psi_{ps}^l\,(1+\frac{c_{\overline{l-1}}}{\delta^l}) - \psi_{ps}^{l-1}\,\frac{c_{\overline{l-1}}}{\delta^l} &= \psi^l + \frac{c_{\overline{l-1}}}{\delta^l}\,\frac{\psi^l-\psi^{l-1}}{2} - \frac{c_{\overline{l}}}{\delta^l}\,\frac{\psi^{l+1}-\psi^l}{2} \\
\Longrightarrow \qquad \psi_{ps}^l &= \frac{\psi^l + \frac{c_{\overline{l-1}}}{\delta^l}\left\{\psi_{ps}^{l-1}+\frac{\psi^l-\psi^{l-1}}{2}\right\} - \frac{c_{\overline{l}}}{\delta^l}\left\{\frac{\psi^{l+1}-\psi^l}{2}\right\}}{1+\frac{c_{\overline{l-1}}}{\delta^l}}
\end{aligned}$$

Defining

$$\mathsf{ZAUX} = \frac{1}{1 + \frac{c_{\overline{l-1}}}{\delta^l}}$$

yields

$$\psi_{ps}^l \quad = \quad \mathsf{ZAUX} \left\{ \psi^l + \frac{1}{\delta_l} \left[c_{\overline{l-1}} \frac{\psi^l - \psi^{l-1}}{2} - c_{\overline{l}} \frac{\psi^{l+1} - \psi^l}{2} + c_{\overline{l-1}} \psi_{ps}^{l-1} \right] \right\} \tag{30}$$

This scheme is conservative, well-conditioned and stable for linear perturbations. To also avoid non linear instability we should have

$$\frac{|c_l - c_{l-1}|}{\delta_l \left| 1 + \frac{c_l}{\delta_l} \right|} < 1 \tag{31}$$

This is obtained by replacing c_l by c_l':

$$c_l' = c_{l-1}' + \left(c_l - c_{l-1}' \right) \frac{1 + \frac{c_{l-1}'}{\delta_l}}{1 + \frac{\left| c_l - c_{l-1}' \right|}{\delta_l}} \tag{32}$$

Having the pseudo subsidence tendency, we can express the corresponding flux:

$$\triangle F_\psi^{ps} = -\frac{\triangle p}{g \triangle t} (\psi_{ps} - \psi) \tag{33}$$

4 Detrainment coefficient derivation

As we suppose that the convective process induces only a vertical redistribution of heat, moisture, rainfall and momentum (Bougeault [1985]'s first hypothesis), we can express that the vertical integral of the moist static energy on the convective column must be conserved by the cumulus components. *If we suppose that the updraught and the downdraught processes are not coupled in the realisation of this balance,* this gives us a relation between K^c and ω^* ($c = u,\, d$):

$$\int_{p_t}^{p_b} \left(Q_1^u - Q_2^u \right) \frac{dp}{g} = 0, \qquad\qquad \int_{p_t}^{p_b} \left(Q_1^d - Q_2^d \right) \frac{dp}{g} = 0 \tag{34}$$

4.1 Updraught

We get:

$$\int_{p_t}^{p_b} \left\{ K_u \left(h_u - h \right) + \hat{\omega^*} \frac{\partial h}{\partial p} + g \frac{\partial J_h}{\partial p} \right\} \frac{dp}{g} = 0$$

which yields the value for the Kuo coefficient K_u:

$$K_u = \frac{\int_{p_t}^{p_b} -\hat{\omega^*} \frac{\partial h}{\partial p} \frac{dp}{g} + J_h(p_t) - J_h(p_b)}{\int_{p_t}^{p_b} \left(h_u - h \right) \frac{dp}{g}} \tag{35}$$

In this view, the detrainment coefficient is constant over the whole vertical.
To introduce some degree of explicit detrainment at the top of our "equivalent single cloud", we introduce a dependency over the vertical through the updraught mass flux divergence, as

$$K^l \quad = \quad K_0 + \beta \max(0, \left(-\frac{\partial \hat{\omega^*}}{\partial p} \right)^l)$$

where the average detrainment rate K_0 is computed as above, leading:

$$K_0^u = \frac{\int_{p_t}^{p_b} -\overset{\wedge}{\omega^*} \frac{\partial h}{\partial p} \frac{dp}{g} - \beta \int_{p_t}^{p_b} \max(0, -\frac{\partial \overset{\wedge}{\omega^*}}{\partial p})(h_u - h) \frac{dp}{g} + J_h(p_t) - J_h(p_b)}{\int_{p_t}^{p_b} (h_u - h) \frac{dp}{g}} \tag{36}$$

The tuning coefficient $\beta \equiv \mathsf{GCVBETA}$ may be reset to 0 to suppress the dependency of the detrainment on the updraught mass flux. Presently recommended value is 0.2.
This enhanced detrainment applies only to the updraughts.

As section (3) gave us $\omega^* \frac{\partial h}{\partial p} = \frac{\psi_{ps} - \psi}{\triangle t}$, we compute a single value of K^u over an entire vertical, keeping also in mind that the introduction of turbulent diffusion fluxes was essentially intended to include boundary layers effects, which would not take much sense if we make independent computations over separate vertical slabs. Discretization of the integrals uses the updraught mass flux and also the layer activity index $\delta^l_{\text{stab}} \equiv \mathsf{KNLAB(JLON, JLEV)}$ which is 1 where appropriate conditions are fulfilled for updraught generation and 0 elsewhere (§6.1.2).

$$\begin{aligned} K_0^u \triangle t &\equiv \mathsf{ZALFP(JLON)} \\ &= \frac{-\sum_{l_t}^{l_b} \delta^l_{\text{stab}} \triangle p^l \left(h^l_{ps} - h^l\right) + \sum_{l_t}^{l_b} \delta^l_{\text{stab}} \triangle p^l \frac{g \triangle t}{\triangle p^l} \left[J_h^{l-1} - J_h^l\right] - \beta \sum_{l_t}^{l_b} \max(0, \left(-\frac{\partial \overset{\wedge}{\omega^*} \triangle t}{\partial p}\right)^l) \delta^l_{\text{stab}} \triangle p^l \left(h^l_c - h^l\right)}{\sum_{l_t}^{l_b} \delta^l_{\text{stab}} \triangle p^l \left(h^l_c - h^l\right)} \end{aligned}$$

For this we compute:

$$\begin{aligned}
\mathsf{ZS2(JLON)} &= \sum_{l_t}^{L} \delta^l_{\text{stab}} \triangle p^l \quad \left\{\left(s^l_c - s^l\right) + L_{\text{bud}} \left(q^l_c - q^l\right)\right\} \\
\mathsf{ZS6(JLON)} &= \sum_{l_t}^{L} \delta^l_{\text{stab}} \triangle p^l L_{\text{bud}} \left(q^l_{ps} - q^l\right) \\
\mathsf{ZS7(JLON)} &= \sum_{l_t}^{L} \delta^l_{\text{stab}} \triangle p^l \quad \left(s^l_{ps} - s^l\right) \\
\mathsf{ZS8(JLON)} &= \sum_{l_t}^{L} \delta^l_{\text{stab}} \triangle p^l \quad L_{\text{bud}} \frac{g \triangle t}{\triangle p^l} \left[J_q^{l-1} - J_q^l\right] \\
\mathsf{ZS9(JLON)} &= \sum_{l_t}^{L} \delta^l_{\text{stab}} \triangle p^l \quad \frac{g \triangle t}{\triangle p^l} \left[J_s^{l-1} - J_s^l\right] \\
\mathsf{ZALFPB(JLON, JLEV)} &= \beta \max(0, (-\frac{\partial \overset{\wedge}{\omega^*} \triangle t}{\partial p})^l) \\
\mathsf{ZS18} &= \sum_{l_t}^{L} \mathsf{ZALFPB}^l \cdot \delta^l_{\text{stab}} \triangle p^l \quad \left\{\left(s^l_c - s^l\right) + L_{\text{bud}} \left(q^l_c - q^l\right)\right\} \\
\mathsf{ZALFP(JLON)} &\equiv K_0 \triangle t = \frac{\max(\mathsf{ZS8} + \mathsf{ZS9} - \mathsf{ZS6} - \mathsf{ZS7} - \mathsf{ZS18}, 0)}{\max(\epsilon_2, \mathsf{ZS2})} \\
K^{ul} \triangle t &\equiv \mathsf{ZALFP} + \mathsf{ZALFPB}^l
\end{aligned}$$

Note that the *budget latent heat* is used here, as we are making budgets between the whole vertical column and the environment.

If ZALFP computed this way would be negative, it would represent a non physical situation, and lead to numerical instabilities.
Comparing equation (35) and equation(100), we see that the vertical gradient of turbulent diffusion fluxes

intervenes in both. In (100), the term $(J_q(p_t) - J_q(p_b))$ should be positive to diagnose convective rather than large scale precipitation: this is confirmed by what happens here.

So the forcing of ZALFP to zero must be traced and cause a stop of the convective treatment: in this case, there is actually no convective contribution to add to the turbulent diffusion and the large scale precipitation schemes.
This is done via variable KNND(JLON), which we reset to 0 when a non-physical situation is met. We then multiply the activity index KNLAB(JLON,JLEV) by KNND(JLON), to remove any convective treatment of the concerned vertical.

The final recombination of the tendencies by CPTEND works as follows:

$$\frac{\partial q}{\partial t} = -\mathbf{V}\nabla q - \omega\frac{\partial q}{\partial p} + \underline{\left(\frac{-c_p Q_2}{L}\right)} - g\frac{\partial J_q}{\partial p} \quad \text{where} \quad \left(\frac{-c_p Q_2}{L}\right) = \omega^{\hat{*}}\frac{\partial q}{\partial p} + K(q_c - q) + g\frac{\partial J_q}{\partial p}$$

Wherever a *convective stop* occurs, we may stop immediately the convective computations (by resetting the convective activity indicator KNLAB to Zero), and just have to reset all convective fluxes to Zero, so that the vertical turbulent diffusion fluxes will no longer be compensated in the recombination.

4.2 Downdraught

We consider a constant detrainment rate over the vertical, and the expressions are slightly simpler:

$$\int_{p_t}^{p_b} \left\{ K_d\,(h_d - h) + \omega^{\check{*}}\frac{\partial h}{\partial p} \right\} \frac{dp}{g} = 0 \qquad \Longrightarrow \qquad K_d = \frac{\int_{p_t}^{p_b} -\omega^{\check{*}}\frac{\partial h}{\partial p}\frac{dp}{g}}{\int_{p_t}^{p_b} (h_d - h)\,\frac{dp}{g}} \tag{37}$$

Using the downdraught activity index $\delta^{\downarrow l}_{\text{stab}} \equiv$ INLAB(JLON, JLEV) (see §6.2.2):

$$\mathsf{ZALFP(JLON)} \equiv K^d\,\triangle t = \frac{-\sum_{l_t}^{l_b} \delta^{\downarrow l}_{\text{stab}} \triangle p^l\,\left(h^l_{pa} - h^l\right)}{\sum_{l_t}^{l_b} \delta^{\downarrow l}_{\text{stab}} \triangle p^l\,(h^l_c - h^l)} \tag{38}$$

For this we compute:

$$\begin{aligned}
\mathsf{ZS2(JLON)} &= \sum_{L}^{l_t} \delta^{\downarrow l}_{\text{stab}} \triangle p^l \quad \left\{ \left(s^l_c - s^l\right) + L_{\text{bud}}\left(q^l_c - q^l\right) \right\} \\
\mathsf{ZS6(JLON)} &= \sum_{L}^{l_t} \delta^{\downarrow l}_{\text{stab}} \triangle p^l L_{\text{bud}}\left(q^l_{pa} - q^l\right) \\
\mathsf{ZS7(JLON)} &= \sum_{L}^{l_t} \delta^{\downarrow l}_{\text{stab}} \triangle p^l \quad \left(s^l_{pa} - s^l\right) \\
\mathsf{ZALFP(JLON)} &\equiv K^d\,\triangle t = \frac{\mathsf{ZS6} + \mathsf{ZS7}}{-\mathsf{ZS2}}
\end{aligned}$$

Negative values of ZALFP are not physical and rejected while setting the corresponding feasibility index INND to 0. In this case, all *downdraught* contributions to the fluxes are reset to zero, which means there is no downdraught activity while the results from the updraught become final.

5 Cloud profiles

5.1 Thermodynamic variables s_c, q_c

5.1.1 Updraught profiles

For s_u and q_u, we know that we have saturation all along the cloud, hence we follow the moist adiabat; but as there is *entrainment* (that we will express by relaxing cloud variables to the environment), things are a little more complex.

5.1.1.1 Entrainment

We will always use Kuo's scheme to parameterize the entrainment:

$$\frac{\partial \psi_u}{\partial \phi} = -\lambda_u (\psi_u - \psi) \quad \text{or} \quad \frac{\partial \psi_u}{\partial p} = \frac{\lambda_u}{\rho} (\psi_u - \psi) \qquad \text{with } \psi = s,\ q,\ \mathbf{V} \tag{39}$$

where $\mathsf{ZENTR} \equiv \lambda_u$ represents the fractional entrainment rate, i.e. the relative variation of the updraught flux with respect to ϕ. We have the relation

$$\frac{\triangle M_u}{M_u} = \lambda_u \triangle \phi = \frac{E_u \triangle p}{M_u}$$

where E_u is the entrained flux over a layer $\triangle p$ and λ_u has the dimension $[1/\phi]$.

The vertical profile of λ_u is the same for all four variables, as the air is entrained with all its characteristics. It is chosen as

$$\lambda_u{}^l = \lambda_{\text{min}} + (\lambda_{\text{max}} - \lambda_{\text{min}})\, \mathsf{ZFRAA}^l \tag{40}$$

where ZFRAA is the uncompleted fraction of the ascent trajectory:

$$\mathsf{ZFRAA}^l = e^{-\lambda_{\text{max}}^{3/4} \lambda_{\text{min}}^{1/4} (\phi^l - \phi_b)} = e^{-\mathsf{ZENEN} \cdot \mathsf{ZS5}^l} \tag{41}$$

with

$$\mathsf{ZENEN} \equiv \lambda_{\text{max}} \left(\frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \right)^{0.25} \quad , \quad \mathsf{ZS5}^l \equiv \int_{p_b}^{p^l} \delta_{\text{stab}}\, d\phi \quad , \quad \mathsf{ZENTRMN} \equiv \lambda_{\text{min}} \quad , \quad \mathsf{ZENTRMX} \equiv \lambda_{\text{max}}$$

δ_{stab} is the layer activity index (§6.1.2), equal to 1 in active layers, 0 elsewhere. If we take constant values for $\lambda_{\text{min}} = E_n \equiv \mathsf{TENTR}$ and $\lambda_{\text{max}} = E_x \equiv \mathsf{TENTRX}$, λ_u is maximum at the cloud base (λ_{max}), and decreases asymptotically to λ_{min} upwards.

It seems that for deep clouds in the tropics there is a loss of buoyancy starting from the cloud basis, that could be related to a too big value of the entrainment rate there. On the other side, experiments carried out for middle latitude active events showed an improvement of squall line structures by increasing the entrainment rates.
An enhanced formulation of the entrainment rate has been proposed, where the minimum and maximum entrainment rates are no longer constant but depend on the vertical integral buoyancy

$$\mathsf{ZS17} \equiv I_b \equiv \int_{\phi_L}^{\phi_1} (h_{\text{ad}} - h) d\phi \tag{42}$$

where the integral extends over all model levels and $h_{\text{ad}} \equiv \mathsf{ZHSE\_AD(JLON, JLEV)}$ is the profile of a not entraining saturated adiabatic ascent:

$$h^l = c_p T^l + \phi^l + L_{\text{bud}} q^l \quad , \quad \mathsf{ZHSE\_AD}^l = \max(\mathsf{ZHSE\_AD}^{l+1}, h) \quad , \quad \mathsf{ZS17}^{\bar{l}} = \sum_L^l (h_{\text{ad}} - h)^{\bar{l}}$$

(i.e. the ascent follows a moist adiabat, with constant h, unless a warmer environment brings the necessary heat to pass on a warmer moist adiabat: in this case, the difference $h_{\text{ad}} - h$ becomes zero, i.e. there is no further contribution to the integrated CAPE, until the environment curve comes back below of the moist adiabat).
The dependency of the entrainment on I_b is chosen so that the same correction applies to the inverses of the minimum and the maximum entrainment rates:

$$\frac{1}{\lambda_{\text{min}}} - \frac{1}{E_n} \quad = \quad \frac{1}{\lambda_{\text{max}}} - \frac{1}{E_x} \qquad \approx \alpha I_b \frac{E_n}{E_x}$$

If E_x/E_n has been chosen large, we avoid this ratio being cancelled too fast by buoyancy corrections.
To avoid numerical problems the RHS is replaced by the expression below, where at the denominator αI_b addresses the case of an excessively small $1/E_x$ while the "1+" avoids problems of a too small $1/E_n$.

At the LHS we also had to introduce a limit preventing the entrainment rates becoming too small, as this induced sometimes oscillations from one step to the following, between a very small and a maximum entrainment.

$$\frac{1}{\lambda_{\min}-\lambda_{\lim}}-\frac{1}{E_n-\lambda_{\lim}} = \frac{1}{\lambda_{\max}-\lambda_{\lim}}-\frac{1}{E_x-\lambda_{\lim}}=\frac{\alpha I_b}{1+\frac{1/E_n}{1/E_x+\alpha I_b}}$$

$$\text{with} \qquad \mathsf{ZENTRN}\equiv\lambda_{\lim} = E_n\left(\frac{E_n}{E_x}\right)^{0.25}$$

or

$$\lambda_{\min}=\lambda_{\lim}+\frac{E_n-\lambda_{\lim}}{1+(E_n-\lambda_{\lim})\frac{\alpha\, I_b E_n}{E_n+\frac{E_x}{1+\alpha\, I_b E_x}}} \quad , \quad \lambda_{\max}=\lambda_{\lim}+\frac{E_x-\lambda_{\lim}}{1+(E_x-\lambda_{\lim})\frac{\alpha\, I_b E_n}{E_n+\frac{E_x}{1+\alpha\, I_b E_x}}} \tag{43}$$

The tunable parameter $\alpha\equiv$ GCVALFA, has the dimension of the inverse of the moist static energy (or $1/\phi$, same as λ), setting it to zero brings back the original formulation, with constant $\lambda_{\max}$ and $\lambda_{\min}$.

This formulation ensures a smooth transition from small entrainment rates for deep clouds to bigger ones for thinner or less buoyant clouds.
Currently ("cycora-bis" tuning of Autumn 2000) recommended values are GCVALFA $=4.5\cdot10^{-5}[s^2/m^2]$, TENTR $=2.5\cdot10^{-6}[s^2/m^2]$, TENTRX $=8.\cdot10^{-5}[s^2/m^2]$.

5.1.1.2 Saturated adiabat computation

For simplicity, we may assume first that the *geopotential inside the cloud is the same as in the environment.* Relaxing of this hypothesis is explained in §5.1.1.4.
Starting at the lowest model level,

- Compute cloud base value T_u^l, q_u^l by solving iteratively

$$h_u^l-\phi^l=c_pT_u^l+Lq_u^l=c_pT^l+Lq^l \qquad \text{so that } q_u^l=q_{sat}\left(T_u^l\right) \tag{44}$$

  This gives you the blue point (i.e. wet bulb temperature and moisture) at this level.

- We need also a diagnostic value of the cloud condensate, obtained with the previously (part II) mentioned formula:

$$\frac{\partial q_u+\ell_u}{\partial\phi}=-\frac{\ell_u}{\phi_0}$$

  where the critical thickness ϕ_0 represents the critical depth above which the cloud starts to precipitate.

- Construct the moist adiabat from level l to level $l-1$, taking into account the entrainment:

$$\begin{aligned} c_pT_u^{l-1}+Lq_u^{l-1}+\phi^{l-1} &= \\ c_pT_u^l+Lq_u^l+\phi^l-\tfrac{\lambda_{l-1}^u+\lambda_l^u}{2}\left(\phi^{l-1}-\phi^l\right)&\left[c_p\left(T_u^l-T^l\right)+L\left(q_u^l-q^l\right)\right] \\ \text{so that } q_u^{l-1}&=q_{sat}\left(T_u^{l-1}\right) \end{aligned} \tag{45}$$

- Test if the cloud moist static energy at level $l-1$ is larger than the environment: if yes, continue, else recompute the blue point at level $l-1$

After this, compute the buoyancy at each level to see if there is or not instability:

$$T_{vu}^l=T_u^l\left[1+\left(\frac{R_v}{R_a}-1\right)q_u^l\right] \tag{46}$$

5.1.1.3 Cloud ensemble entrainment

Our parameterization reduces the convective clouds inside a mesh to one single updraught and one single downdraught.
In reality, a grid box may contain several types of clouds at the same time. Observations show that for a single plume, the entrainment rate is inversely proportional to the plume section: less entraining clouds reach higher

and are more buoyant. So, as less entraining clouds find themselves alone at higher levels, the mesh-averaged cloud moist static energy could increase with height, not through a non physical energy creation process, but through this selective sampling effect.

To parameterize this effect in a simple manner, we may apply a relaxation of the cloud moist static energy towards that of a fictive non entraining ascent.
We computed above (42)the non entraining ascent moist static energy h_{ad}. The effective entraining profile constructed afterwards is put into $\mathsf{ZHS\_REE} \equiv h_u$.
The relaxation term will actually be hidden in one block inside the cloud geopotential increment from one level to the next:

$$\triangle'\phi = \frac{\triangle\phi}{1 + \mathsf{GCVNU}(1 - \mathsf{ZFRAA})\max(0, (h_{\mathrm{ad}} - h_u))}$$

$1 - \mathsf{ZFRAA}$ being the *completed* fraction of the ascent.

So the effect is applied through reducing the in-cloud thickness of lower pressure (=higher) buoyant layers of the ascent used in the cloud profile computation, yielding a slower decrease of the moist static energy with height.

The relaxation coefficient GCVNU has the same physical dimensions as an entrainment rate, and should be of the same order of magnitude: presently recommended value is $\mathsf{GCVNU} \sim 2.5 \cdot 10^{-5}[s^2/m^2]$ ("cycora bis", Autumn 2000).

Nothing is done to try to simulate the interaction of this change with the momentum convective redistribution, as it would be very difficult to estimate the cloud base velocity for a non entraining plume, and the effect is anyhow likely to have no systematic direction, unlike what happens for T and q.

This development is also quasi-irrelevant for downdraughts, as it would require very high entrainment rates to prevent any type of downdraught to reach the surface: this is also a justification for keeping downdraught entrainment and detrainment rates independent of height up to now (see below).

5.1.1.4 Cloud-environment pressure gradient

(Note: This development is also related to the momentum profile, see §5.2)

In the saturated adiabat $c_pT + L\,q + \phi = \text{const}$, $d\phi$ is actually computed (see appendix) taking the temperature and moisture values of the cloud, so that the cloud merges the environment at equal pressure but will have a different geopotential.

The opposite situation is to take for $d\phi$ the value of the environment, so that the cloud will merge the environment at equal geopotential but will have a different pressure.
A continuous transition between "equi-geopotential" and "equi-pressure" treatments is allowed by a free parameter: $0 \le \mathsf{GCVADS} \le 1$, the value 0 corresponding to the "equi-pressure" situation, and the value 1 to the "equi-geopotential" situation.

When taking into account the cloud-environment pressure difference in the momentum entrainment parameterization (§5.2), together with the effect of the *"ensembling"* entrainment (§5.1.1.3), the intermediate solution seems more logical.

Practical use of GCVADS appears in §5.1.3.

When using the *non hydrostatic* large scale dynamical equations, the model variable $\widehat{\mathcal{P}}$ represents the (reduced) pressure departure from the hydrostatic value. It would then seem logical to make use of this variable within this part of the convection parameterization.

5.1.2 Downdraught profiles

For s_d and q_d we follow the moist adiabat downwards, assuming entrainment of the environmental air, but this time with a constant value of the parameter λ_d, namely the minimum entrainment computed for the updraught, and passed as argument to the downdraught routine.

$$\lambda_d = \lambda_t \equiv \lambda_{\min} \tag{47}$$

We never introduce the buoyancy dependency GCVALFA for the downdraught.

5.1.3 Construction of the updraught profile for thermodynamical variables

We need to compute the three local variables ZQN $\equiv q_u$, ZTN $\equiv T_u$, ZLN $\equiv \ell_u$.
The updraught condensed water contents is presently simply diagnosed with

$$\frac{\partial\left(q_u + \ell_u\right)}{\partial \phi} = -\frac{\ell_u}{\phi_0} \tag{48}$$

where the critical thickness $\phi_0 \equiv$ ECMNP is the equivalent depth around which the convective clouds start to be fully precipitating ones. Recommended value is around ECMNP $= 3000$ [J/kg].

The idea of this diagnostic equation originated from Arakawa and Schubert [1974]'s appendix; physically, it writes that the total water decreases of an amount equal to its condensed fraction ℓ_u over a depth ϕ_0, which is conceivable if the precipitation flux is proportional to the condensate ℓ_u.
For the entrainment effect, we have the 3 equations:

$$\frac{\partial q_u}{\partial \phi} = \lambda^u\left(q - q_u\right) \quad , \qquad \frac{\partial T_u}{\partial \phi} = \lambda^u\left(T - T_u\right) \quad , \qquad \frac{\partial \ell_u}{\partial \phi} = -\lambda^u\left(\ell_u\right)$$

with λ^u given by (40) and (43).
We want to construct the moist pseudo-adiabat from model level b to the level $h \equiv b-1$, immediately above it.
To compute the saturated values T_h, q_h, we must follow the saturated pseudo-adiabat $q = q_s(T)$ which is non linear. To solve this, we use a *Newton algorithm* which linearizes q_s in the neighbourhood of the preceding iteration:

$$q^{k+1} = q_s\left(T^k\right) + \frac{\partial q_s}{\partial T^k}\left(T^{k+1} - T^k\right) \tag{49}$$

Of course, we must have saturation at level h: $q_h = q_s$, and the lower level b considered at one step is the previously computed level $l+1$ taking into account the entrainment of environmental air. This mixing is performed by writing:

$$\begin{aligned}
\xi^l &= \lambda^l\left(\phi^l - \phi^{l+1}\right) \quad , \qquad \xi^l = \tfrac{\xi^l}{1+\xi^l} \qquad \text{for numerical stability} \\
T_b &= T_u^{l+1} + \xi^{l+1}(T^{l+1} - T_u^{l+1}) \\
q_b &= q_u^{l+1} + \xi^{l+1}(q^{l+1} - q_u^{l+1}) \\
\ell_b &= \ell_u^{l+1} \cdot (1 - \xi^{l+1})
\end{aligned} \tag{50}$$

We know that along the pseudo-adiabat, h is conserved:

$$dh = c_p dT + L dq + d\phi = 0 \tag{51}$$

and we express $c_p \equiv c_p(q)$ and $L \equiv L(T)$ by (see Part II)

$$c_p = c_{pb} + \left(c_{pv} - c_{w|i}\right)(q - q_b) = c_{pb} + \gamma\, \triangle q \tag{52}$$

$$L = L_b + \left(c_{pv} - c_{w|i}\right)(T - T_b) = L_b + \gamma\, \triangle T \tag{53}$$

where we defined $\gamma \equiv \left(c_{pv} - c_{w|i}\right)$.
Note that those relations suppose that

$$q + \ell \equiv q + \ell_w + \ell_i = q_b + \ell_{wb} + \ell_{ib}$$

i.e. that all moisture stays in the system, but only for the *crossing* of the layer; (48) is then applied for "cross layer" calculations of the condensate's departure: in other words, the total water for a layer is estimated using (48) but the result is then assumed constant along the whole layer height while calculating the moist adiabat segment crossing it.

$$dc_p = \gamma dq \ , \qquad dL = \gamma dT \qquad \Longrightarrow \gamma dh = c_p \, dL + L \, dc_p + \gamma \, d\phi = d(Lc_p) + \gamma d\phi = 0$$

$$L \, c_p - L_b \, c_{pb} + \gamma \, \triangle\phi = 0 \tag{54}$$

Multiplying (52) by (53), and combining with (54) yields

$$Lc_p = L_b c_{pb} + \gamma \, \triangle T \, c_{pb} + \gamma \, \triangle q \, L_b + \gamma^2 \triangle T \, \triangle q \qquad \Longrightarrow \quad \triangle T \, c_{pb} + \triangle q \, L_b + \gamma \triangle T \, \triangle q + \triangle\phi = 0$$

hence

$$\begin{aligned} c_{pb} \, (T - T_b) + L_h \, (q - q_b) + \triangle\phi &= 0 \\ c_{ph} \, (T - T_b) + L_b \, (q - q_b) + \triangle\phi &= 0 \end{aligned} \tag{55}$$

if we neglect the second order term.

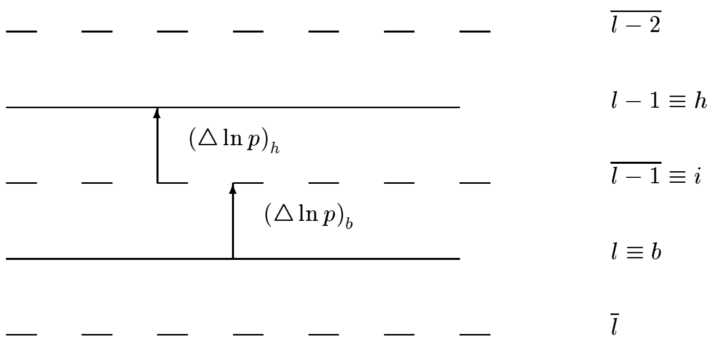
To estimate $\triangle\phi$ we have, for the *"equi-pressure cloud"* approach ($\mathsf{GCVADS} = 0$):

$$d\phi = -\frac{dp}{\rho} = -RT \frac{dp}{p}$$

Noting $i \equiv \bar{h}$ the interface between the two full model levels b and $h \equiv b-1$:

$$\begin{aligned} (\triangle \ln p)_b &= \ln \frac{p_b}{p_i} \qquad (\triangle \ln p)_h = \ln \frac{p_i}{p_h} \\ \triangle\phi &= R_b \, T_b \, \left(\triangle \ln p\right)_b + R_h \, T_h \, \left(\triangle \ln p\right)_h \\ &= R_b \, T_b \, \left(\triangle \ln p\right)_b + \left(R_b + R_v(q_h - q_b)\right) T_h \left(\triangle \ln p\right)_h \\ &\equiv \widetilde{R}_b^- \, T_b + \widetilde{R}_b^+ \, T_h + \widetilde{R}_v^+ \, T_h \, (q_h - q_b) \end{aligned} \tag{56}$$

The three coefficients $\widetilde{R}_b^-$, $\widetilde{R}_b^+$, $\widetilde{R}_v^+$ are independent of the subsequent computations of q_h and T_h. Practically in the routine, we have:



$$\begin{aligned} \mathsf{PALPH(KLON,KLEV)} &\equiv \ln \frac{p_{\overline{l-1}}}{p_{l-1}} = (\triangle \ln p)_h \qquad \text{and} \qquad \ln \frac{p_l}{p_{\overline{l-1}}} = (\triangle \ln p)_b \\ \mathsf{PLNPR(KLON,KLEV)} &\equiv \ln \frac{p_{\bar{l}}}{p_{\overline{l-1}}} = (\triangle \ln p)_b - (\triangle \ln p)_{h+1} \\ \mathsf{ZRBB(KLON)} &\equiv \widetilde{R}_b^- = R_b \, (\triangle \ln p)_b = R_b \ln \frac{p_l}{p_{\overline{l-1}}} \\ \mathsf{ZRBH(KLON)} &\equiv \widetilde{R}_b^+ = R_b \, (\triangle \ln p)_h = R_b \ln \frac{p_{\overline{l-1}}}{p_{l-1}} \\ \mathsf{ZRVH(KLON)} &\equiv \widetilde{R}_v^+ = R_v \, (\triangle \ln p)_h = R_v \ln \frac{p_{\overline{l-1}}}{p_{l-1}} \\ \text{with} \ \ R_b &= R_a \, (1 - l_b - q_b) + R_v \, q_b \\ &= R_a \, (1 - l_b) + (R_v - R_a) \, q_b \end{aligned}$$

For the *"equi-geopotential cloud"* approach ($\mathsf{GCVADS} = 1$), we take directly $\triangle\phi$ from the environment. Modulation between both cases with parameter GCVADS, is obtained by

$$\begin{aligned} \mathsf{ZRBB} &= (1 - \mathsf{GCVADS}) \cdot \mathsf{ZRBB} + \mathsf{GCVADS} \cdot \frac{\phi^l - \phi^{l+1}}{T_b} \\ \mathsf{ZRBH} &= (1 - \mathsf{GCVADS}) \cdot \mathsf{ZRBH} \\ \mathsf{ZRVH} &= (1 - \mathsf{GCVADS}) \cdot \mathsf{ZRVH} \end{aligned}$$

For the *"ensemblist"* formulation, the relaxation to the not entraining profile is performed by multiplying by the fraction of buoyancy-excess with respect to the not entraining, undiluted plume:

$$\mathsf{ZFFAND} = \frac{1}{1 + \mathsf{GCVNU}(1 - \mathsf{ZFRAA}) \max(0, h_{\mathrm{ad}} - h_u)} \tag{57}$$

$$\mathsf{ZRBB} = \mathsf{ZRBB} \cdot \mathsf{ZFFAND} \quad , \qquad \mathsf{ZRBH} = \mathsf{ZRBH} \cdot \mathsf{ZFFAND} \quad , \qquad \mathsf{ZRVH} = \mathsf{ZRVH} \cdot \mathsf{ZFFAND}$$

We need to diagnose the cloud liquid water ℓ_b in order to compute the effective gas constant R_b.
Using equation(48) we get:

$$\begin{aligned} \mathsf{ZLN} \quad &\equiv \quad \ell_u^l = \ell_b e^{-1/\chi} - \left(q_u^l - q_b\right) \chi (1 - e^{-1/\chi}) \\ \text{with } \mathsf{ZLIQ} \quad &\equiv \quad \chi \equiv \frac{\phi_0}{\triangle \phi_u^l} = \frac{\phi_0}{\widetilde{R}_b^- T_b + \left(\widetilde{R}_b^+ + \widetilde{R}_v^+ (q_u^l - q_b)\right) T_u^l} \end{aligned} \tag{58}$$

and equation (50) gives us ℓ_b.
Combining (55) and (56):

$$\begin{aligned} c_{pb}\left(T - T_b\right) + L\left(q - q_b\right) + \widetilde{R}_b^-\ T_b + \widetilde{R}_b^+\ T + \widetilde{R}_v^+\ T\left(q - q_b\right) \quad &= \quad 0 \\ \left(c_{pb} + \widetilde{R}_b^+\right)\left(T - T_b\right) + \left[\widetilde{R}_v^+\ T_b + \widetilde{R}_v^+\ \left(T - T_b\right) + L\right]\left(q - q_b\right) + \left(\widetilde{R}_b^+ + \widetilde{R}_b^-\right) T_b \quad &= \quad 0 \end{aligned} \tag{59}$$

Let be

$$\begin{array}{llcll} \mathsf{ZCP} & \equiv & \widetilde{C}_p & \equiv & c_p + \widetilde{R}_b^+ + \widetilde{R}_v^+\left(q - q_b\right) \\ \mathsf{ZLH} & \equiv & \widetilde{L} & \equiv & L + \widetilde{R}_v^+\ T_b + \widetilde{R}_v^+\left(T - T_b\right) = L + \widetilde{R}_v^+ T \\ \text{we still have :} & & \frac{\partial \widetilde{C}_p}{\partial q} & = & \frac{\partial \widetilde{L}}{\partial T} = \gamma + \widetilde{R}_v^+ \end{array} \tag{60}$$

Introducing an iterative process (Newton's loop), represented by control variable k:

$$\begin{array}{lclclcl} \widetilde{C}_{pb}\left(T^k - T_b\right) & + & \widetilde{L}^k\left(q^k - q_b\right) & + & \left(\widetilde{R}_b^+ + \widetilde{R}_b^-\right) T_b & = & 0 \\ \widetilde{C}_{pb}\left(T^{k+1} - T_b\right) & + & \widetilde{L}^{k+1}\left(q^{k+1} - q_b\right) & + & \left(\widetilde{R}_b^+ + \widetilde{R}_b^-\right) T_b & = & 0 \\ \hline \widetilde{C}_{pb}\left(T^{k+1} - T^k\right) & + & \widetilde{L}^{k+1}\left(q^{k+1} - q_b\right) & - & \widetilde{L}^k\left(q^k - q_b\right) & = & 0 \end{array} \tag{61}$$

For the iterative process, we make a first guess with:

$$\begin{aligned} T^{k=0} \quad &= \quad T_b + \left(T^l - T^{l+1}\right) \\ q^{k=0} \quad &= \quad q_b - \tfrac{1}{\widetilde{L}}\left\{\widetilde{C}_{pb}\left(T^{k=0} - T_b\right) + \left(\widetilde{R}_b^+ + \widetilde{R}_b^-\right) T_b\right\} \end{aligned} \tag{62}$$

Remark that setting $q = q_b$, $T = T_b$ in the first equation yields:

$$\widetilde{R}_b^+ + \widetilde{R}_b^- = 0 \tag{63}$$

And we have also

$$\widetilde{L}^{k+1} \quad = \quad L^{k+1} + \widetilde{R}_v^+\ T^{k+1} = L^k + \gamma\left(T^{k+1} - T^k\right) + \widetilde{R}_v^+\ T^{k+1} = \widetilde{L}^k + \left(\gamma + \widetilde{R}_v^+\right)\left(T^{k+1} - T^k\right) \tag{64}$$

$$\begin{aligned} \widetilde{C}_p^{k+1} \quad &= \quad c_p^{k+1} + \widetilde{R}_b^+ + \widetilde{R}_v^+\ \left(q^{k+1} - q^k\right) = c_p^k + \gamma\left(q^{k+1} - q^k\right) + \widetilde{R}_b^+ + \widetilde{R}_v^+\ \left(q^{k+1} - q^k\right) \\ &= \quad \widetilde{C}_p^k + \left(\gamma + \widetilde{R}_v^+\right)\left(q^{k+1} - q^k\right) \end{aligned} \tag{65}$$

Replacing b by k in (61) yields

$$\widetilde{C}_p^k\left(T^{k+1} - T^k\right) + \widetilde{L}^{k+1}\left(q^{k+1} - q^k\right) \quad = \quad 0 \tag{66}$$

$$\widetilde{C}_p^{k+1}\left(T^{k+1} - T^k\right) + \widetilde{L}^k\left(q^{k+1} - q^k\right) \quad = \quad 0 \tag{67}$$

Transforming equation (59) for iteration $k + 1$, with b=k and (63):

$$\begin{aligned} &\left(c_p^k + \widetilde{R}_b^+\right)\left(T^{k+1} - T^k\right) \\ &\quad + \left[\widetilde{R}_v^+\ T^k + \widetilde{R}_v^+\ \left(T^{k+1} - T^k\right) + L^k + \gamma\left(T^{k+1} - T^k\right)\right]\left(q_s(T^k) + \frac{\partial q_s}{\partial T^k}\left(T^{k+1} - T^k\right) - q^k\right) = 0 \end{aligned}$$

Dismissing the second degree terms in $\left(T^{k+1}-T^k\right)$

$$
\begin{aligned}
& \left(T^{k+1}-T^k\right)\left[\widetilde{R}_b^{+}+c_p^k+\left(L^k+\widetilde{R}_v^{+}\,T^k\right)\tfrac{\partial q_s}{\partial T^k}+\left(\widetilde{R}_v^{+}+\gamma\right)\left(q_s(T^k)-q^k\right)\right] \\
& \quad +\left(L^k+\widetilde{R}_v^{+}\,T^k\right)\left(q_s(T^k)-q^k\right)=0 \\
\Longleftrightarrow \quad & \left(T^{k+1}-T^k\right)\left[\widetilde{C}_p^k+\widetilde{L}^k\tfrac{\partial q_s}{\partial T^k}+\left(\widetilde{R}_v^{+}+\gamma\right)\left(q_s(T^k)-q^k\right)\right]+\widetilde{L}^k\left(q_s(T^k)-q^k\right)=0
\end{aligned}
$$

Using (64):

$$\left(T^{k+1}-T^k\right)\left[\widetilde{C}_p^k+\widetilde{L}^k\frac{\partial q_s}{\partial T^k}\right]+\widetilde{L}^{k+1}\left(q_s(T^k)-q^k\right)=0 \tag{68}$$

Using (66) to replace $\widetilde{L}^{k+1}$ yields

$$
\begin{aligned}
\left(T^{k+1}-T^k\right)\left[\widetilde{C}_p^k+\widetilde{L}^k\frac{\partial q_s}{\partial T^k}-\widetilde{C}_p^k\frac{q_s(T^k)-q^k}{q^{k+1}-q^k}\right] &= 0 \\
q^{k+1}\left(\widetilde{C}_p^k+\widetilde{L}^k\frac{\partial q_s}{\partial T^k}\right)-\widetilde{C}_p^k q_s(T^k)-\widetilde{L}^k\frac{\partial q_s}{\partial T^k}q^k &= 0
\end{aligned}
$$

$$\mathsf{ZDELQ}\equiv\left(q^{k+1}-q^k\right)=\frac{q_s(T^k)-q^k}{1+\frac{\widetilde{L}^k}{\widetilde{C}_p^k}\frac{\partial q_s}{\partial T^k}} \tag{69}$$

(67) gives

$$\mathsf{ZDELT}\equiv\left(T^{k+1}-T^k\right)=-\frac{\widetilde{L}^k}{\widetilde{C}_p^{k+1}}\left(q^{k+1}-q^k\right) \tag{70}$$

We use also equations (64) and (65):

$$\widetilde{C}_p^{k+1}-\widetilde{C}_p^k = \left(\gamma+\widetilde{R}_v^{+}\right)\left(q^{k+1}-q^k\right) \tag{71}$$

$$\widetilde{L}^{k+1}-\widetilde{L}^k = \left(\gamma+\widetilde{R}_v^{+}\right)\left(T^{k+1}-T^k\right) \tag{72}$$

$\left(\gamma+\widetilde{R}_v^{+}\right)$ is the variable ZDCP in the code.

So the Newton algorithm is given by using successively (69), (71), (70), (72) in NBITER iterations.
The above somehow complex development has the big advantage of its precision, so that NBITER may be kept as low as 2, as it was for the calculation of condensation without vertical motion.

5.2 Momentum profiles

5.2.1 Theory

Applying the development of §1.1, equation (19) writes, for momentum:

$$\left(\frac{\partial \mathbf{V}}{\partial t}\right)^{\text{conv}} = \underbrace{\overset{\wedge}{\omega^*}\frac{\partial \mathbf{V}}{\partial p}}_{\text{pseudo subs}}+\underbrace{K_u(\mathbf{V}_u-\mathbf{V})}_{\text{Detrainment}}+\underbrace{\overset{\vee}{\omega^*}\frac{\partial \mathbf{V}}{\partial p}}_{\text{pseudo asc}}+\underbrace{K_d(\mathbf{V}_d-\mathbf{V})}_{\text{Detrainment}}\equiv -g\frac{\partial F_{\mathbf{V}}^u}{\partial p}-g\frac{\partial F_{\mathbf{V}}^d}{\partial p} \tag{73}$$

In Bougeault's initial approach, it was decided to subtract the vertical turbulent diffusion scheme at this stage, in order to compensate it in the final recombination. However, it doesn't seem very physical to replace friction (expressed by this turbulent diffusion in arrays PSTRTU, PSTRTV of APLPAR) by an acceleration term due to convection. So following Ph. Bougeault's advice, we do not include those terms.

The vertical budget over the vertical (using Bougeault's hypothesis that the convective process only produces a reorganization of moisture, heat and momentum over a same vertical), writes:

$$\int_{p_t}^{p_b}\left(\frac{\partial \mathbf{V}}{\partial t}\right)^{\uparrow}_{\text{conv}}\frac{dp}{g}=0=\int_{p_t}^{p_b}\overset{\wedge}{\omega^*}\frac{\partial \mathbf{V}}{\partial p}+K_u\left(\mathbf{V}_u-\mathbf{V}\right)\frac{dp}{g}$$

$$\int_{p_t}^{p_b}\left(\frac{\partial \mathbf{V}}{\partial t}\right)^{\downarrow}_{\text{conv}}\frac{dp}{g}=0=\int_{p_t}^{p_b}\overset{\vee}{\omega^*}\frac{\partial \mathbf{V}}{\partial p}+K_d\left(\mathbf{V}_d-\mathbf{V}\right)\frac{dp}{g}$$

and gives us one element of the velocity profile, namely the velocity at its initial point.

To build the profile, we must consider the effect of the entrainment of environmental air into the draught, and the effect of a possible pressure gradient between the cloud and the environment:

$$\overset{\wedge}{\omega^*}\frac{\partial \mathbf{V}_u}{\partial p} = -(\nabla\phi)_u + \frac{\lambda_u}{\rho}\overset{\wedge}{\omega^*}(\mathbf{V}_u - \mathbf{V}) \qquad \overset{\vee}{\omega^*}\frac{\partial \mathbf{V}_d}{\partial p} = -(\nabla\phi)_d + \frac{\lambda_d}{\rho}\overset{\vee}{\omega^*}(\mathbf{V}_d - \mathbf{V}) \tag{74}$$

The entrainment coefficients λ_u and λ_d are the same as for the thermodynamical variables s and q, as the air is entrained with all its properties together, see above equations (40) and (47).
Kershaw and Gregory [1997] have shown that the horizontal pressure gradient between the cloud and the environment is proportional to the draught mass flux and to the large scale vertical shear. With this, we parameterize (74):

$$\frac{\partial \mathbf{V}_u}{\partial p} = \frac{\lambda_u}{\rho}(\mathbf{V}_u - \mathbf{V}) + \mathcal{G}_u\frac{\partial \mathbf{V}}{\partial p} \qquad \text{or} \qquad \frac{\partial \mathbf{V}_u}{\partial \phi} = -\lambda_u(\mathbf{V}_u - \mathbf{V}) + \mathcal{G}_u\frac{\partial \mathbf{V}}{\partial \phi} \tag{75}$$

for the updraught, and

$$\frac{\partial \mathbf{V}_d}{\partial p} = -\frac{\lambda_d}{\rho}(\mathbf{V}_d - \mathbf{V}) + \mathcal{G}_d\frac{\partial \mathbf{V}}{\partial p} \qquad \text{or} \qquad \frac{\partial \mathbf{V}_d}{\partial \phi} = +\lambda_d(\mathbf{V}_d - \mathbf{V}) + \mathcal{G}_d\frac{\partial \mathbf{V}}{\partial \phi} \tag{76}$$

for the downdraught.
Kershaw and Gregory [1997] found nearly the same proportionality coefficient for the updraught and the downdraught: $\mathcal{G}_d \simeq \mathcal{G}_u \simeq 0.7$. The adequate value might however be affected by the context (other parameterisations and feedbacks) of the particular model where the parameterisation is integrated.

5.2.2 Building the momentum profile

We develop here the calculation for the updraught, the same method being applied to the downdraught, with a few appropriate sign differences.

To discretize equation (75) in Arpège-Aladin we must beware that the entrainment equations for q and s were discretized as:

$$\frac{\partial \psi_c}{\partial \phi} = -\lambda(\psi_c - \psi) \tag{77}$$

$$\psi_c^l = \psi_c^{l+1} + \left(\phi^l - \phi^{l+1}\right)\lambda^{l+1}\left(\psi^{l+1} - \psi_c^{l+1}\right) \tag{78}$$

instead of using values at the interface level:

$$\psi_c^l = \psi_c^{l+1} + \left(\phi^l - \phi^{l+1}\right)\lambda^{\bar{l}}\left(\psi^{\bar{l}} - \psi_c^{\bar{l}}\right)$$

To avoid this approximation, we would have to compute first a value of ψ^l, based itself on the value of ψ_c^l into the *Newton* loop, so a double iterative process would be necessary.

To keep consistent, we merely do the same approximation for equation (75):

$$\mathbf{V}_c^l - \mathbf{V}_c^{l+1} - \mathcal{G}_u\left(\mathbf{V}^l - \mathbf{V}^{l+1}\right) = -\left(\phi^l - \phi^{l+1}\right)\ \lambda^{l+1}\ \left(\mathbf{V}_c^{l+1} - \mathbf{V}^{l+1}\right)$$

Let be

$$\xi^l \equiv \left(\phi^l - \phi^{l+1}\right)\ \lambda^{l+1} > 0 \tag{79}$$

$$\Longrightarrow \mathbf{V}_c^l = \mathbf{V}_c^{l+1}\left(1 - \xi^l\right) + \mathcal{G}_u\mathbf{V}^l - (\mathcal{G}_u - \xi_l)\,\mathbf{V}^{l+1}$$

We impose the speed at the cloud base $\mathbf{V}_c^b = \mathbf{V}_{c0}$.

Searching for an expression in the form:

$$\mathbf{V}_c^l = \beta_l\mathbf{V}_{c0} + (1 - \beta_l)\,\overline{\mathbf{V}}^l \qquad \text{or} \qquad \mathbf{V}_c^{l+1} = \beta_{l+1}\mathbf{V}_{c0} + (1 - \beta_{l+1})\,\overline{\mathbf{V}}^{l+1} \tag{80}$$

we get

$$
\begin{aligned}
\mathbf{V}_c^l &= (1-\xi_l)\left\{\beta_{l+1}\mathbf{V}_{c0} + (1-\beta_{l+1})\overline{\mathbf{V}}^{l+1}\right\} + \mathcal{G}_u\mathbf{V}^l - (\mathcal{G}_u - \xi_l)\,\mathbf{V}^{l+1} \\
\Longrightarrow \qquad \beta_l &= (1-\xi_l)\,\beta_{l+1} \qquad (81)\\
\beta_b &= 1 \\
(1-\beta_l)\,\overline{\mathbf{V}}^l &= (1-\xi_l)\,(1-\beta_{l+1})\,\overline{\mathbf{V}}^{l+1} + \mathcal{G}_u\mathbf{V}^l - (\mathcal{G}_u-\xi_l)\,\mathbf{V}^{l+1} \\
\Longrightarrow \qquad \overline{\mathbf{V}}^l &= (1-\xi_l)\,\frac{(1-\beta_{l+1})}{(1-\beta_l)}\overline{\mathbf{V}}^{l+1} + \frac{\mathcal{G}_u\mathbf{V}^l - (\mathcal{G}_u-\xi_l)\,\mathbf{V}^{l+1}}{(1-\beta_l)} \\
&= \overline{\mathbf{V}}^{l+1}\frac{(1-\xi_l)-\beta_l}{(1-\beta_l)} + \frac{\mathcal{G}_u\mathbf{V}^l - (\mathcal{G}_u-\xi_l)\,\mathbf{V}^{l+1}}{(1-\beta_l)} \\
&= \overline{\mathbf{V}}^{l+1} + \frac{\xi_l\left(\mathbf{V}^{l+1} - \overline{\mathbf{V}}^{l+1}\right) + \mathcal{G}_u\left(\mathbf{V}^l - \mathbf{V}^{l+1}\right)}{(1-\beta_l)} \qquad (82)
\end{aligned}
$$

At the cloud base:

$$
\begin{aligned}
\beta_b &= 1 \qquad \Longrightarrow \qquad \beta_{b-1} = (1-\xi_b) \qquad \Longleftrightarrow \qquad (1-\beta_{b-1}) = \xi_{b-1} \\
\overline{\mathbf{V}}^{b-1} &= \mathbf{V}^b + \frac{\mathcal{G}_u\left(\mathbf{V}^{b-1} - \mathbf{V}^b\right)}{\xi_{b-1}} \qquad (83)
\end{aligned}
$$

if $\mathcal{G}_u = 0$ we find:

$$
\overline{\mathbf{V}}^{b-1} = \mathbf{V}^b \qquad \text{and} \qquad \overline{\mathbf{V}}^l = \overline{\mathbf{V}}^{l+1} + \frac{\xi_l\left(\mathbf{V}^{l+1} - \overline{\mathbf{V}}^{l+1}\right)}{(1-\beta_l)} \qquad (84)
$$

which is exactly the parameterization implemented before we introduced the horizontal pressure gradient effect. The auxiliary variables are

$\mathsf{ZRMIX(JLON,JLEV)} \equiv \xi^l$

$\mathsf{ZBET(JLON,JLEV)} \equiv \beta^l \qquad (\mathsf{ZUM(JLON,JLEV)},\mathsf{ZVM(JLON,JLEV)}) \equiv \overline{\mathbf{V}}$

The pressure gradient coefficients are in the code $\mathcal{G}_u \equiv$ TUDGP and $\mathcal{G}_d \equiv$ TDDGP.
It appears logical to take the same value for both, and also for GCVADS (see §5.1.1.4):

$$
\mathsf{TUDGP} = \mathsf{TDDGP} = \mathsf{GCVADS} = 0.8
$$

5.2.3 Computing the cloud base velocity

We write the conservation of momentum along the vertical:

$$
\begin{aligned}
\int_{p_t}^{p_b}\left(\frac{\partial\mathbf{V}}{\partial t}\right)^u_{\text{conv}} dp &= 0 = \int_{p_t}^{p_b}\overset{\wedge}{\omega^*}\frac{\partial\mathbf{V}}{\partial p} + K_u\,(\mathbf{V}_u - \mathbf{V})dp \\
&= \int_{p_t}^{p_b}\left(\frac{\partial\mathbf{V}}{\partial t}\right)^u_{ps} dp + \left(\int_{p_t}^{p_b} K_u\mathbf{V}_u dp - \int_{p_t}^{p_b} K_u\mathbf{V}dp\right) \\
\Longrightarrow \qquad \mathbf{V}_{c0} &= \frac{-\int_{p_t}^{p_b}\left(\frac{\partial\mathbf{V}}{\partial t}\right)^u_{ps} dp + \int_{p_t}^{p_b} K_u\mathbf{V}\;dp - \int_{p_t}^{p_b} K_u(1-\beta)\overline{\mathbf{V}}\;dp}{\int_{p_t}^{p_b} K_u\beta\;dp} \qquad (85)
\end{aligned}
$$

Note that the use of a detrainment rate varying over the vertical:

$$
K_u^l\triangle t = \mathsf{ZALFP(JLON)} + \mathsf{ZALFPB(JLON,JLEV)}
$$

imposes to leave it under the integrals. The routine computes:

$$
\begin{aligned}
(\mathsf{ZS3},\mathsf{ZS4}) &= \sum_L^{l_t}\delta_{\text{stab}}\triangle p^l(\mathbf{V}^l_{ps} - \mathbf{V}^l) & (\mathsf{ZS10},\mathsf{ZS11}) &= \sum_L^{l_t}\delta_{\text{stab}}\triangle p^l\mathbf{V}^l K_u^l \\
(\mathsf{ZS13},\mathsf{ZS14}) &= \sum_L^{l_t}\delta_{\text{stab}}\triangle p^l(1-\beta^l)\overline{\mathbf{V}}^l & \mathsf{ZS12} &= \sum_L^{l_t}\delta_{\text{stab}}\triangle p^l\beta^l K_u^l
\end{aligned}
$$

5.2.4 Practical implementation: CAS calculation

The vertical integrals in previous sections concern actually the convectively active layers.
For the thermodynamic variables, the active layers alone were contributing to vertical integrals through the use of the activity index $\delta_{\text{stab}} \equiv$ KNLAB(JLON, JLEV) equal to 1 in the active layers and 0 elsewhere.
But as the active regions may not be connected over the vertical, this would be very hazardous in the momentum calculation, as we need to accumulate the auxiliary variables β and $\overline{\mathbf{V}}$, the latter depending on the vertical shear when we take into account the pressure gradient term ($\max(\mathcal{G}_u, \mathcal{G}_d) > 0$).
In this case, a global treatment of the vertical leads to some oscillating uncomfortable behaviours.

The remedy we found was to introduce a specific treatment for momentum terms, considering individual *Connected Active Segments* over the vertical, and reinitializing the auxiliary β and $\overline{\mathbf{V}}$ at the bottom of each connected active segment. Computation proceeds as follows:

- The entrainment terms ξ^l were saved at the time of entrainment computations for the thermo-dynamical variables in the first vertical loop, into variable ZRMIX(JLON,JLEV).

- We build an array of activity transitions ZTRAN(JLON,JLEV) such that

  $$\mathsf{ZTRAN} = \begin{cases} 1 & \text{at the lowest level of each \textit{recognized} connected active segment over the vertical} \\ 0 & \text{elsewhere} \end{cases}$$

  The CAS computation can be switched off (LCVCAS=.FALSE.), in this case we impose

  $$\mathsf{ZTRAN} = \begin{cases} 1 & \text{at the lowest model level} \\ 0 & \text{elsewhere} \end{cases}$$

  This should *not* be done as soon as pressure gradient coefficients TUDGP and TDDGP are not both zero.

- We perform an upward vertical loop, building the profile:

  - reset ZBET = 1 and (ZUM, ZVM) = (PU, PV) each time we meet ZTRAN = 1 over the vertical, else apply equations (81) and (82).
  - compute provisional values for cloud base velocities with equation (85) into arrays (ZA13(JLON,JLEV), ZA14(JLON,JLEV)): both are reset to zero at the lowest level of the active segments, and they reach the needed value of the cloud base velocity when arriving at the top level of an active segment; for the inactive layers above, they keep their current value.
  - propagate downwards the values of (ZA13(JLON,JLEV), ZA14(JLON,JLEV)) reproducing the values obtained just below the transition levels, so that they keep the connected segment base velocity along the whole height of each connected active segment.

Equation (80) defines completely the cloud velocity profile from variables β = ZBET, $\overline{\mathbf{V}}$ = (ZUM, ZVM), and $\mathbf{V}_{c0}$ = (ZA13, ZA14).
We developed the above calculation – which may not seem the most direct – to get best performance of the parallelized and vectorized code.

6 Cloud mass flux: the closure hypotheses

6.1 Updraught mass flux by Kuo closure

6.1.1 Formulation

To go further we need to introduce an additional expression for the updraught and downdraught mass fluxes. Following developments in [Levine , 1959], [Simpson and Wiggert, 1969] and [Simpson, 1971], we can express the vertical velocity in a single cumulus tower (actually the rate of rise of the tower) as

$$\frac{dW}{dt} = \underbrace{\frac{gB}{1+\gamma}}_{\text{buoyancy}} - \underbrace{\frac{1}{M}\frac{dM}{dz}W^2}_{\text{entrainment}} - \underbrace{\mathcal{K}_d\, W^2}_{\text{drag}} = -\frac{1}{g}\frac{d\omega_c/\rho_c}{dt} \tag{86}$$

Where gB is the buoyancy per unit mass, γ the apparent mass coefficient due to acceleration of the surrounding fluid, M the mass of the rising tower, and $\mathcal{K}_d$ a drag coefficient.
Applying this to our context in pressure coordinates,

$$\rho_c \frac{\partial \omega_c/\rho_c}{\partial t} + \rho_c \omega_c \cdot \frac{\partial \omega_c/\rho_c}{\partial p} = -\rho_c g^2 \frac{B}{1+\gamma} - \frac{1}{M_c}\frac{dM_c}{dp}\,\omega_c^2 - \mathcal{K}_d\,\omega_c^2$$

The buoyancy term is evaluated as follows:

$$gB = g\frac{T_{vc} - \overline{T}_v - \triangle T_v(\mathsf{LWC})}{\overline{T}_v} \tag{87}$$

where $\triangle T_v(\mathsf{LWC})$ is the buoyancy reduction due to the weight of suspended liquid water.
Using $M_c = -\sigma_c \cdot \omega_c$ we write:

$$\frac{\partial \omega_c}{\partial t} + \omega_c \cdot \frac{\partial \omega_c}{\partial p} \simeq -\beta \cdot \frac{T_{vc} - \overline{T}_v}{(1+\gamma')} - \omega_c \frac{\partial \omega_c}{\partial p} - \mathcal{K}_d \cdot \omega_c^2 \tag{88}$$

$$\frac{\partial \omega_c}{\partial t} \simeq -\beta \cdot \frac{T_{vc} - \overline{T}_v}{(1+\gamma')} - \frac{\partial \omega_c^2}{\partial p} - \mathcal{K}_d \cdot \omega_c^2 \tag{89}$$

$$\beta = \frac{\rho_c \cdot g^2}{\overline{T}_v} \tag{90}$$

Chen and Bougeault [1990] express the relative updraught mass flux as

$$\hat{\omega^*} = \sigma_c(\omega_c - \omega_e) = \sigma_c \cdot \omega_c^* \tag{91}$$

They then approximate roughly equation (88) with

$$\frac{\partial \omega_c^*}{\partial t} \simeq C_f \cdot \frac{\omega_c^{*2}}{P_b - P_t} - \beta \cdot \frac{\left(T_{vc} - \overline{T}_v\right)}{1+\gamma'} \tag{92}$$

where $P_b - P_t$ is the total thickness of the convective layer, and the dissipation coefficient C_f includes effects of drag and entrainment (optimal value $C_f \sim 50$).
In the buoyancy term, $\gamma' = 0.5$ is the so-called virtual mass parameter (linked to the acceleration of the surrounding fluid).

The time change of the cloud scale vertical velocity results from a balance between the dissipative processes at smaller scales and the buoyancy effects (the CAPE is the vertical integral of the buoyancy term).

The convective mass flux $\hat{\omega^*} = \alpha^* \cdot \omega_c^*$, where $\alpha^* \equiv \sigma_c$ is the fractional area of the grid box covered by updraughts.

The lifetime of a single cell is about 1 hour (say half an hour for development, half an hour for dissipation): for a large scale NWP ($\triangle t \geq 20\ min$) it is acceptable to neglect storage and vertical acceleration terms within cumulus clouds. *This is not true for a meso-scale model.*

To estimate α^* consistently with the closure hypothesis and supposing that the convective updraughts are not in stationary equilibrium with the large-scale forcing, we need to introduce an equation representing the storage of moist static energy in the cloud updraughts:

$$\frac{\partial \alpha^*}{\partial t} \cdot \int_{p_t}^{p_b} \left(h_c - \overline{h}\right)\frac{dp}{g} = L\int_{p_t}^{p_b} \alpha^* \omega_c^* \frac{\partial \overline{q}}{\partial p}\frac{dp}{g} + L \cdot \int_{p_t}^{p_b} \mathsf{CVGQ}\frac{dp}{g} \tag{93}$$

Note that this equation does not embody the conservation of moist static energy by the convective process, which will be insured by the computation of the K^c factor (equation 35).

In the case there is no storage in the closure (which is Arpège-Aladin's current status), $\frac{\partial \alpha^*}{\partial t} = 0$ and (93) reduces to Kuo's 1965 hypothesis:

Total ***M***oisture ***C***onvergence = rate of cloud water production
= ***Rained out*** water + ***Detrained*** water

expressing that cloud water has either to be disposed by precipitation or recycled in the environment by the detrainment term (detrainment of moisture means that the environmental air is evaporating some cloud water, moistening by this the environment).

In equation (92), assuming that the convective updraughts are stationary ($\frac{\partial \omega_c^*}{\partial t} = 0$) and replacing $(T_{vc} - \overline{T}_v)$ by $\left(\frac{h_c - \overline{h}}{c_p}\right)$, yields

$$\hat{\omega^*}^2 \simeq \alpha^{*2} \rho_c \left(h_c - \overline{h}\right) \left\{ \frac{p_b - p_t}{T_v} \; \frac{g^2}{c_p C_f (1+\gamma')} \right\} = \rho_c \left(h_c - \overline{h}\right) \alpha^2 \tag{94}$$

or

$$\hat{\omega^*} = \alpha \sqrt{\rho_c \left(h_c - \overline{h}\right)} \tag{95}$$

In the code, we neglect furthermore the difference between ρ and ρ_c, and use actually $\frac{p}{T}$, rejecting a value of R_c into the α coefficient.

$$\hat{\omega^*} = \alpha' \sqrt{\rho \left(h_c - \overline{h}\right)} = \alpha \sqrt{\frac{p}{T} \left(h_c - \overline{h}\right)} \tag{96}$$

Coefficient α is dimensional, and is determined by expressing Kuo's 1965 hypothesis:

Total ***M***oisture ***C***onvergence = rate of cloud water production
= ***Rained out*** water + ***Detrained*** water

expressing that as there is no storage within the closure, cloud water has either to be disposed by precipitation or recycled in the environment by the detrainment term (detrainment of moisture means that the environmental air is evaporating some cloud water, moistening by this the environment).
We may express those two effects as:

- Rainfall rate (rough value produced by the updraught, before evaporating some of it either below the cloud or in the downdraught):

$$\mathcal{P} \equiv \int_{p_t}^{p_b} Q_2^{cu} \frac{c_p}{L} \frac{dp}{g} = - \int_{p_t}^{p_b} \left[\hat{\omega^*} \frac{\partial q}{\partial p} + K \left(q_u - q\right) \right] \frac{dp}{g} - \left(J_q(p_b) - J_q(p_t)\right) \tag{97}$$

- Detrained moisture: environment moistening + moisture taken away by the divergence of the turbulent diffusion flux:

$$M_{\mathrm{detr}} = \int_{p_t}^{p_b} K \left(q_u - q\right) \frac{dp}{g} + \left(J_q(p_b) - J_q(p_t)\right) \tag{98}$$

The turbulent vertical diffusion term, if used, has to appear in both expressions, in order to close the moisture budget. So it disappears when adding $\mathcal{P} + M_{\mathrm{detr}}$.

For the total moisture convergence, the scheme's important idea is to add the effect of turbulent vertical diffusion to the dynamical convergence. This was suggested by Kuo and validated by several long term measurements over the tropics.

$$TMC = \int_{p_t}^{p_b} \mathsf{CVGQ} \frac{dp}{g} = - \int_{p_t}^{p_b} \mathcal{R} \left[\mathbf{V} \cdot \nabla q + \omega \frac{\partial q}{\partial p} \right] \frac{dp}{g} + \left(J_q(p_t) - J_q(p_b)\right) + \delta_{\mathrm{srcon}} \left(\mathcal{P}_{\mathrm{LS}}(p_t) - \mathcal{P}_{\mathrm{LS}}(p_b)\right) \tag{99}$$

where the modulation factor $\mathcal{R}$ was introduced in §2 to take into account mesh size effects, and δ_{srcon} is 1 if LSRCON = .TRUE. and 0 otherwise.

Writing $TMC = \mathcal{P} + M_{\mathrm{detr}}$ yields

$$- \int_{p_t}^{p_b} \hat{\omega^*} \frac{\partial q}{\partial p} \frac{dp}{g} = - \int_{p_t}^{p_b} \mathcal{R} \left[\mathbf{V} \cdot \nabla q + \omega \frac{\partial q}{\partial p} \right] \frac{dp}{g} + \left(J_q(p_t) - J_q(p_b)\right) + \delta_{\mathrm{srcon}} \left(\mathcal{P}_{\mathrm{LS}}(p_t) - \mathcal{P}_{\mathrm{LS}}(p_b)\right) \tag{100}$$

In a convective situation, J_q is pointing upwards, i.e. has a negative value, more negative near the surface, and the divergence term $(J_q(p_t) - J_q(p_b))$ should be positive.

As the horizontal advection terms for moisture are generally small, if we consider the reference case where $\mathcal{R} = 1$, we can see that it is this vertical turbulent diffusion term that will render $|\hat{\omega^*}| > |\omega|$, allowing some absolute subsidence outside the convective clouds, which is physically needed for having convection rather than stratiform rain.

In this way, we simulate the (observed) fact that it is the surface evaporation (or rather its part not converted in stratiform precipitation) which is the main responsible item for the cloud vertical velocity excess, rather than a mere channelling of the large scale mass convergence.

The "smart" method (LSRCON=.TRUE.) for taking into account the mesh size effects subtracts the (downward) divergence of large scale precipitation from the moisture convergence CVGQ (so reducing the moisture convergence amount actually feeding the convective process). When the large scale precipitation increases downwards, $\mathcal{P}_{\mathrm{LS}}(p_t) - \mathcal{P}_{\mathrm{LS}}(p_b) < 0$ and the effect will be to reduce available moisture convergence as intended.

α is then obtained from (100):

$$\alpha = \frac{\int_{p_t}^{p_b} \mathsf{CVGQ}\frac{dp}{g}}{\int_{p_t}^{p_b} \left[\rho(h_u - h)\right]^{1/2} \frac{\partial q}{\partial p} dp} \tag{101}$$

6.1.2 Convective activity index

For Kuo's closure, the layer is declared active when the two conditions: moisture convergence and positive buoyancy, are both satisfied. We write the buoyancy force as

$$\mathsf{ZKUO1} \equiv [R_a(1 - \ell_u) + (R_v - R_a)q_u]T_u - RT = R_aT_{vu} - R_aT_v > 0 \tag{102}$$

while the convergence is assessed through

$$\mathsf{ZKUO2} \equiv \sum_{\mathsf{KLEV}}^{l} \triangle p^l L^l_{\mathrm{eff}} \mathsf{CVGQ}^l \tag{103}$$

and the activity index is defined by

$$\mathsf{KNLAB(JLON,JLEV)} \equiv \delta^l_{\mathrm{stab}} = \max\left(\delta^{l+1}_{\mathrm{stab}}, \begin{cases} 1 & \text{where } \min(\mathsf{ZKUO1},\mathsf{ZKUO2}) > 0 \\ 0 & \text{elsewhere} \end{cases}\right) \tag{104}$$

Note that there is at this stage of the computation no test of the capacity to reach the Lifting Condensation Level: see §7.3.

6.1.3 Kuo closure: Discretization

The first vertical loop computes

$$\mathsf{ZHCMHL} \equiv (h_c - h)^l = c^l_p(T^l_c - T^l_w) + L^l_{\mathrm{eff}}(q^l_c - q^l_w) \tag{105}$$

and keeps the memory of $(h_c - h)^{l+1} = \mathsf{ZHCMH(JLON)}$, so we can estimate the value at the interface level $\bar{l}$. We store then

$$\mathsf{ZFORM(JLON,JLEV)} = \delta^l_{\mathrm{stab}} \sqrt{\left(\mathsf{ZHCMH}^l + \mathsf{ZHCMH}^{l+1}\right)\frac{p^{\bar{l}}}{T^l + T^{l+1}}} \tag{106}$$

$$= \mu\, \delta^l_{\mathrm{stab}} \sqrt{(h_c - h)^{\bar{l}} \rho^{\bar{l}}}\,, \qquad \mu = \sqrt{2R^{\bar{l}}} \tag{107}$$

To compute α we use local integrals:

$$\mathsf{ZFMDQL} = \frac{\mathsf{ZFORM}^{\bar{l}}}{2}\left(q^{l+1} - q^l\right)$$

$$\mathsf{ZS6} = \sum_L^l \delta_{\text{stab}}^{l+1} \, (\mathsf{ZFMDQ}^{\overline{l}} + \mathsf{ZFMDQ}^{\overline{l+1}}) L_{\text{eff}}^{l+1} = \sum_L^l \delta_{\text{stab}}^{l+1} \, \left\{ \mu \sqrt{(h_c - h)\,\rho} \, \triangle q \right\}^{l+1}$$

$$\mathsf{ZS1} = \sum_L^l \delta_{\text{stab}}^{l+1} \mathsf{ZICVG}^{l+1} = \sum_L^l \delta_{\text{stab}}^{l+1} \, \{ L_{\text{eff}} \; \mathsf{CVGQ} \; \triangle p \}^{l+1}$$

yielding finally

$$\mathsf{ZALF(JLON)} \equiv \alpha = \triangle t_{\text{phys}} \frac{\mathsf{ZS1}}{\mathsf{ZS6}} \tag{108}$$

with usual precautions in the code against a null value of ZS6. The final value of the updraught mass flux is then

$$\mathsf{ZFORM(JLON, JLEV)} \equiv (-\overset{\wedge}{\omega^*} \triangle t)^{\overline{l}} = \mathsf{ZALF(JLON)} * \mathsf{ZFORM(JLON, JLEV)} \tag{109}$$

Note that we use here the local effective latent heat, as we are assessing intra cloud water transformations.

6.2 Downdraught mass flux by Kuo Closure

6.2.1 Formulation

The magnitude of the downdraught mass flux $\overset{\vee}{\omega^*}$ should depend on *static stability* and of the amount of *available precipitation* to initiate and maintain the downdraught. We postulate an expression similar to the one obtained for the updraught:

$$\overset{\vee}{\omega^*} \propto (h_d - h)^{1/2} \tag{110}$$

To modulate the discontinuity as the downdraught reaches the ground, a shaping function will be applied.

The closure hypothesis states now that a fraction ϵ of the total available precipitation for the downdraught is used to moisten the environment through the pseudo ascent:

$$\int_{p_t}^{p_b} \overset{\vee}{\omega^*} \frac{\partial q_d}{\partial p} \frac{dp}{g} = \epsilon \, \mathcal{P} \tag{111}$$

The resulting net precipitation is therefore $(1-\epsilon)\mathcal{P}$, and the evaporation rate ϵ represents a degree of freedom of the parameterization (GDDEVA).

A second tunable parameter was introduced: GDDSDE, the exponent of the modulation function applied to the downdraught mass flux to attenuate the full breaking of the draught as it meets the ground:

$$\mathcal{F}(p) = \left(\frac{p_s - p}{p_s - p_t} \right)^{\mathsf{GDDSDE}}$$

where p_t is the pressure at the top level of the downdraught and p_s the surface pressure.
Experiments showned a better behaviour when applying the shaping function before the computations of (111).
The recommended values for the two parameters are: GDDEVA $= 0.25$ and GDDSDE $= 0.5$.
Remark: In presence of downdraught parameterization (LCVDD=.TRUE.), the sub-cloud evaporation correction has to be switched off (LCVEVAP=.FALSE.) (§7.1.2).

6.2.2 Downdraught activity index

Symmetrically to the updraught, we diagnose the downdraught activity through a double Kuo test: negative buoyancy and precipitation flux convergence. Negative buoyancy is diagnosed by

$$\mathsf{ZKUO1} = (R_a + (R_v - R_a) q_d^l) T_d^l - R^l T^l = R_a T_{vd} - R_a T_v < 0$$

and precipitation convergence with

$$\mathsf{ZKUO2} = \sum_{k=1}^{l} g \left(\mathcal{P}^{k-1} - \mathcal{P}^k \right) L_{\text{eff}}^k$$

where both KUO1 and KUO2 are estimated at full model levels. The activity index is then given by

$$\mathsf{INLAB(JLON, JLEV)} \equiv \delta_{\text{stab}}^{\downarrow l} = \max \left(\delta_{\text{stab}}^{\downarrow l-1}, \, \delta_{\text{stab}}^{l-1} \cdot \left\{ \begin{array}{ll} 1 & \text{where } \max(\mathsf{ZKUO1}, \mathsf{ZKUO2}) < 0 \\ 0 & \text{elsewhere} \end{array} \right. \right)$$

so if present, downdraught activity extends down to the ground.

6.3 Updraught mass flux by CAPE Closure

6.3.1 Theory

$$\mathsf{CAPE} = -\int_{p_t}^{p_b} \frac{T_{vc} - \overline{T}_v}{\overline{T}_v} \frac{dp}{\rho} = -\int_{p_t}^{p_b} R_a (T_{vc} - \overline{T}_v) \frac{dp}{p} \tag{112}$$

The cloud-scale virtual temperature T_{vc} should take into account the liquid water suspended into the cloud:

$$T_{vc} = T_c \left[1 + \frac{R_c - R_a}{R_a}\, q_c \right] - \frac{L}{c_p}\, \ell \tag{113}$$

Note that Bougeault [1985]'s scheme uses an approximation of the CAPE based on moist static energy: $T_{vc} - \overline{T}_v$ is replaced by $\frac{h_c - h}{c_p}$, yielding a much larger estimate than the previous expression, which results in an adaptation of the magnitude of the parameters.

We write:

$$\begin{aligned} T_{vc} &= T_c(1 - \ell_c + \mu q_c) \qquad & T_v = T(1+\mu q) \\ \mu &= \frac{R_v - R_a}{R_a} \qquad & R = R_a(1 - q - l) + R_v q \end{aligned}$$

From developments of Fritsch and Chappel [1980] and Nordeng [1994]:

$$\frac{\partial \mathsf{CAPE}}{\partial t} = -\frac{\mathsf{CAPE}}{\tau}$$

Assessing orders of magnitude, yields:

$$Q_1^{cu} = (\underbrace{\overset{\wedge}{\omega^*} \frac{\partial s}{\partial p}}_{10^3} + \underbrace{K(s_c - s)}_{10} + \underbrace{g\frac{\partial J_s}{\partial p}}_{10^{-1}}) \frac{1}{c_p} \implies \frac{\partial T}{\partial t} \simeq \frac{\overset{\wedge}{\omega^*}}{c_p} \frac{\partial s}{\partial p}$$

$$-\frac{Q_2^{cu}}{L} = (\underbrace{\overset{\wedge}{\omega^*} \frac{\partial q}{\partial p}}_{10} + \underbrace{K(q_c - q)}_{10^{-6}} + \underbrace{g\frac{\partial J_q}{\partial p}}_{10^{-10}}) \frac{1}{c_p} \implies \frac{\partial q}{\partial t} \simeq \overset{\wedge}{\omega^*} \frac{\partial q}{\partial p}$$

We have then

$$\begin{aligned} \frac{\partial \mathsf{CAPE}}{\partial t} &= -\int_b^t \frac{\partial}{\partial t} \frac{T_{vc} - T_v}{T_v} g dz \simeq \int_b^t \frac{1}{T_v} \frac{\partial T_v}{\partial t} g dz = \int_{p_t}^{p_b} R_a \frac{\partial T_v}{\partial t} \frac{dp}{p} \\ \frac{\partial T_v}{\partial t} &= (1+\mu q) \frac{\partial T}{\partial t} + \mu T \frac{\partial q}{\partial t} \approx (1+\mu q)(\frac{\overset{\wedge}{\omega^*}}{c_p} \frac{\partial s}{\partial p}) + \mu T \overset{\wedge}{\omega^*} \frac{\partial q}{\partial p} \\ &= \overset{\wedge}{\omega^*} \left[(1+\mu q)(\frac{1}{c_p}\frac{\partial s}{\partial p}) + \mu T \frac{\partial q}{\partial p} \right] \end{aligned}$$

Keeping the form

$$\overset{\wedge}{\omega^*} = -\alpha \sqrt{\rho(h_c - h)}$$

and integrating over the cloud:

$$-\alpha \int_{p_t}^{p_b} \left[(1+\mu q)\sqrt{\rho(h_c - h)}(\frac{1}{c_p}\frac{\partial s}{\partial p}) + \mu \sqrt{\rho(h_c - h)} T \frac{\partial q}{\partial p} \right] \frac{dp}{p} = \int_{p_t}^{p_b} \frac{\partial T_v}{\partial t} \frac{dp}{p}$$

$$\alpha = \frac{1}{\tau} \frac{\mathsf{CAPE}}{-\int_{p_t}^{p_b} \left[(1+\mu q)\sqrt{\rho(h_c - h)}(\frac{1}{c_p}\frac{\partial s}{\partial p}) + \mu \sqrt{\rho(h_c - h)} T \frac{\partial q}{\partial p} \right] \frac{dp}{p}}$$

6.3.2 Convective activity indexes

Unlike Kuo's closure, we diagnose the layer activity solely through its buoyancy, so the test of equation(104) uses now ZKUO1 only to diagnose δ_{stab}.

6.3.3 Discretization

We compute

$$\begin{aligned}
\mathsf{ZS15} &\equiv \frac{\mathsf{CAPE}}{R_a} = \sum_{L-1}^{1} \delta_{\text{stab}}^{l+1} \frac{\triangle p^{l+1}}{p^{l+1}} \left[T_{vc}^{l+1} - T_v^{l+1}\right] \\
&= \sum_{L-1}^{1} \delta_{\text{stab}}^{l+1} \frac{\triangle p^{l+1}}{p^{l+1}} \left[T_c^{l+1}(1 - \ell_c^{l+1} + q_c^{l+1}\mu) - T^{l+1}(1 + q^{l+1}\mu)\right] \\
\mathsf{ZS16} &= -\sum_{L-1}^{1} \delta_{\text{stab}}^{l+1} \left\{(1 + q^{l+1}\mu)\frac{1}{c_p^{l+1}}\left[\mathsf{ZFORM} * \frac{\triangle s}{\triangle p}\right]^{l+1} + T^{l+1}\mu\left[\mathsf{ZFORM} * \frac{\triangle q}{\triangle p}\right]^{l+1}\right\}\left(\frac{\triangle p^{l+1}}{p^{l+1}}\right) \\
&= -\sum_{L-1}^{1} \delta_{\text{stab}}^{l+1} \left\{(1 + q^{l+1}\mu)\left(\frac{\partial T}{\partial t}\right)^{l+1} + T^{l+1}\mu\left(\frac{\partial q}{\partial t}\right)^{l+1}\right\}\left(\frac{\triangle p^{l+1}}{p^{l+1}}\right) \\
&= -\sum_{L-1}^{1} \delta_{\text{stab}}^{l+1} \frac{\triangle p^{l+1}}{p^{l+1}} \frac{\partial T_v}{\partial t}
\end{aligned}$$

In the code,

- RETV $\equiv \mu = \frac{R_v - R_a}{R_a}$
- PDELP $\equiv \triangle p^l \equiv (p^{\bar{l}} - p^{\overline{l-1}}) > 0$ is layer l's pressure thickness
- PAPRSF $\equiv p^l$ is the pressure at the full level
- ZTNL $\equiv T_c^{l+1}$, ZQNL $\equiv q_c^{l+1}$, ZLNL $\equiv \ell_c^{l+1}$ are the updraught temperature moisture and condensed phase at the level below current level;
- ZFORM represents the updraught mass flux (not yet normalized)
- PTAUX $\equiv \tau$ is defined as

$$\mathsf{PTAUX(JLON)} = \mathsf{RTCAPE} * \left(\mathsf{PGM(JLON)} * \frac{\triangle x_{\text{ref}}}{\triangle x_{\text{equiv}}}\right)^{\mathsf{GCOMOD}}$$

so

$$(\mathsf{ZFMDQ} + \mathsf{ZFMDQL}) = \frac{(\mathsf{ZFORM} * \triangle q)^{\overline{l+1}} + (\mathsf{ZFORM} * \triangle q)^{\bar{l}}}{2} = \mathsf{ZFORM}^{l+1} * \frac{\triangle q^{l+1}}{\triangle p^{l+1}} \triangle p^{l+1} \sim \frac{\partial q}{\partial t} \triangle p^{l+1}$$

The mass flux normalisation factor is then

$$\alpha = \frac{\triangle t_{\text{phys}}}{\mathsf{PTAUX}} \begin{cases} \left(\frac{\mathsf{ZS15}}{\mathsf{ZS16}}\right) & \text{where } \mathsf{ZS15} > 0 \text{ and } \mathsf{ZS16} > 10^{-6} \\ \left(\frac{\mathsf{ZS15}}{10^{-6}}\right) & \text{where } \mathsf{ZS15} > 0 \text{ and } 0 < \mathsf{ZS16} < 10^{-6} \\ 0 & \text{elsewhere} \end{cases} \tag{114}$$

7 Tendency and flux calculations

7.1 Updraught fluxes

The routine has to return the liquid and solid precipitation fluxes: PFPLCL, PFPLCN, the convective moisture and dry static energy fluxes (linked to pseudo subsidence): PDIFCQ, PDIFCS and the convective horizontal momentum fluxes: PSTRCU, PSTRCV.

7.1.1 Moisture budget

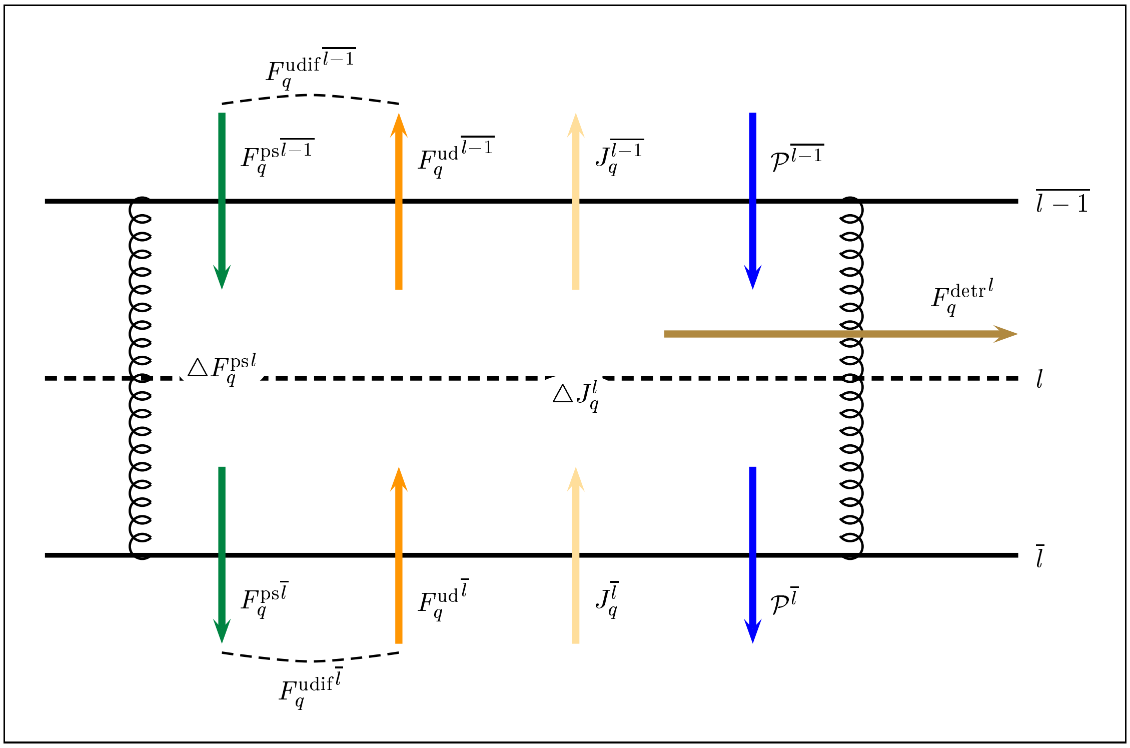


Figure 6: Updraught water budget for a layer

The different fluxes represented on figure 6 (note that the arrows indicate the *probable direction* of the fluxes, while the positive direction is always downwards) are grouped as follows in the calculations:

- The moisture advection flux resulting of the updraught and its pseudo subsidence (referred as *"convective diffusion flux"* by analogy with the *"turbulent diffusion flux"*).
  Note that at this stage the array PDIFCQ contains only the contribution of the updraught (as required by our budget), while at the end of the routine it will receive the sum of updraught and downdraught contributions.

$$\mathsf{PDIFCQ}^{\overline{l-1}} = \left(F_q^{\mathrm{udif}}\right)^{\overline{l-1}} = +\frac{(\hat{\omega^*})^{\overline{l-1}}}{g}\, q_u^{\overline{l-1}} - \frac{(\hat{\omega^*})^{\overline{l-1}}}{g}\, q_{ps}^{\overline{l-1}} = \frac{(-\hat{\omega^*})^{\overline{l-1}}}{g}\, (q_{ps}^{\overline{l-1}} - q_u^{\overline{l-1}}) \qquad (115)$$

  There is downwards moisture advection (PDIFCQ > 0) when the environment corrected by the pseudo subsidence is moister than the updraught. Reversely, a negative value of PDIFCQ means that pseudo subsidence brings drier air downwards.

- The precipitation flux from the upper layers

$$\mathcal{P}^{\overline{l-1}} = \mathcal{P}_w^{\overline{l-1}} + \mathcal{P}_i^{\overline{l-1}}$$

- The flux resulting from the updraught activity in layer l (only if it is active, i.e if $\delta_{\mathrm{stab}} = 1$).

$$F_{q_c}^l = \triangle F_q^{\mathrm{ps}\,l} + F_q^{\mathrm{detr}\,l} - \triangle J_q^l \qquad (116)$$

  where we find:

  - the generation of pseudo subsidence (33)

$$\triangle F_q^{ps\,l} = -\delta_{\mathrm{stab}}^l \frac{\triangle p^l}{g\triangle t}\,(q_{ps}^l - q^l)$$

– the detrainment effects

$$F_q^{\mathrm{detr}^l} = -\delta_{\mathrm{stab}}^l \frac{\triangle p^l}{g\triangle t} K^u \;\left(q_u^l - q^l\right)$$

– subtraction of the local divergence of the vertical turbulent diffusion flux (PDIFTQ):

$$-\triangle J_q^l = -\delta_{\mathrm{stab}} \left[J_q\right]_{\overline{l-1}}^{\overline{l}}$$

This term could be replaced by its mean value over the convective layer, as was found useful for the divergence of dry static energy turbulent diffusion flux, in order to keep a symmetry between moisture and energy treatments (see below). Simple averaging is not appropriate as it would mix very different moisture tendencies between the bases and the tops of the cumulus towers: therefore, it was proposed to apply a normalization by the saturating humidity:

$$\mathsf{ZS8S} = \frac{\mathsf{ZS8}}{L_{\mathrm{bud}}\mathsf{ZS5S}} = \frac{\sum_1^L \delta_{\mathrm{stab}}^l \triangle p^l \;\; \frac{g\triangle t}{\triangle p^l} q_{\mathrm{sat}}^l L_{\mathrm{bud}} \left[J_q^{l-1} - J_q^l\right]}{L_{\mathrm{bud}} \sum_1^L \delta_{\mathrm{stab}}^l \triangle p^l q_{\mathrm{sat}}^l}$$

and we introduce a parameter $0 \leq \mathsf{ZCVPSI} \leq 1$ modulating the respective effects of the local divergence and the mean divergence of the moisture vertical turbulent diffusion flux. Control of ZCVPSI is discussed farther.

The water substance budget may be written as

$$\begin{aligned}
\mathsf{ZFTOTQ} \;\; &\equiv \;\; F_{\mathrm{w_tot}}^{\overline{l}} = (F_q^{\mathrm{udif}})^{\overline{l}} + \mathcal{P}^{\overline{l}} \\
&= \;\; (F_q^{\mathrm{udif}})^{\overline{l-1}} + \mathcal{P}^{\overline{l-1}} \\
&\quad -\delta_{\mathrm{stab}} \left\{ \frac{\triangle p^l}{g\triangle t} \left\{ (q_{ps}^l - q^l) + K^u (q_u^l - q^l) - \mathsf{ZS8S} \cdot q_{\mathrm{sat}}^l (1 - \mathsf{ZCVPSI}) \right\} + \mathsf{ZCVPSI} \cdot \left(J_q^{\overline{l}} - J_q^{\overline{l-1}} \right) \right\}
\end{aligned}$$

ZFTOTQ must be output at the layer bottom, through pseudo subsidence diffusion and precipitation (downwards) or a moisture correction upwards in case the budget would yield negative precipitation:

$$0 \leq \mathsf{ZFCORQ} = \begin{cases} 0 \\ F_q^{\mathrm{udif}^{\overline{l}}} - F_{\mathrm{w\_tot}}^{\overline{l}} \end{cases} \qquad \text{or} \qquad 0 \leq \mathcal{P}^{\overline{l}} = \begin{cases} F_{\mathrm{w\_tot}}^{\overline{l}} - F_q^{\mathrm{udif}^{\overline{l}}} \\ 0 \end{cases} \tag{117}$$

The correction flux ZFCORQ should be generally zero, but it has to be evaluated exactly to apply the suitable correction in the enthalpy budget.

The precipitation flux is then parted between solid ($\mathcal{P}_i$) and liquid ($\mathcal{P}_w$) precipitation through the snow rate $\alpha_{\mathrm{snow}} \equiv$ ZSNP:

$$\mathsf{PFPLCL} \equiv \mathcal{P}_w^{\overline{l}} = \mathcal{P}^{\overline{l}}(1 - \alpha_{\mathrm{snow}}^{\overline{l}}) \qquad \mathsf{PFPLCN} \equiv \mathcal{P}_i^{\overline{l}} = \mathcal{P}^{\overline{l}} \alpha_{\mathrm{snow}}^{\overline{l}} \tag{118}$$

$$\alpha_{\mathrm{snow}} = \begin{cases} 1 & \text{if } T^{\overline{l}} \leq T_t \\ 1 - \mathsf{USDMLT} \frac{\left(T^{\overline{l}} - T_t\right)^2}{p^{\overline{l}}} & \text{if } \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{119}$$

where $T_t \equiv$ RTT is the triple point temperature, and USDMLT is a constant parameter ($\sim 12500[Pa/K^2]$) representing the speed of solid to liquid transition.
As α_{snow} depends only of large scale pressure and temperature at the given level, it ignores the nature of the precipitation coming from the upper levels.
Seen the various computations for the preservation of integral quantities, it was impossible to write even a simplified microphysical scheme, that would have followed the phase of the precipitation along its fall, as was done for the large scale precipitation (part II).

Following some problems of unwished freezing of the falling precipitation in the case of inversions, the snow precipitating fraction α_{snow} is now computed as

$$\frac{d\alpha_{\text{snow}}}{d\phi} = \mathsf{GCVMLT} \cdot (T - T_t) \tag{120}$$

This equation is integrated from the top of the atmosphere to the surface:

$$\alpha^l_{\text{snow}} = \alpha^{l-1}_{\text{snow}} + \mathsf{GCVMLT} \cdot (T - T_t) \cdot (\phi^l - \phi^{l-1})$$

imposing at each step of the integration $0 \le \alpha_{\text{snow}} \le 1$.

7.1.2 Evaporation of the precipitation

In absence of downdraught parameterization (LCVDD=.FALSE.), precipitation evaporation in the sub-cloud (inactive) layers may be introduced, under control of the model key LCVEVAP:

$$\left\{\mathsf{LCVEVAP} \text{ and } \delta^l_{\text{stab}} = 0 \text{ and } \mathcal{P}^{\bar{l}} > 0\right\} \Longrightarrow \mathcal{P}^{\bar{l}} = \mathcal{P}^{\bar{l}} - \frac{\triangle p^l}{g\triangle t} \underbrace{(1 - \delta^l_{\text{stab}}) K'^u \left(q^l_u - q^l\right)}_{\mathsf{ZECORQ}}$$

i.e we remove the detrained moisture flux from the precipitation flux (as long as the result stays positive). When the downdraught parameterization is active, the precipitation evaporation flux is estimated (see §7.2.1) over the whole downdraught height, which normally extends down to the surface: the simplified scheme must then be disabled, by setting LCVEVAP = .FALSE..
ZECORQ here represents yet another moisture correction, the removed water being merely evacuated to the ground, which is very similar to the treatment of ZFCORQ above or the similar corrections in the downdraught part.

In the updraught ZFCORQ represented a moisture correction to prevent negative precipitation (for instance after excessive moisture detrainment) in the active layers: ZFCORQ was a local addition to the precipitation flux to prevent it to be negative. The corresponding water added to the vertical column was taken from the ground (coherently with the moist adiabatic method), and the the column has to exchange heat to integrate this new water, so ZFCORQ is multiplied by the budget latent heat in the enthalpy budget (§7.1.3).

In the downdraught (§7.2.1), another ZFCORQ prevents negative precipitation evaporation (for instance after excessive downdraught air detrainment) by removing excess water and reversely ZFCORQ1 brings the missing water when the the evaporation would exceed the available precipitation. Again, corresponding corrections in the enthalpy budget use the budget latent heat - as long as the additions or subtraction to the precipitation flux occur in draught-active layers.

The final correction of the enthalpy flux where precipitation falls to zero in equation (122) ensures the budget correction corresponding to excess evaporation.

7.1.3 Enthalpy budget

The enthalpy flux is computed via the flux latent heat.
The same components are intervening as in the moisture budget, so we have:

$$\begin{aligned}
\mathsf{ZFTOTS} \equiv F^{\bar{l}}_{\text{s_tot}} &= (F^{\text{udif}}_s)^{\overline{l-1}} - L^{\overline{l-1}}_{\text{flux}}(\mathcal{P})^{\overline{l-1}} - \delta_{\text{stab}} \left\{ \frac{\triangle p^l}{g\triangle t} \left((s^l_{ps} - s^l) + K^u(s^l_u - s^l)\right) + \underline{[J_s]^{\bar{l}}_{\overline{l-1}}} \right\} \\
&= (F^{\text{udif}}_s)^{\bar{l}} - L^{\bar{l}}_{\text{flux}}(\mathcal{P})^{\bar{l}} - L_{\text{bud}}\ \mathsf{ZFCORQ}
\end{aligned}$$

Unlike ZFTOTQ, ZFTOTS represents the total *sensible* heat, hence the minus sign of $-L_{\text{flux}}\mathcal{P}$ as its convergence through evaporation implies a reduction of sensible heat.
For the peace of the computation, we could replace the underlined term by its mean value, in order to avoid competition between dry instability and convection: without this, the two enthalpy sources compensate but

diverge, leading to explosion. The mean is obtained by dividing the already used ZS9 by the cumulated pressure thickness of the active layers stored into ZS5(JLON):

$$\mathsf{ZS9} = \frac{\mathsf{ZS9}}{\mathsf{ZS5}} = \frac{\sum_1^L \delta^l_{\mathrm{stab}} \triangle p^l \quad \frac{g\triangle t}{\triangle p^l}\left[J_s^{l-1} - J_s^l\right]}{\sum_1^L \delta^l_{\mathrm{stab}} \triangle p^l}$$

We introduce a free parameter $0 \leq \mathsf{GCVPSI} \leq 1$ allowing a continuous transition from non-averaging state ($\mathsf{GCVPSI} = 1$) to fully averaging state ($\mathsf{GCVPSI} = 0$).
The complete formulation is then:

$$\begin{aligned} \mathsf{ZFTOTS} &\equiv F^{\bar{l}}_{\mathrm{s\_tot}} \\ &= (F_s^{\mathrm{udif}})^{\overline{l-1}} - L^{\overline{l-1}}_{\mathrm{flux}}(\mathcal{P})^{\overline{l-1}} \\ &\quad -\delta_{\mathrm{stab}}\left\{\frac{\triangle p^l}{g\triangle t}\left\{(s^l_{ps} - s^l) + K^u(s^l_u - s^l) - \mathsf{ZS9}\cdot(1-\mathsf{GCVPSI})\right\} + \mathsf{GCVPSI}\cdot\left(J_s^{\bar{l}} - J_s^{\overline{l-1}}\right)\right\} \end{aligned}$$

It seems logical to apply the apply the same averaging to fluxes J_s and J_q, i.e. to ZCVPSI to GCVPSI. This is controlled by the model key LCVLIS:

$$\mathsf{LCVLIS} \implies \{\mathsf{ZCVPSI} = \mathsf{GCVPSI}\}$$

The historical situation, where the smoothing was applied only to the enthalpy (which was more critical), is still available:

$$.\mathsf{NOT.LCVLIS} \implies \{\mathsf{GCVPSI} = 0 \text{ and } \mathsf{ZCVPSI} = 1\}$$

With the weighting of the specific moisture by saturating humidity, the symmetrical treatment seems much more logical, as well as the the choice $\mathsf{GCVPSI} = 0.5$ which avoids a fictive time step dependency in the case either GCVPSI or $1 - \mathsf{GCVPSI}$ would be close to zero.

If the scheme is precipitating, the dry static energy convective diffusion flux is given by

$$\mathsf{PDIFCS} \equiv (F_s^{\mathrm{udif}})^{\bar{l}} = F^{\bar{l}}_{\mathrm{s\_tot}} + L^{\bar{l}}_{\mathrm{flux}}\mathcal{P}^{\bar{l}} \tag{121}$$

but if not, we have to correct:

$$(F_s^{\mathrm{udif}})^{\bar{l}} = F^{\bar{l}}_{\mathrm{s\_tot}} + L_{\mathrm{bud}}\left[F^{\bar{l}}_{\mathrm{w\_tot}} - \mathcal{P}^{\bar{l}}\right] \tag{122}$$

The convective enthalpy flux due to precipitation is obtained through the flux latent heat.

7.1.4 Momentum fluxes

Rewriting equation (73) for the updraught only, and introducing equation (80) for the updraught velocity profile,

$$\begin{aligned} \left(\frac{\partial \mathbf{V}}{\partial t}\right)^u_{\mathrm{conv}} = -g\frac{\partial F^u_{\mathbf{V}}}{\partial p} &= \left(\frac{\partial \mathbf{V}}{\partial t}\right)_{ps} + K_u(\mathbf{V}_u - \mathbf{V}) \\ &= \frac{\mathbf{V}_{ps} - \mathbf{V}}{\triangle t} - K_u\ \mathbf{V} + K_u(1-\beta)\overline{\mathbf{V}} + \beta\ K_u\ \mathbf{V}_{c0} \\ &= \frac{\mathbf{V}_{ps} - \mathbf{V}(1+K'_u) + K'_u(1-\beta)\overline{\mathbf{V}} + \beta\ K'_u\ \mathbf{V}_{c0}}{\triangle t} \end{aligned} \tag{123}$$

using the adimensional K'_u

$$K'_u \equiv \mathsf{ZALFS} = \frac{K_u\ \triangle t}{1 + K_u\ \triangle t}$$

for stability purpose, i.e. the explicit equation:

$$\mathbf{V}^+ - \mathbf{V} = K\triangle t(\mathbf{V}_u - \mathbf{V})$$

is replaced by an implicit:

$$\mathbf{V}^{+} - \mathbf{V} = K\triangle t(\mathbf{V}_u - \mathbf{V}^{+}) \Longrightarrow (\mathbf{V}^{+} - \mathbf{V})(1 + K\triangle t) = K\triangle t(\mathbf{V}_u - \mathbf{V})$$

The horizontal momentum vertical convective diffusion flux (PSTRCU, PSTRCV) $\equiv F^u_{\mathbf{V}}$ is then

$$(F^u_{\mathbf{V}})^{\overline{l}} = (F^u_{\mathbf{V}})^{\overline{l-1}} - \delta^l_{\text{stab}} \frac{\triangle p^l}{g\triangle t} \left\{ \mathbf{V}^l_{ps} - \mathbf{V}^l(1 + K'_u) + K'_u(1-\beta^l)\overline{\mathbf{V}}^l + \beta^l\ K'_u\ \mathbf{V}_{c0} \right\} \tag{124}$$

7.2 Downdraught fluxes

The downdraught routine ACCVIMPD receives on input the diffusive moisture, static energy and momentum fluxes and the precipitation fluxes computed by the updraught; it has to modify their values, according to downdraught effects.

7.2.1 Moisture budget

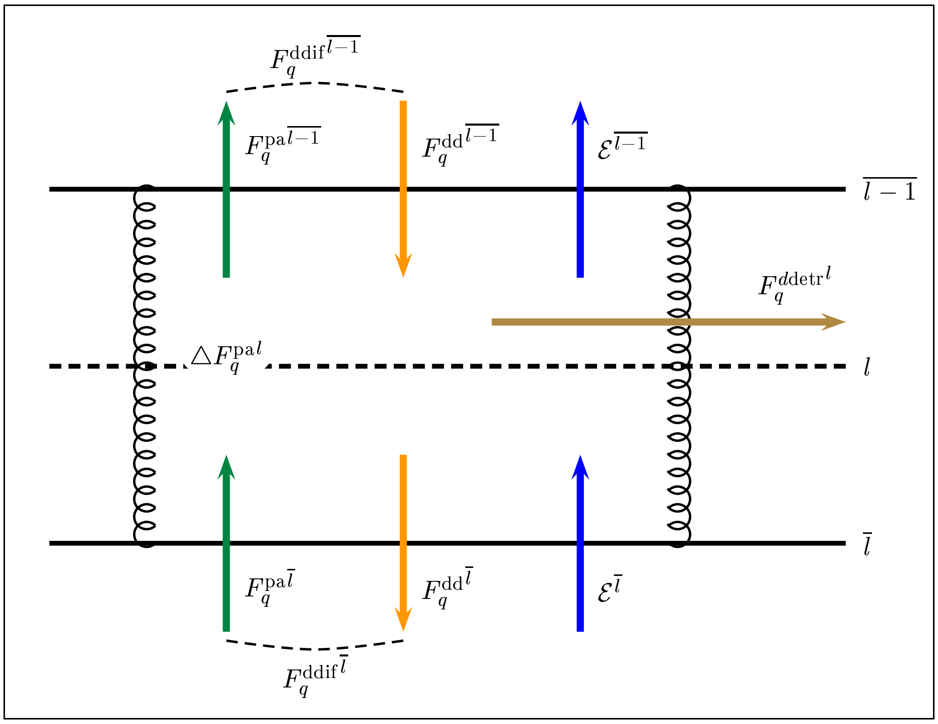


Figure 7: Downdraught water budget for a layer

The different fluxes represented on figure 7 (again, the arrows indicate the probable direction of the fluxes, while the positive direction is always downwards) are grouped as follows:

- The moisture advection flux resulting from the downdraught and its associated pseudo ascent, towards the upper layers

$$\text{ZDIFCQD}^{\overline{l}} \equiv \left(F_q^{\mathrm{ddif}}\right)^{\overline{l}} = \frac{\check{\omega}^{*\,\overline{l}}}{g}\, q_d^{\overline{l}} - \frac{\check{\omega}^{*\,\overline{l}}}{g}\, q_{pa}^{\overline{l}} = -\frac{\check{\omega}^{*\,\overline{l}}}{g}\, (q_{pa}^{\overline{l}} - q_d^{\overline{l}}) \tag{125}$$

  There is upwards moisture advection ($F_q^{\mathrm{ddif}} < 0$) when the environment corrected by the pseudo ascent is moister than the downdraught. Reversely, the rare case of a positive value of F_q^{ddif} means that the pseudo ascent brings drier air upwards.

- The precipitation evaporation flux $\mathcal{E}$ from the lower layers.
  As dry air is entrained upwards to compensate for the downward motion of the rain drop, this air gets in close contact with the drop and evaporates part of it, inducing the upwards precipitation evaporation flux. Furthermore, if $\mathcal{E}$ was not directed upwards, we would soon have a saturation preventing further

downdraught activity.
$\mathcal{E}$ produced at the lower levels enters the layer at the lower side. It doesn't seem easy to express a boundary condition at the surface, while $\mathcal{E}$ has clearly to be zero at the top of the downdraught, being near the source of the precipitation hence near to saturation. Therefore, we will perform the computation from the top downwards along the active layer.

- The downdraught activity generated in current layer (as soon as the downdraught is active in this layer, i.e. when INLAB $\equiv \delta^{\downarrow}_{\text{stab}} = 1$)

$$F_{q_d} = \triangle F_c^{\text{pal}^l} + F_q^{\text{ddetr}^l} = -\delta^{\downarrow}_{\text{stab}} \left(\frac{\Delta p}{g\triangle t}[(q^l_{pa} - q^l) + \left\{ \begin{array}{l} K(q_d - q) \\ \\ 0 \end{array} \right.] \right) \tag{126}$$

where we apply the detrainment term only while the downdraught is dryer than the environment: the detrainment of downdraught air into the environment implies a reduction of the downdraught section and the moistening of the corresponding liberated area to the environmental value.

The water substance budget for the layer may be written as

$$\text{ZFTOTQ} \equiv F^{\bar{l}}_{\text{w\_tot}} = F_q^{\text{ddif}^{\overline{l-1}}} + \mathcal{E}^{\overline{l-1}} - \delta^{\downarrow}_{\text{stab}} \left(\frac{\Delta p}{g\triangle t}[(q^l_{pa} - q^l) + \left\{ \begin{array}{l} K^d(q^l_d - q^l) \\ \\ 0 \end{array} \right.] \right) = F_q^{\text{ddif}^{\bar{l}}} + \mathcal{E}^{\bar{l}}$$

$F^{\bar{l}}_{\text{w\_tot}}$ is the water flux you must input at the lower interface of the layer, as pseudo ascent and precipitation evaporation fluxes, the saldo of $F^{\overline{l-1}}_{\text{w\_tot}}$ being provided by local downdraught activity. This way you get the upward precipitation evaporation flux at the lower interface, or a downwards moisture correction flux, depending of the sign of the budget:

$$0 \geq \text{ZFCORQ} = \left\{ \begin{array}{l} 0 \\ \\ F_q^{\text{ddif}^{\bar{l}}} - F^{\bar{l}}_{\text{w\_tot}} \end{array} \right. \qquad \text{or} \qquad 0 \geq \mathcal{E}^{\bar{l}} = \left\{ \begin{array}{l} F^{\bar{l}}_{\text{w\_tot}} - F_q^{\text{ddif}^{\bar{l}}} \\ \\ 0 \end{array} \right.$$

ZFCORQ is actually a physical security against a downward precipitation evaporation flux: as soon as ZFCORQ $<$ 0 there is no further precipitation evaporation. In this case, the increase of the pseudo ascent between the bottom and the top of the layer is not compensated completely by the downwards moisture flux in the downdraught:

- $\|q^{l+1}_{pa} - q^{l+1}\|$ small: generation of compensative ascent is small in the layer, and the mass flux increase is small: doesn't seem plausible.
- $\|K(q^{l+1}_d - q^{l+1})\|$ small: small entrainment of ambient moisture, downdraught moisture being close or even bigger than the environment.

In the case the precipitation evaporation flux obtained from the budget would exceed (in absolute value) the precipitation flux, we introduce a correction avoiding the physical impossibility:

$$0 \leq \text{ZFCORQ1} = \left\{ \begin{array}{l} 0 \\ \\ -\mathcal{E} - \mathcal{P} \end{array} \right. \qquad\qquad \mathcal{E} = \mathcal{E} + \text{ZFCORQ1}$$

Of course, both ZFCORQ and ZFCORQ1, representing corrections respectively against negative and excessive evaporation, will intervene in the enthalpy budgets.

7.2.2 Enthalpy budget

With the same ingredients as the moisture budget, we get:

$$\begin{aligned} \text{ZFTOTS} \equiv F^{\bar{l}}_{\text{s\_tot}} &= (F_s^{\text{ddif}})^{\overline{l-1}} - L^{\overline{l-1}}_{\text{flux}} \mathcal{E}^{\overline{l-1}} - \delta^{\downarrow}_{\text{stab}} \left\{ \frac{\Delta p^l}{g\triangle t} \left((s^l_{pa} - s^l) + K^d(s^l_d - s^l) \right) \right\} \\ &= (F_s^{\text{ddif}})^{\bar{l}} - L^{\bar{l}}_{\text{flux}} \mathcal{E}^{\bar{l}} - L_{\text{bud}}\ (\text{ZFCORQ} + \text{ZFCORQ1}) \end{aligned}$$

7.2.3 Momentum fluxes

Similarly to equation(124) the horizontal momentum vertical downdraught diffusion flux $(\mathsf{ZSTRCUD}, \mathsf{ZSTRCVD}) \equiv F^d_{\mathbf{V}}$ is given by

$$(F^d_{\mathbf{V}})^{\bar{l}} = (F^d_{\mathbf{V}})^{\overline{l-1}} - \delta^{\downarrow l}_{\mathrm{stab}} \frac{\triangle p^l}{g\triangle t} \left\{ \mathbf{V}^l_{pa} - \mathbf{V}^l(1+K'_d) + K'_d(1-\beta^l)\overline{\mathbf{V}}^l + \beta^l\ K'_d\ \mathbf{V}_{d0} \right\} \tag{127}$$

where $K'_d \equiv \mathsf{ZALFS} = \frac{K_d \triangle t}{1+K_d\triangle t}$ and $\mathbf{V}_{d0} \equiv (\mathsf{ZA13}, \mathsf{ZA14})$ is the velocity at the top of the downdraught.

7.3 Lifting Condensation Level

No test was done about the capacity for the ascent (i.e. at least one of the ascents within the grid box) to actually reach the LCL.
Up to now, we proceeded as follows:

- The cloud profile was built by following an entraining moist adiabat from the blue point of the lowest "inactive" layer
- the layer activity required:
  - that the arrival point (T_u, q_u) was warmer than the wet bulb temperature $T_u \geq T_w$, $q_u \geq q_w$;
  - the buoyancy, i.e. $T_{vu} > T_v$;
  - large scale moisture convergence.

This does not necessarily imply that $T_u > T$, so we could have $q_u = q_{\mathrm{sat}}(T_u) < q_{\mathrm{sat}}(T)$.
In this case, we would have little chance to reach the lifting condensation level, except at some places within the grid box because of the subgrid variability. The bigger $q_{\mathrm{sat}} - q_u$, the smaller the chance of the convection to take place; but it also depends on the intensity of the motions induced by the potential convection.
From this, it was proposed to compare the corresponding normalized surface precipitation to the minimum saturation default of the computed profile:

$$\frac{\mathcal{P}_s}{\mathcal{P}_{\mathrm{ref}}} \quad \leftrightarrow \quad \min(q_{\mathrm{sat}} - q_u)$$

The routine computes the minimum over the vertical of $q_{\mathrm{sat}} - q_u$.

- If it is lower than zero, everything is all right.
- If not, we say that convection may take place only if there is sufficient activity to allow subgrid inhomogeneities to reach the LCL, and we cut the convective scheme when the computed convective precipitation at the surface is smaller than a threshold, computed by multiplying the minimum of $q_{\mathrm{sat}} - q_u$ over the vertical by a reference precipitation flux ($-\mathsf{SCO} \approx 20\ kg\ m^{-2}s^{-1}$).

7.4 Net convective fluxes

Contributions from both updraught and downdraught are summed at the end of ACCVIMPD:

$$\begin{aligned}
\mathsf{PFPLCL} \equiv \mathcal{P}_w &= \mathsf{INND} \cdot \mathcal{E}_w + \mathcal{P}_w \\
\mathsf{PFPLCN} \equiv \mathcal{P}_i &= \mathsf{INND} \cdot \mathcal{E}_i + \mathcal{P}_i \\
\mathsf{PDIFCQ} \equiv F_q^{\mathrm{cdif}} &= \mathsf{INND} \cdot F_q^{\mathrm{ddif}} + F_q^{\mathrm{udif}} \\
\mathsf{PDIFCQ} \equiv F_s^{\mathrm{cdif}} &= \mathsf{INND} \cdot F_s^{\mathrm{ddif}} + F_s^{\mathrm{udif}} \\
(\mathsf{PSTRCU}, \mathsf{PSTRCV}) \equiv F_{\mathbf{V}} &= \mathsf{INND} \cdot F^d_{\mathbf{V}} + F^u_{\mathbf{V}}
\end{aligned}$$

where INND is the feasibility index equal to zero where the downdraught is not physical and 1 elsewhere (see §4.2).

7.5 Variable mass effects

A correction is applied to the turbulent diffusion fluxes J_q and J_s in order to preserve the boundary condition at the surface; a correction is also applied above, to keep a reasonable repartition of the correction.
At the surface, the convective diffusion flux must be zero, so the correction writes:

$$\begin{aligned} J_s^{\overline{L}'} &= J_s^{\overline{L}} - \delta_m \; q_s \; (c_{pv} - c_{pa}) \; T_s \; J_q^{\overline{L}} \\ J_q^{\overline{L}'} &= J_q^{\overline{L}} \; (1 - \delta_m q_s) \end{aligned}$$

Higher up, we correct also the sole turbulent fluxes, but the correction is computed from the sum of turbulent and convective diffusion fluxes:

$$\begin{aligned} J_s^{\bar{l}} &= J_s^{\bar{l}} - \delta_m \; q^{\bar{l}} \; (c_{pv} - c_{pa}) \; T^{\bar{l}} \; (F_q^{\mathrm{cdif}^{\bar{l}}} + J_q^{\bar{l}}) \\ J_q^{\bar{l}} &= J_q^{\bar{l}} - \delta_m \; q^{\bar{l}} \; (F_q^{\mathrm{cdif}^{\bar{l}}} + J_q^{\bar{l}}) \end{aligned}$$