

**University of Connecticut LES  
(some) GABLS4 results and  
perspectives on modeling of stable boundary layers**

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# Introduction

- Stable boundary layers (SBL) are ubiquitous
  - Typically forming at night and in polar regions throughout the day
- The dynamics of the SBL are poorly understood, with implications for night forecasts, e.g., of temperature and fog, and pollution dispersion
- LES remains challenging
  - Stable stratification leads to reduction of the energetic scales and large anisotropy
- LES modeling challenges are often attributed to inadequacies of subgrid-scale (SGS) models
  - Most LES investigations employ sophisticated SGS models
  - Simple SGS models, e.g., the constant coefficient Smagorinsky–Lilly, are not typically used (in contrast to convective flows)
- Hypothesis: numerical model error is key for accurate predictions
  - Strong integration between the discrete approximation (order of accuracy or resolving power, grid spacing) and the physical model (turbulence closure)



image by R. Beare from Fernando & Weil (2010)

# Problem description

- Moderately stable atmospheric boundary layer case of Beare et al. (2004)
- Governing equations (anelastic approximation)

– Mass: 
$$\frac{\partial \bar{\rho}_0 \tilde{u}_i}{\partial x_i} = 0$$

– Momentum: 
$$\frac{\partial \bar{\rho}_0 \tilde{u}_i}{\partial t} + \frac{\partial (\bar{\rho}_0 \tilde{u}_i \tilde{u}_j)}{\partial x_j} = -\theta_0 \bar{\rho}_0 \frac{\partial \bar{\pi}_2}{\partial x_i} + \delta_{i3} g \frac{\bar{\rho}_0 (\tilde{\theta} - \langle \tilde{\theta} \rangle_x)}{\theta_0} - \epsilon_{ijk} \bar{\rho}_0 f_j (\tilde{u}_k - u_{g,k}) - \frac{\partial \tau_{ij}}{\partial x_j}$$

– Potential temperature: 
$$\frac{\partial \bar{\rho}_0 \tilde{\theta}}{\partial t} + \frac{\partial \bar{\rho}_0 \tilde{\theta} \tilde{u}_j}{\partial x_j} = -\frac{\partial \sigma_j}{\partial x_j}$$

- Subgrid scale models

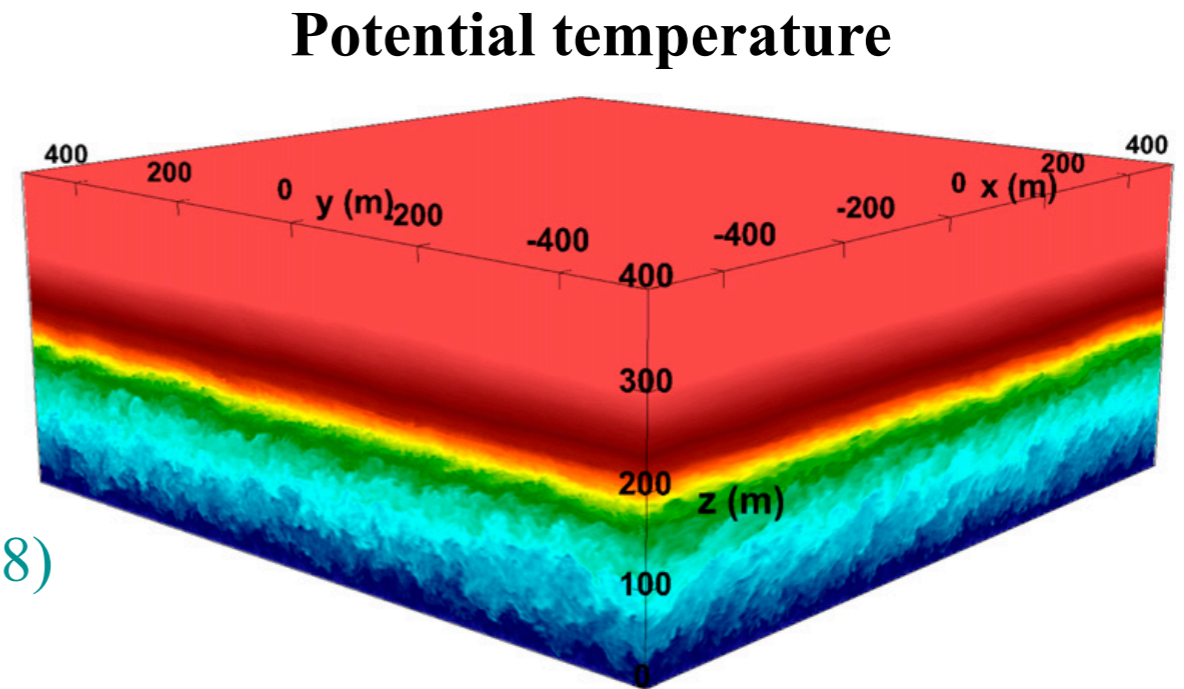
– Smagorinsky–Lilly 
$$\tau_{ij} = -2 \bar{\rho}_0 \nu_t \tilde{D}_{ij} \quad \sigma_j = -\bar{\rho}_0 \frac{V_t}{\text{Pr}_t}$$

eddy diffusivity:  $\nu_t = \Delta^2 | \tilde{D} | f_m(\text{Ri})$ , where  $\Delta = C_s \Delta x$  is the SGS eddy scale

- Buoyancy adjusted stretched vortex model (Chung & Matheou 2014) is used as reference
- Surface cooling rate of 0.25 K/h
  - Surface fluxes computed dynamically using Monin–Obukhov similarity theory (MOST)
- Periodic boundary conditions in the horizontal and sponge layer at domain top
- Flow attains a stationary state after 8 hours

# Parametric study

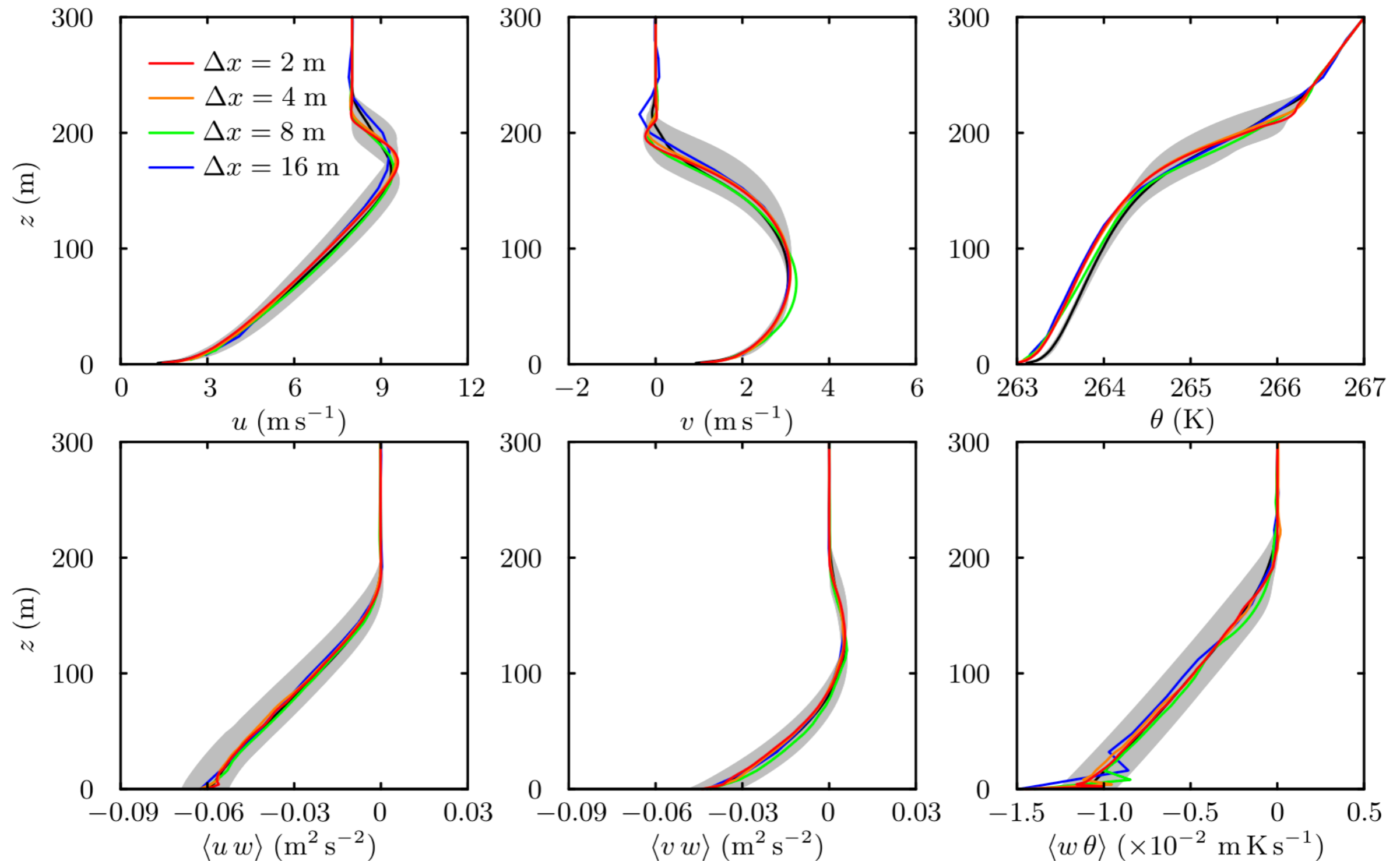
- Smagorinsky model constant
  - $C_s = 0.10 - 0.24$ , with 0.01 increments
  - Theoretical value  $C_s \approx 0.18$
- Advection scheme
  - Fully conservative non-dissipative family of schemes of Morinishi et al. (1998) adapted to the anelastic approximation
  - Second-, fourth- and sixth-order approximations
- Grid resolution
  - $\Delta x = 4 - 8$  m, with 1 m increments (5 values)
  - All runs have  $128 \times 128$  grid points in the horizontal and 400 m vertical domains
- Reference run:  $\Delta x = 2$  m, sixth-order advection, buoyancy adjusted stretched vortex model
  - Grid resolution independent results
- Total of 485 LES runs





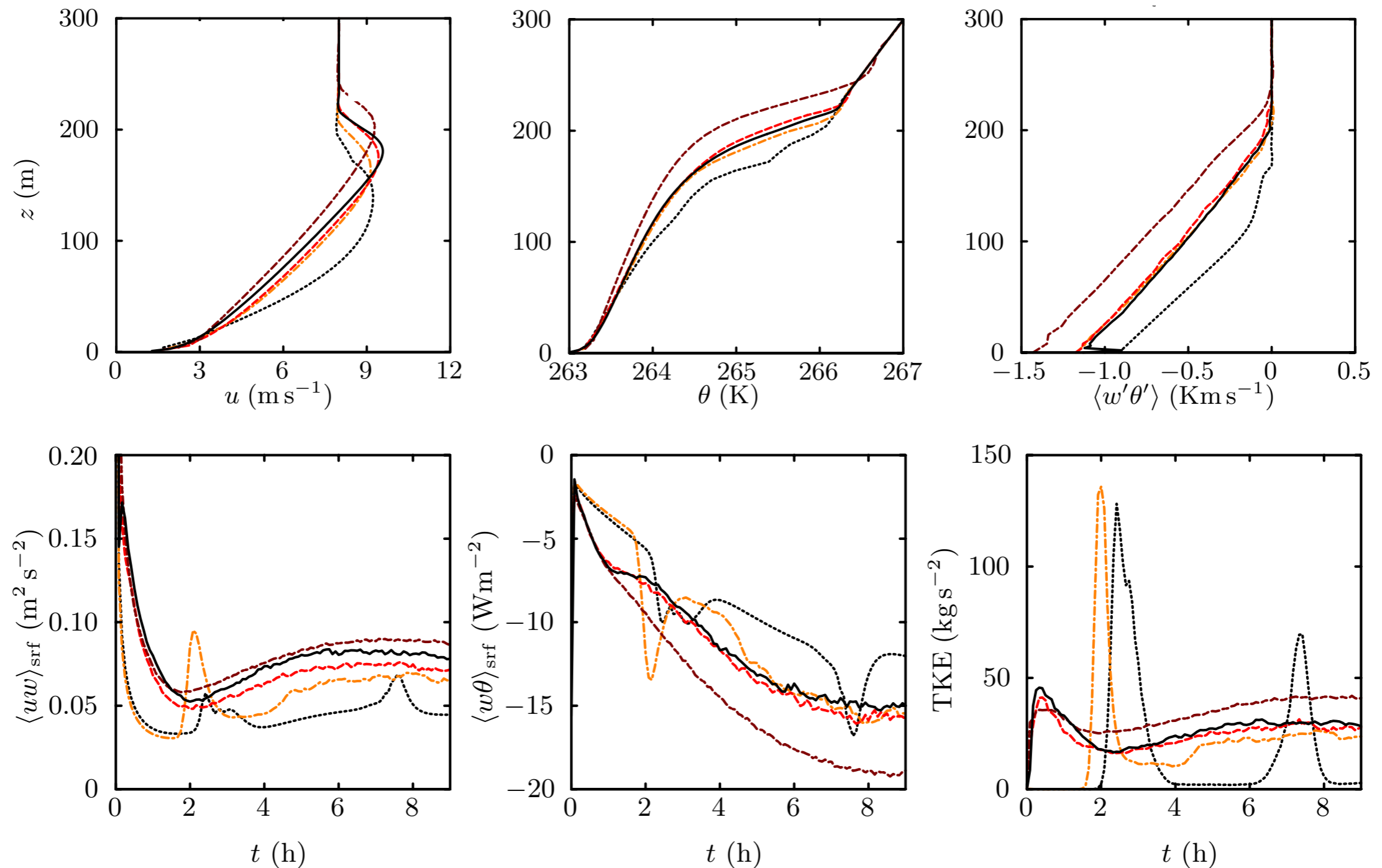
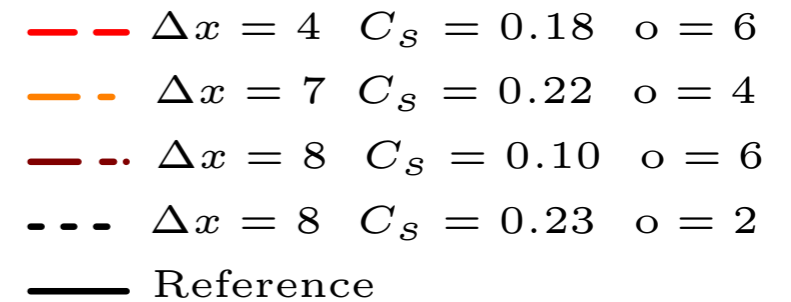
# Reference model grid convergence

- Buoyancy adjusted stretched vortex SGS model (figure from Matheou & Chung 2014)



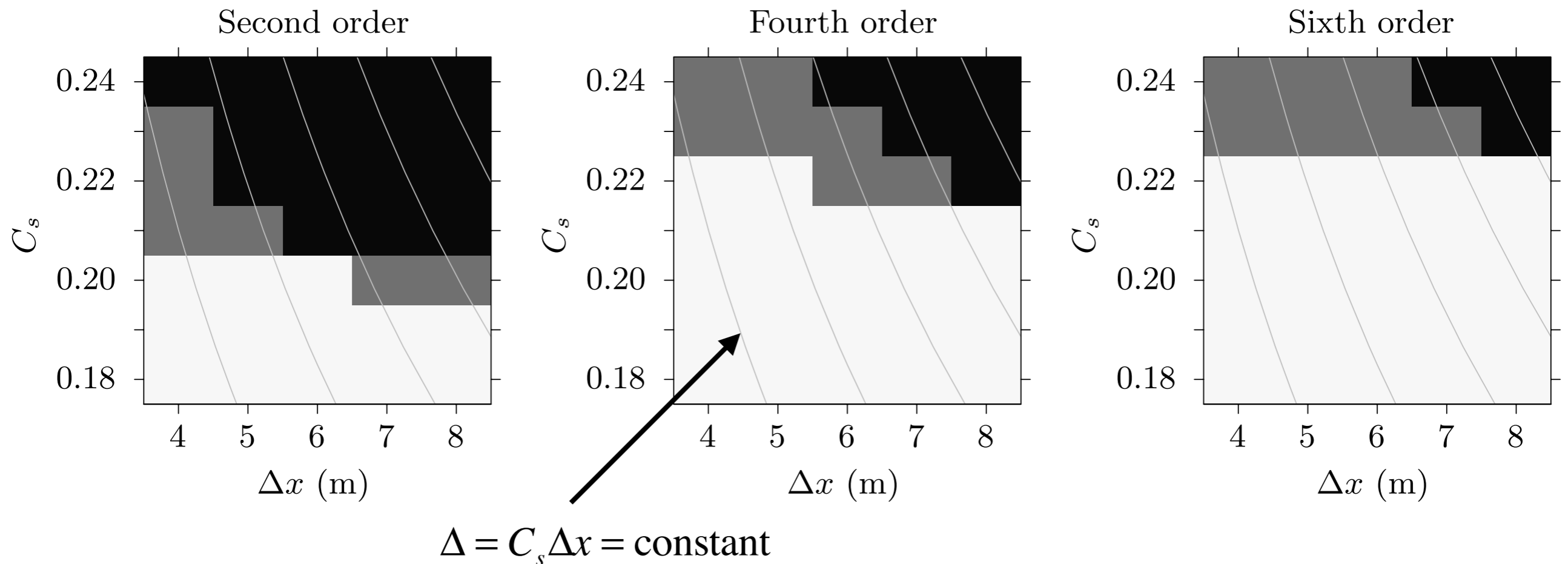
# Smagorinsky results – Overview

- Range of LES results is large
- Spurious turbulence collapse observed in some runs



# Spurious turbulence collapse

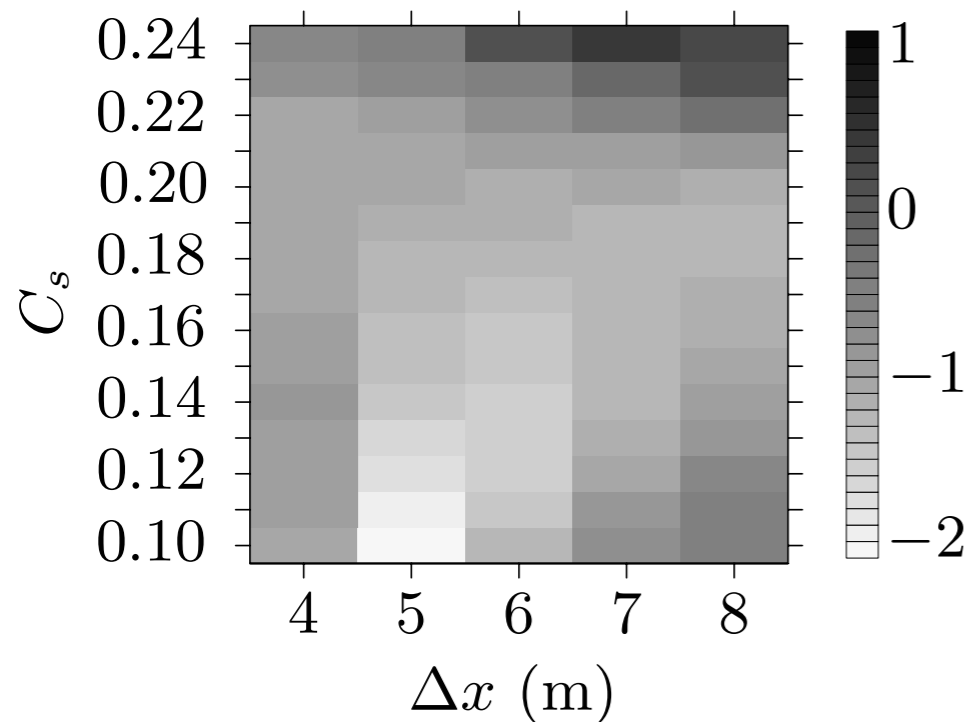
- Turbulence collapse flags
  - Global collapse, entire boundary layer laminarization (black rectangles)
  - Collapse during model spin-up: negligible TKE in  $0 < t < 0.5$  h, and subsequent recovery (gray rectangles)
- Spurious turbulence collapse depends on advection scheme order



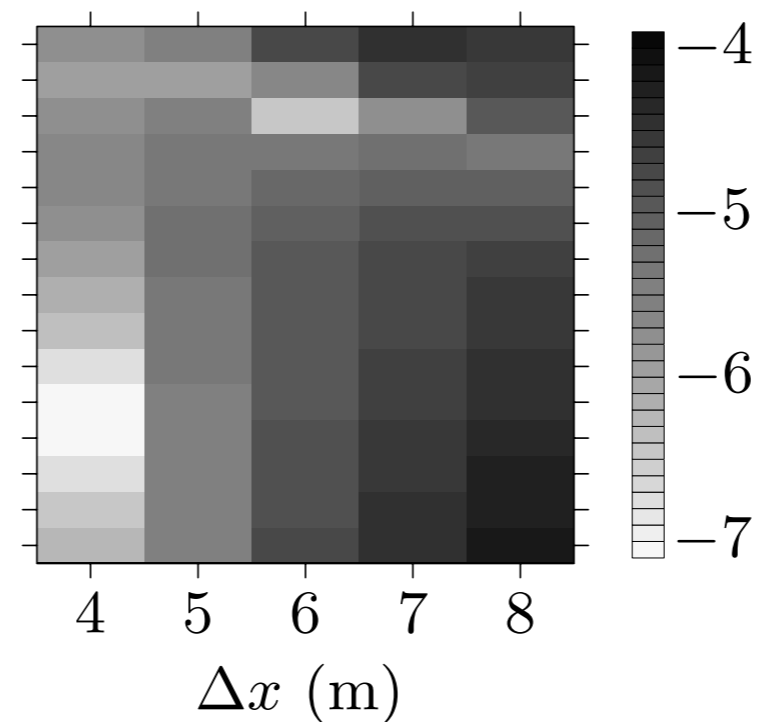
# “Error” norms with respect to reference LES

- Define error (or distance) between parametric study runs and reference LES
  - $l^2$ -norm for mean profiles (i.e., wind and temperature)
  - Difference of time-mean values for surface fluxes
- Error does not converge
- Large differences in surface heat flux ( $\sim 20\%$  difference)

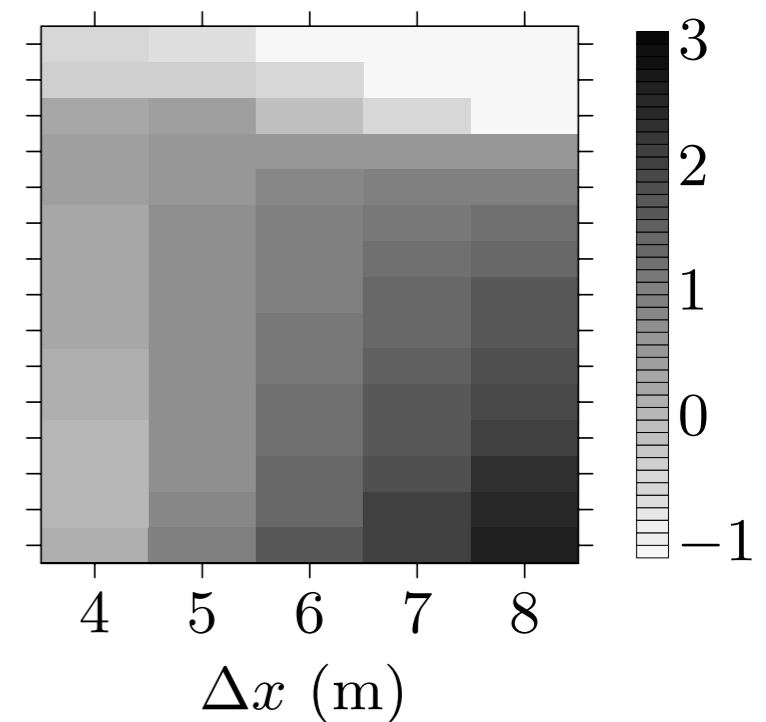
**Zonal wind**  
Logarithm  $l^2$ -norm



**Potential temperature**  
Logarithm  $l^2$ -norm



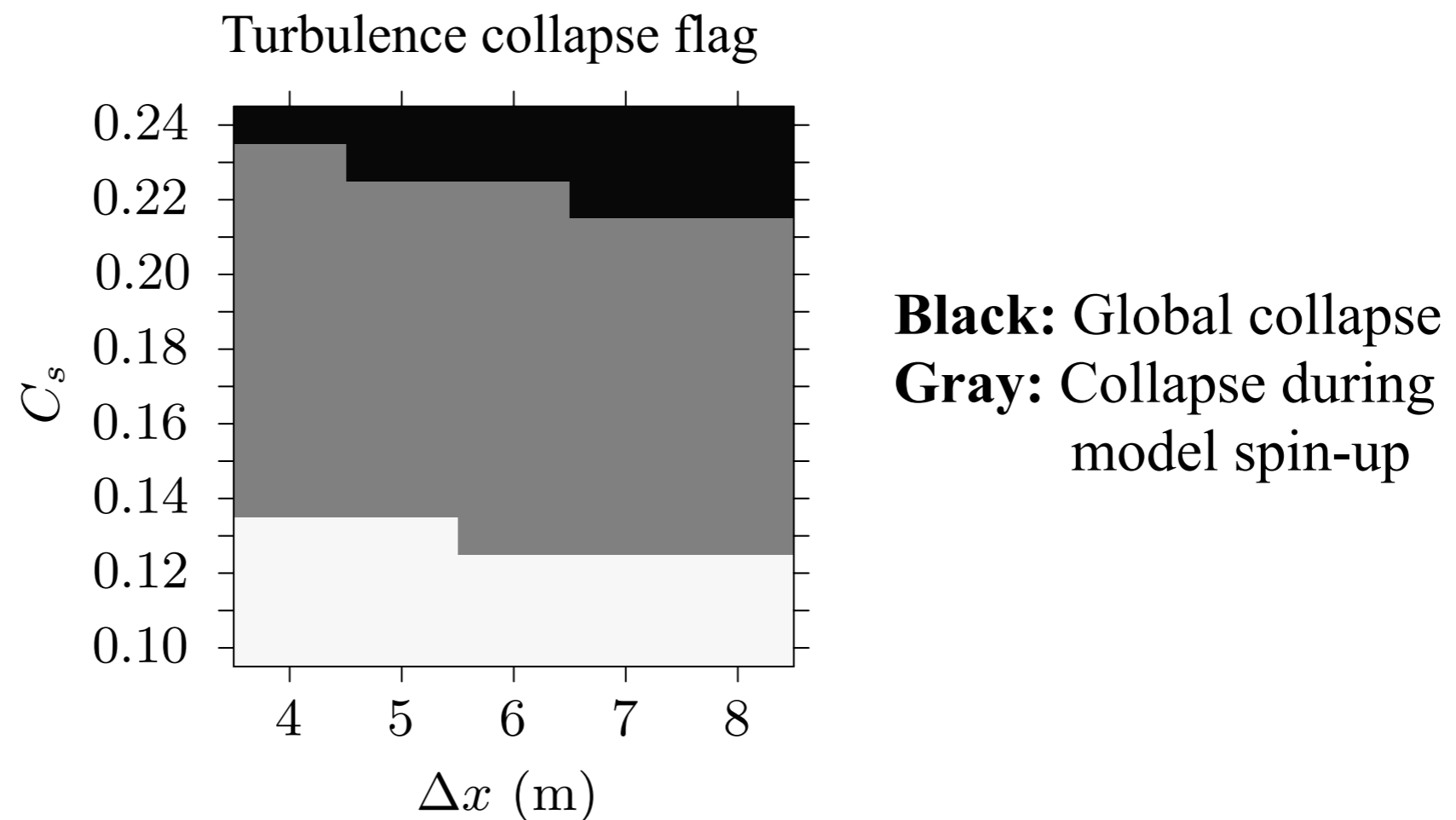
**Surface heat flux**  
ref. – Smagorinsky ( $\text{W}/\text{m}^2$ )



**only fourth-order advection shown**

# Prescribed heat flux simulations

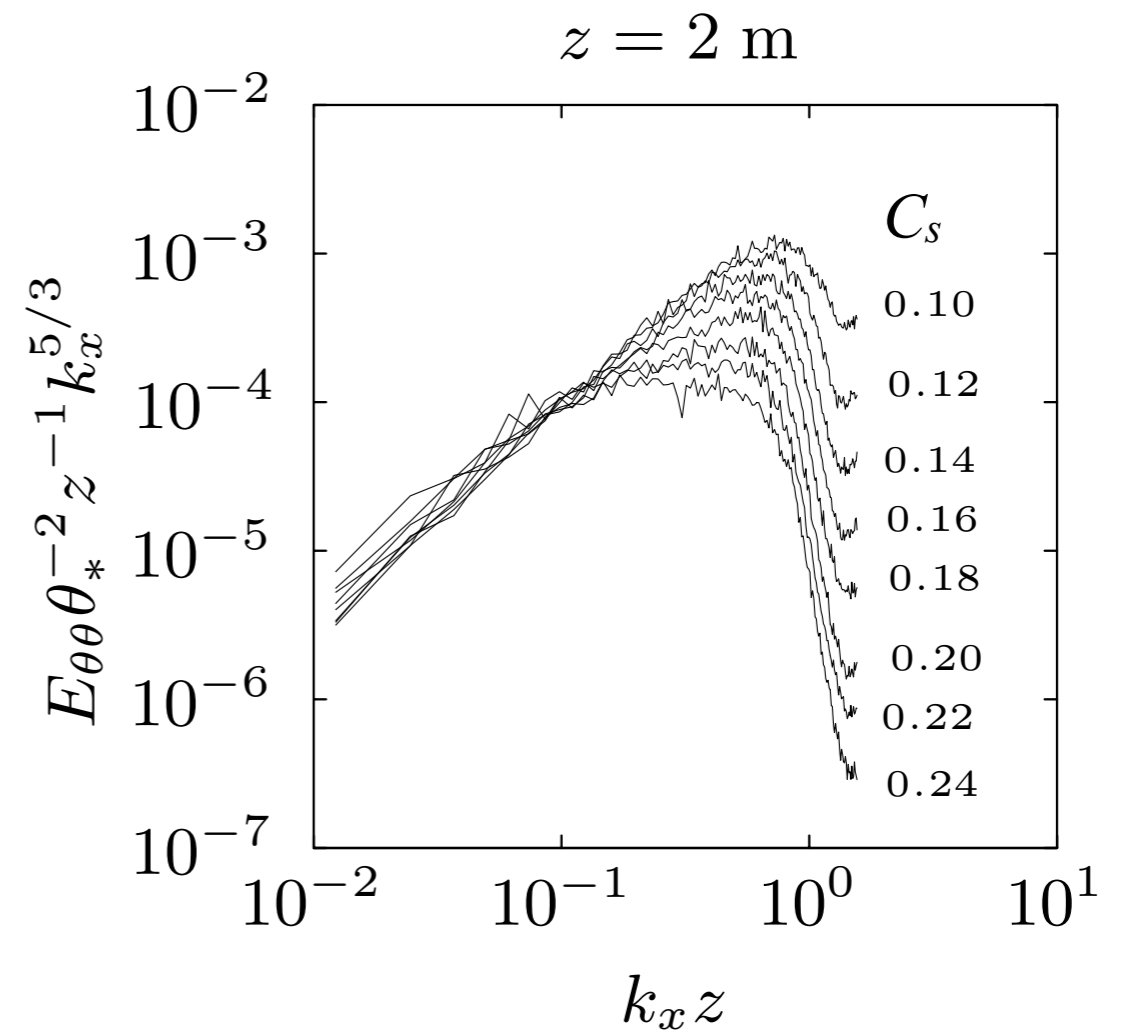
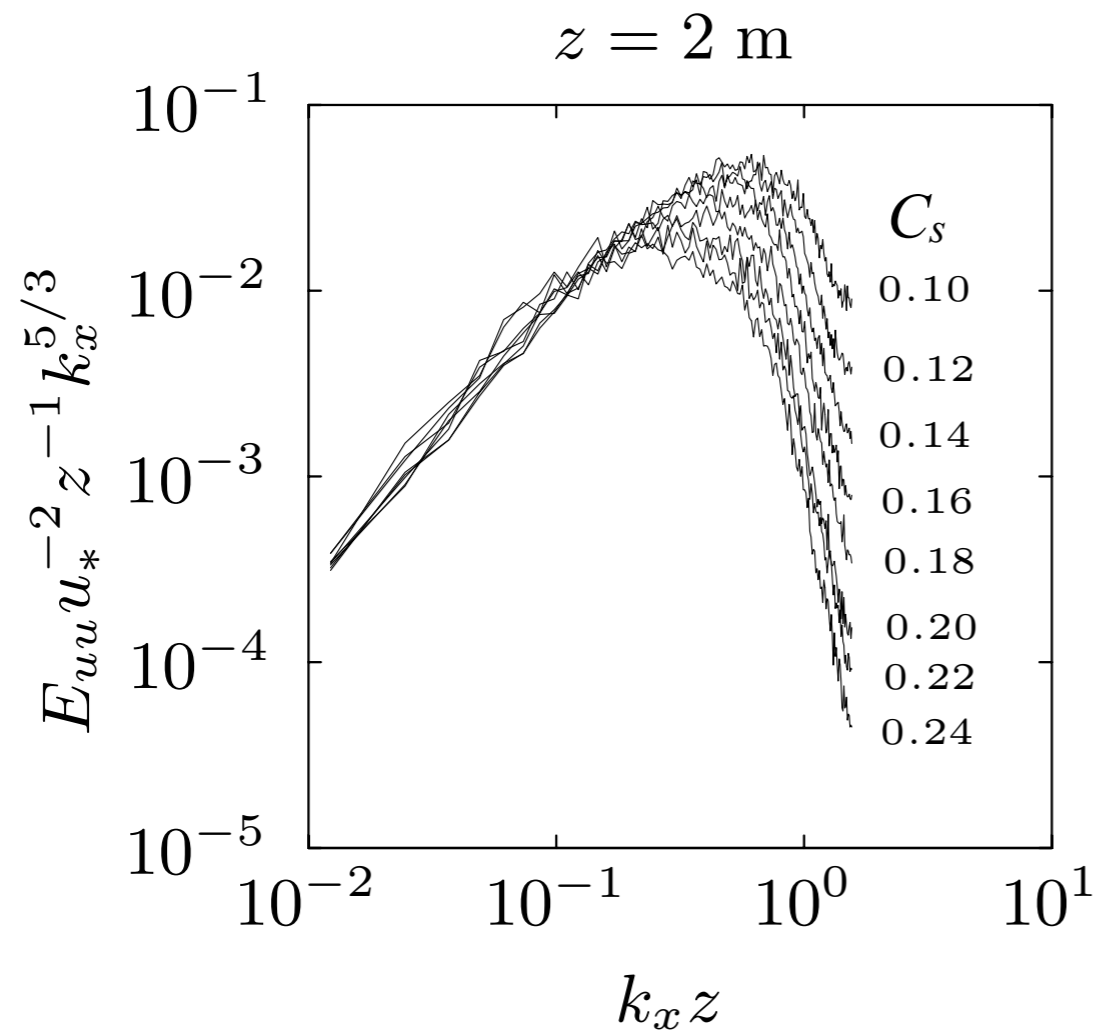
- The time-dependent heat flux from the reference run is used uniformly in all surface grid cells of the Smagorinsky runs
- The momentum flux is computed dynamically using MOST
- Spurious turbulence collapse regime expands
- Error increases when heat flux is not dynamically computed





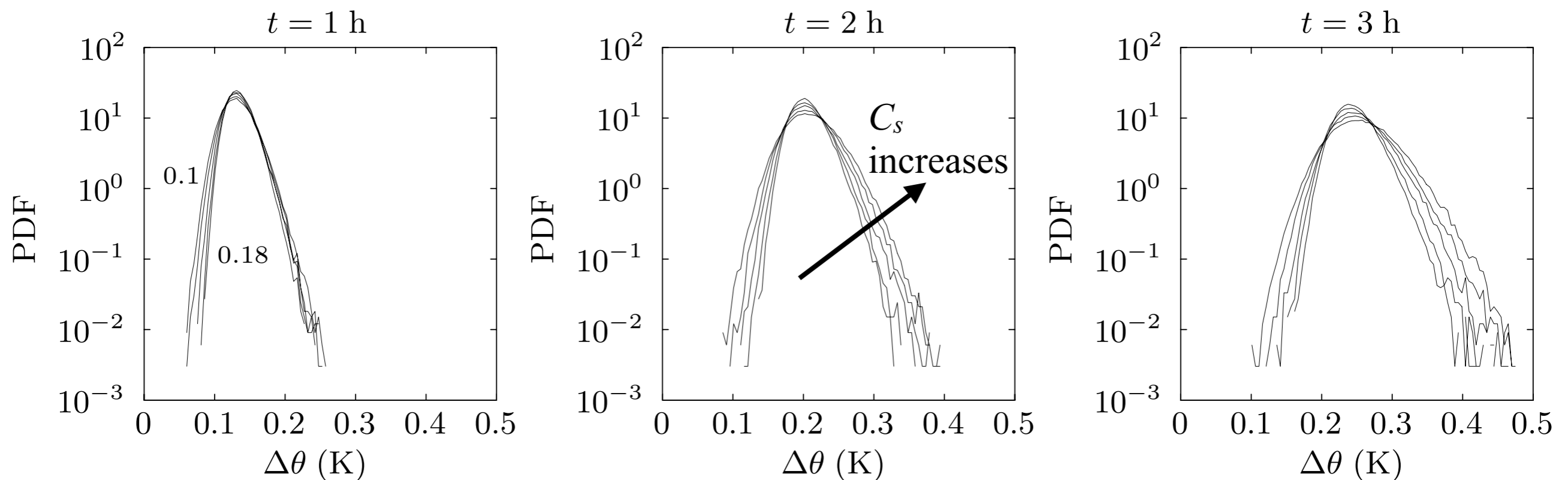
# Spectra

- Energy accumulation at small scales as model constant is reduced
- Runs with  $\Delta x = 4$  m, sixth-order advection at  $t = 9$  h (end of the run)



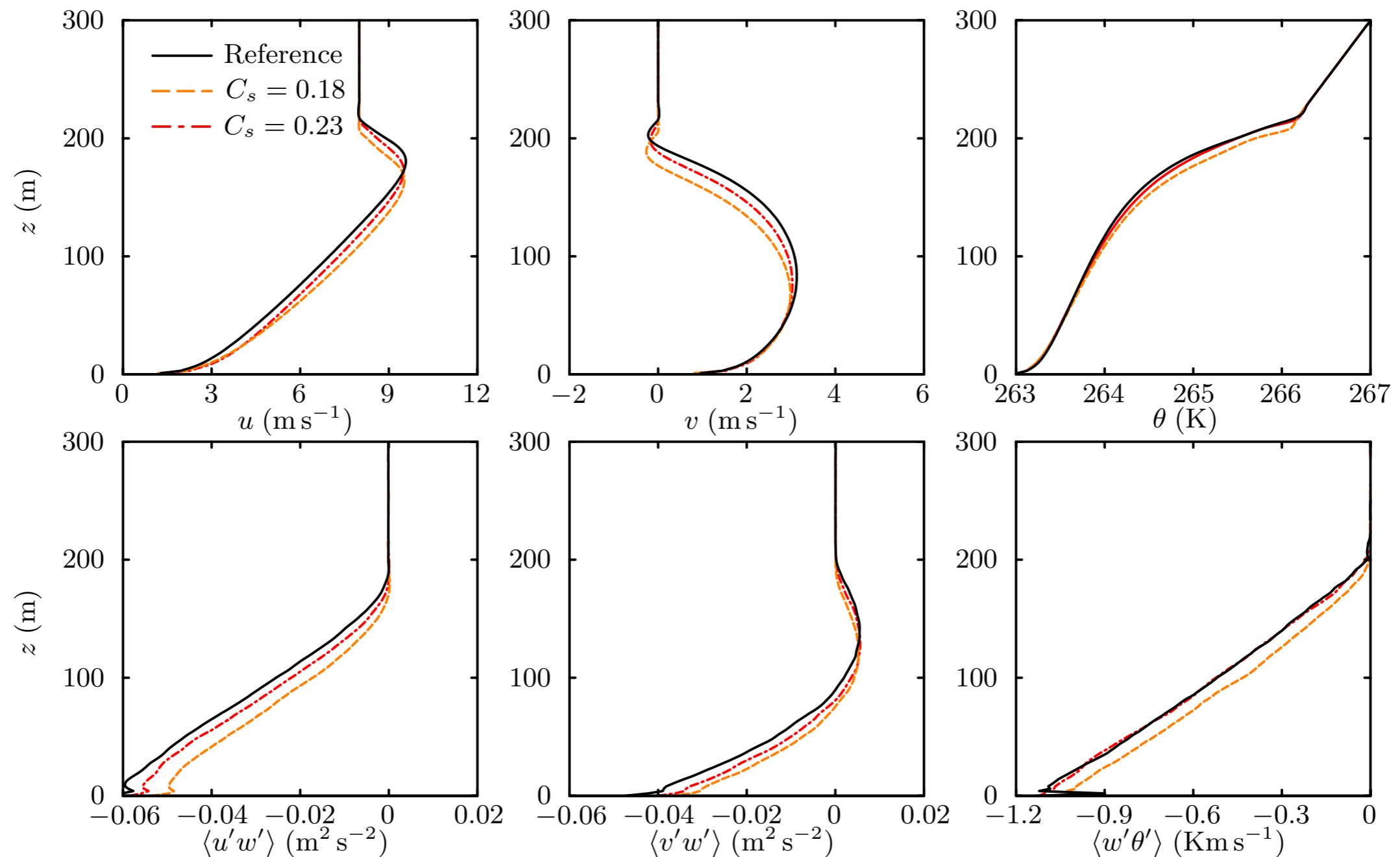
# Distributions of near-surface temperature difference

- Probability density functions (PDF) of temperature difference,  $\Delta\theta$ , between surface and first grid cell
- PDFs become broader as model constant,  $C_s$ , decreases
- Surface heat flux depends on  $\Delta\theta^3$
- Broader PDFs result in larger in magnitude mean surface fluxes (increased cooling)



# A posteriori comparison with reference

- Smagorinsky can accurately capture the boundary layer structure
- ...but the value of the constant is not known a priori and may be flow dependent



# Part 1: Summary and conclusions

- Aspects of a large-eddy simulation model are studied in simulations of a moderately stable atmospheric boundary layer
- Three model parameters are considered: the grid spacing, the value of the SGS model constant, and the order of accuracy (resolving power) of the advection discretization
- Two main error-producing mechanisms are identified:
  - For high values of the model constant spurious turbulence collapse is observed
  - For low values of the model constant, numerical discretization errors dominate, leading to accumulation of energy at the small scales and over-prediction of the magnitude of the surface heat flux
- The constant coefficient Smagorinsky–Lilly model can accurately capture moderately stable flows, in contrast to the conclusions of previous studies
  - Judicious choice of parameters is necessary (but not known *a priori*)
- Surface fluxes depend on model constant
  - The observed differences are relatively large given that the flow and model configuration is identical
  - The feedback between boundary-layer turbulence and surface flux is important

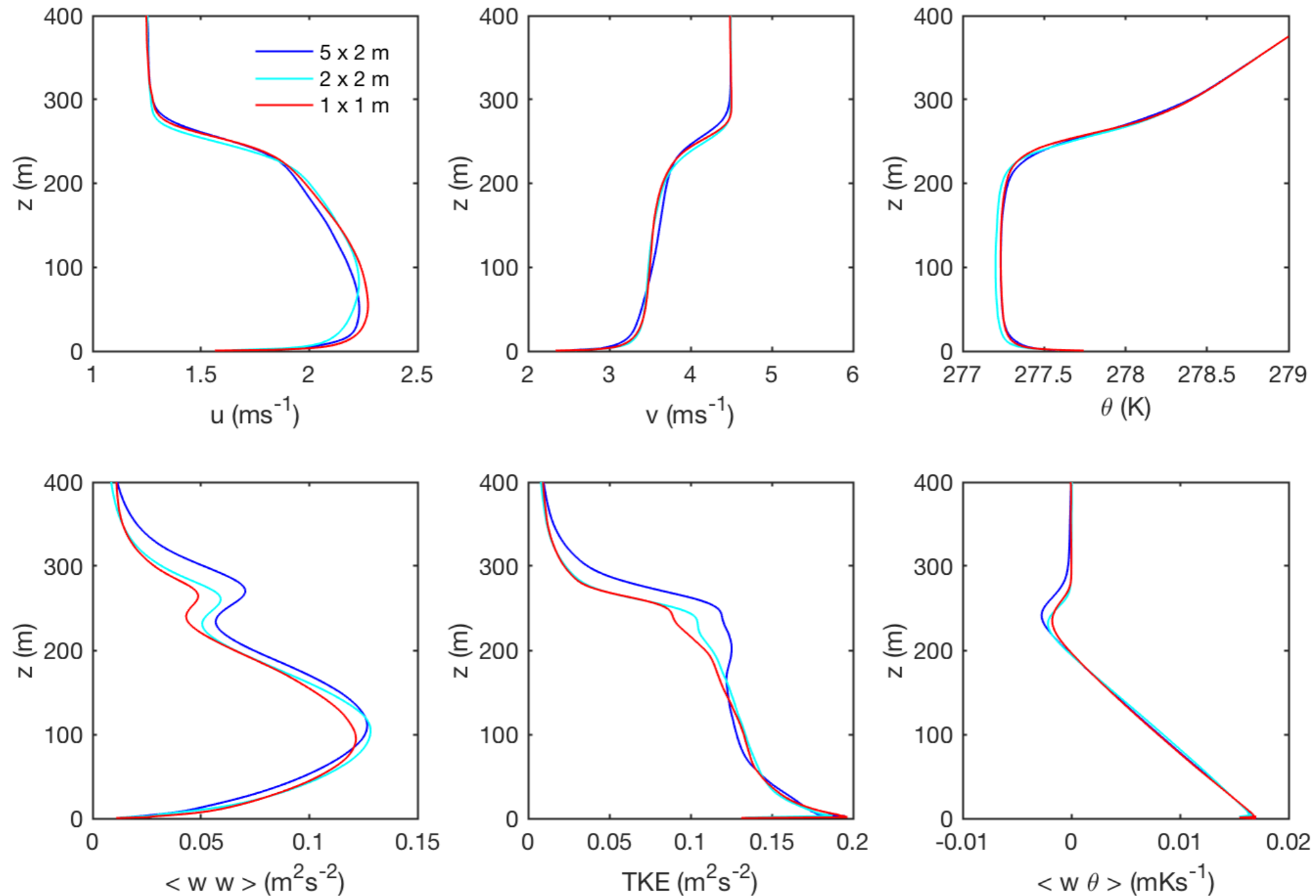
## Part 2: GABLS4 LES results

- LES model of Matheou & Chung (2014)
- Buoyancy adjusted stretched-vortex model (Chung & Matheou 2014)
- Advection scheme
  - Same for momentum and temperature
  - Fully conservative non-dissipative family of schemes of Morinishi et al. (1998) adapted to the anelastic approximation
  - Sixth-order approximation
  - No numerical dissipation



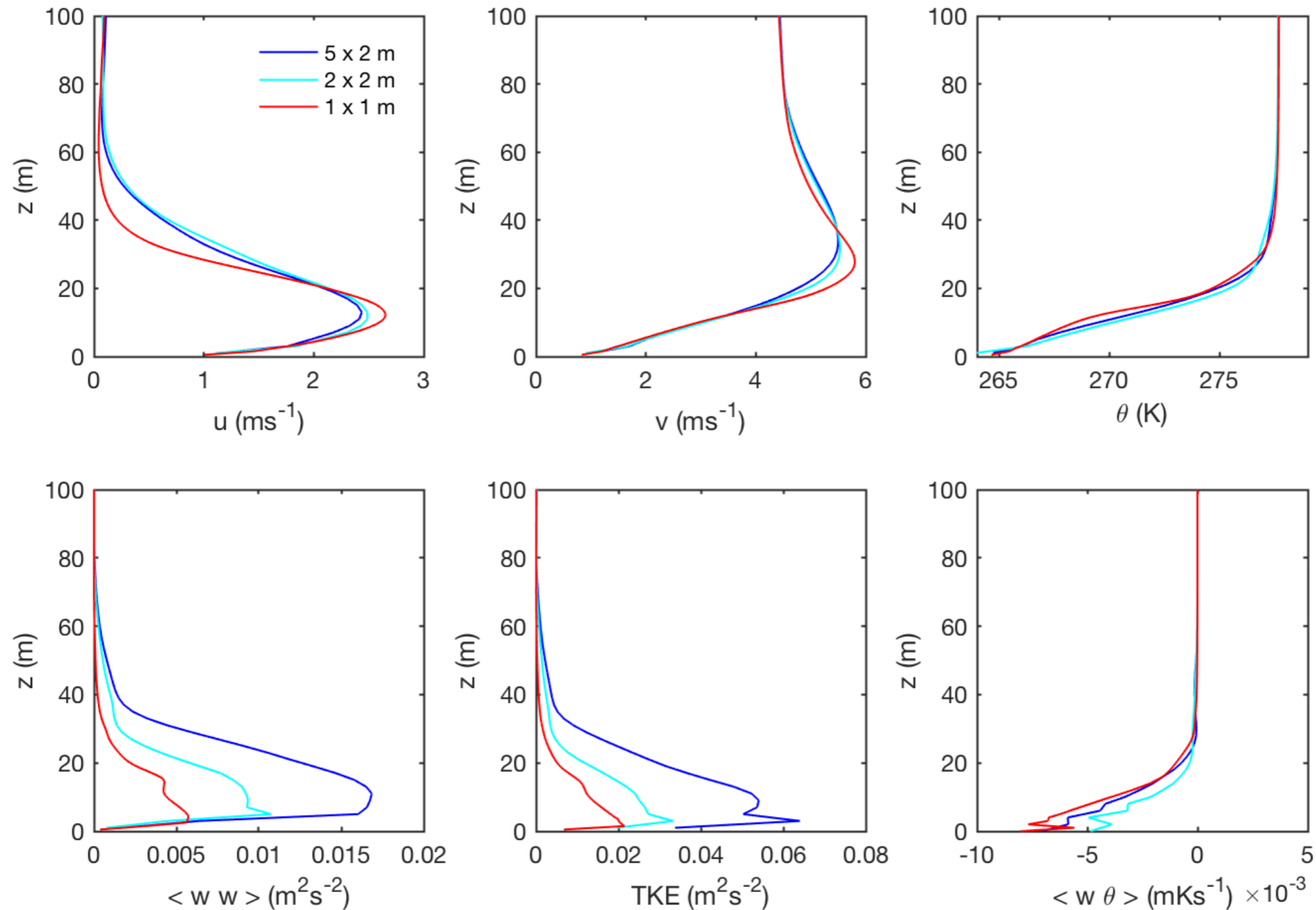
# GABLS4: Grid convergence, diurnal LES, daytime $t = 6$ h

- Small differences with respect to grid resolution, fluxes are converged



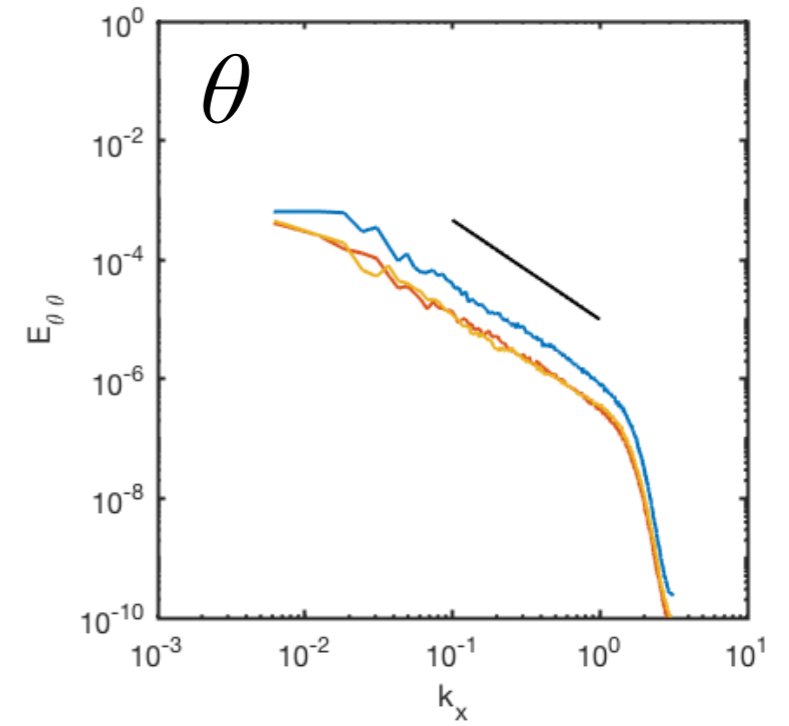
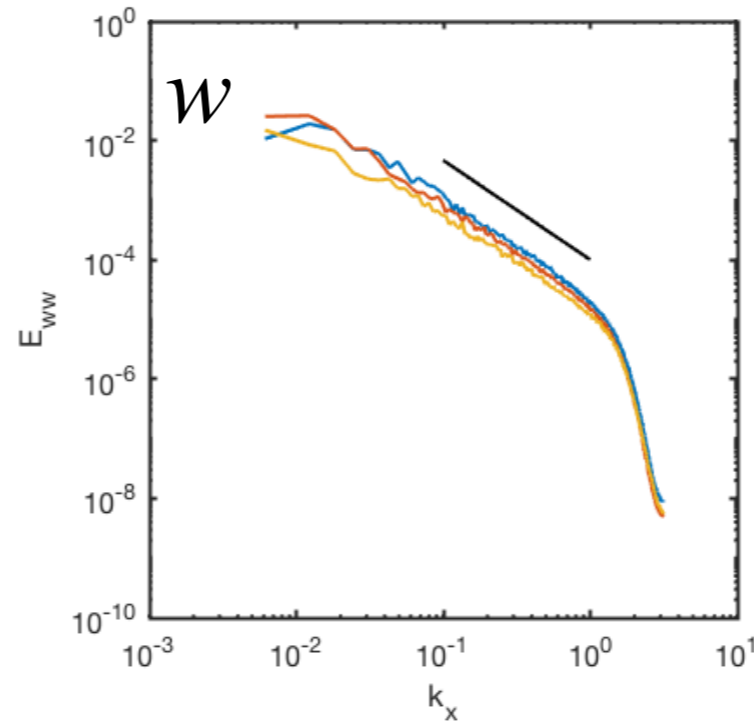
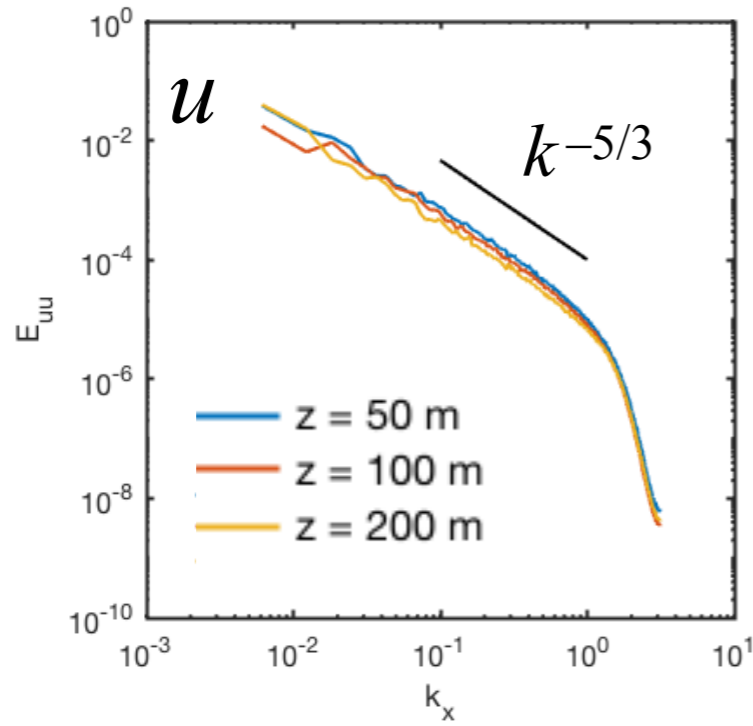
# GABLS4: Grid convergence, diurnal LES, “nighttime” $t = 16$ h

- Mean profiles somewhat change decreasing  $\Delta x$
- Fluxes exhibit larger differences, model is more energetic at coarse resolution

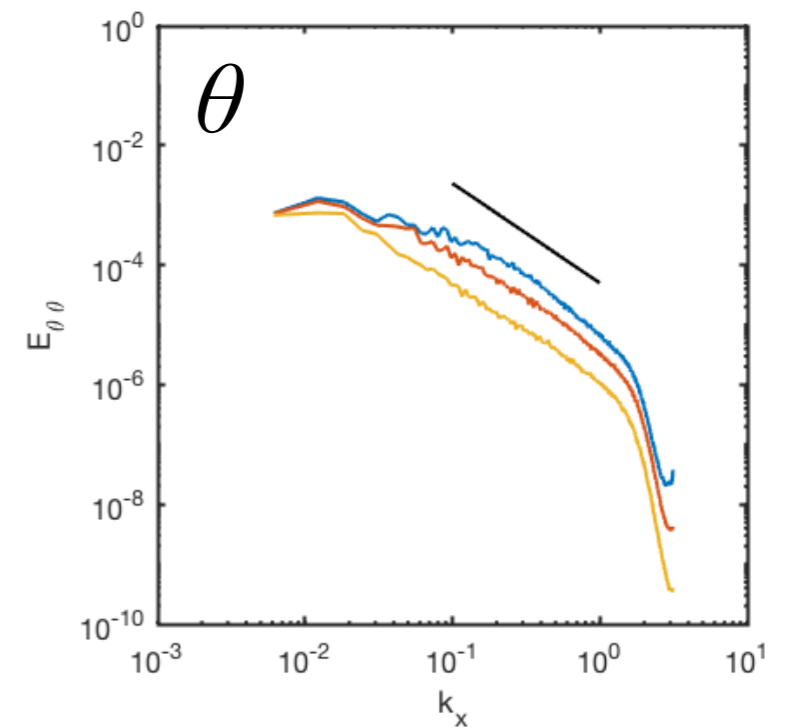
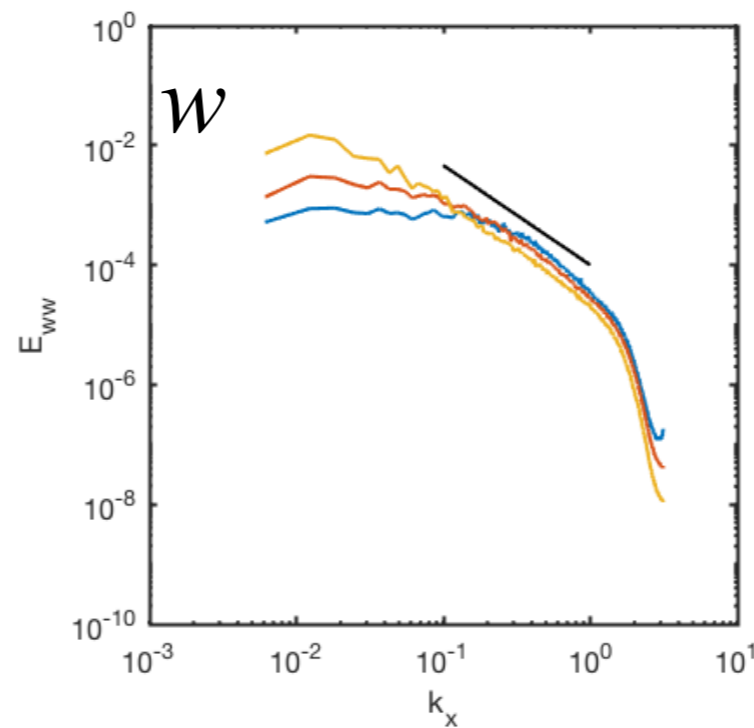
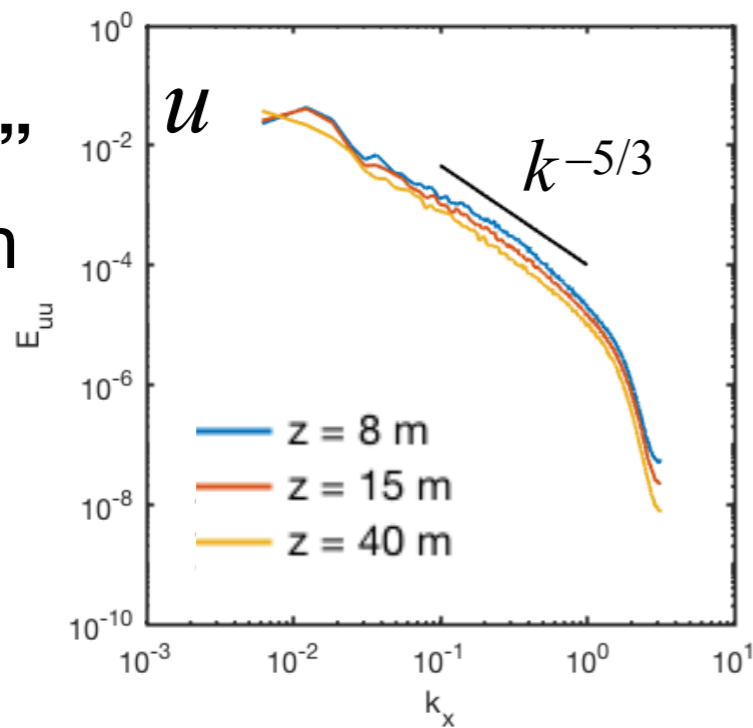


# Spectra: convective and stable conditions $\Delta x = 1$ m

Day  
 $t = 6$  h

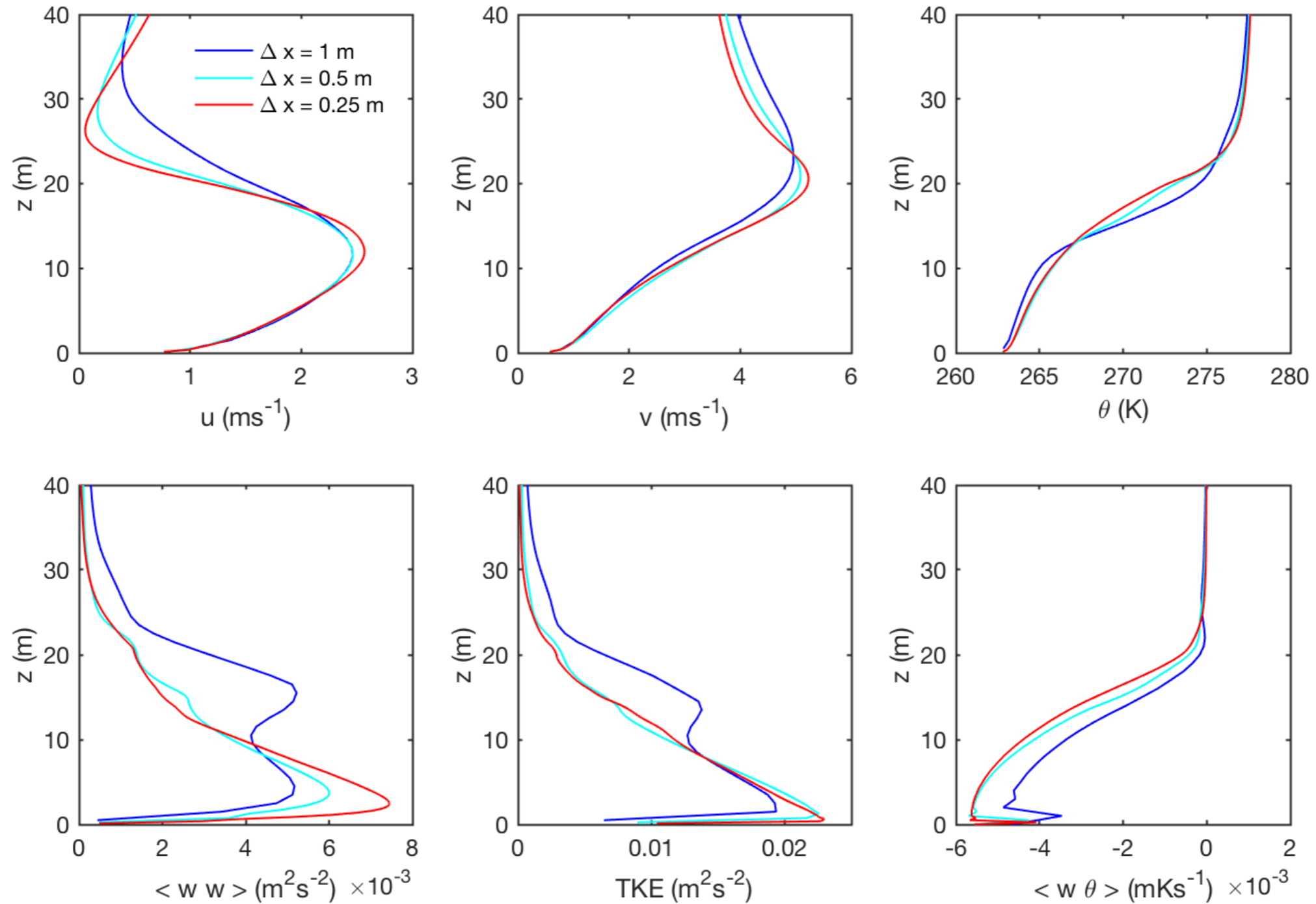


“Night”  
 $t = 16$  h



# Grid convergence, “nighttime” LES $t = 18$ h

- Grid converged results at  $\Delta x = 0.5$  m



# Backup



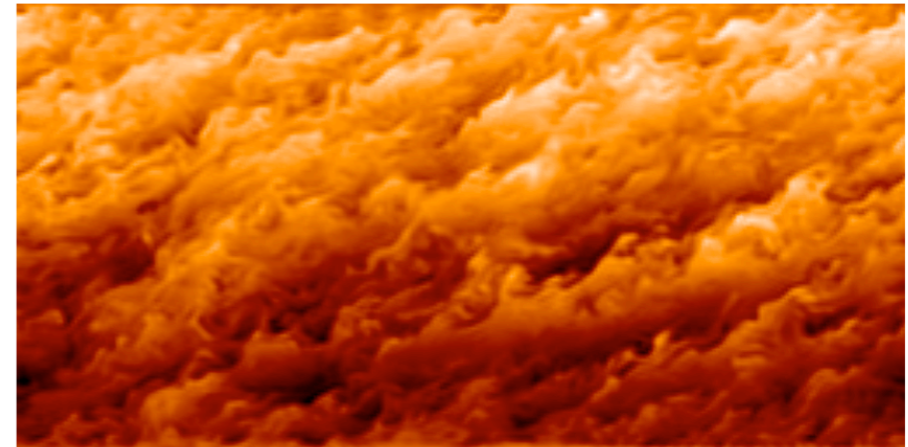
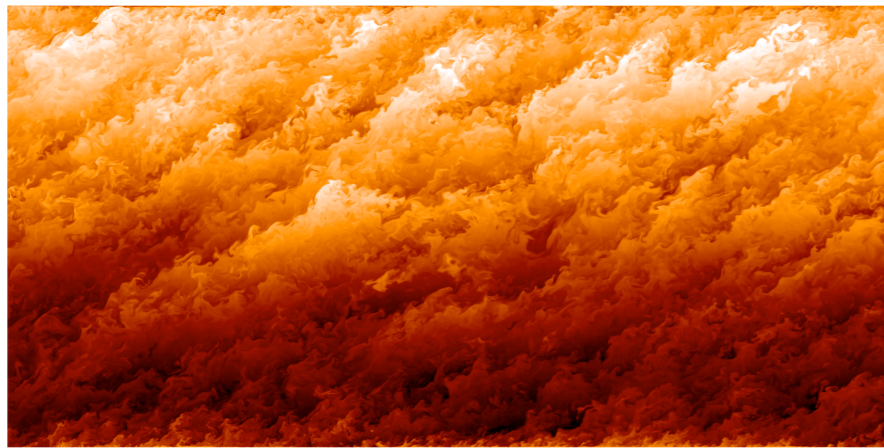
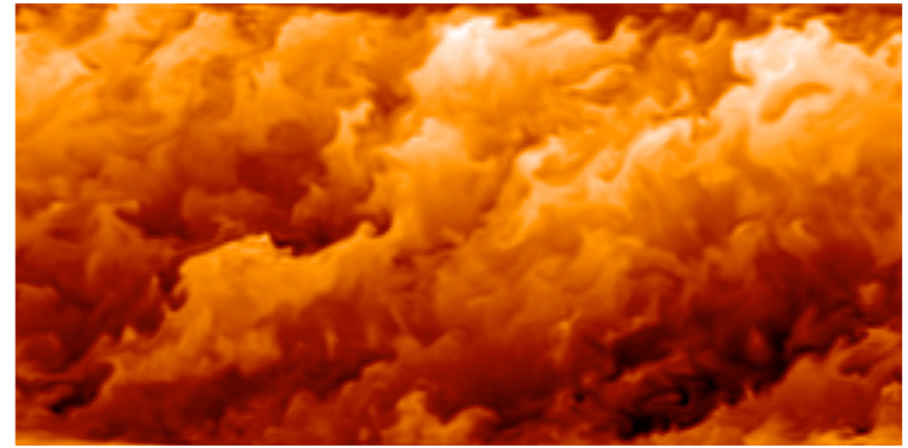
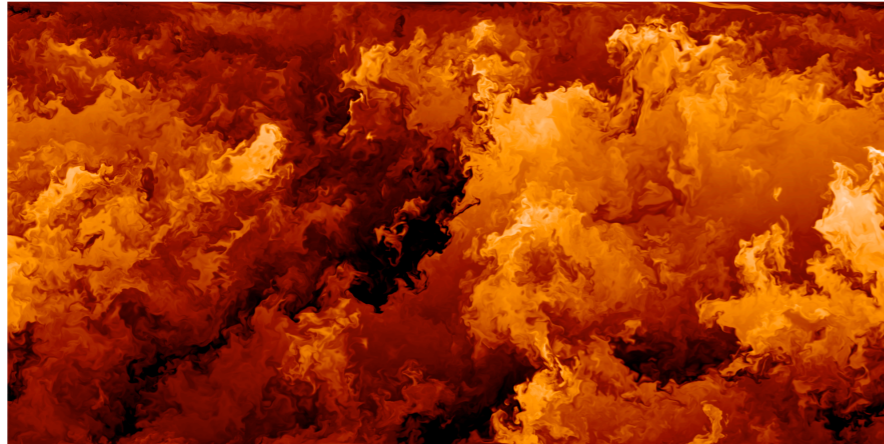
# Homogeneous stratified sheared turbulence

DNS:  $2048 \times 1024^2$

Flux  
Richardson  
number

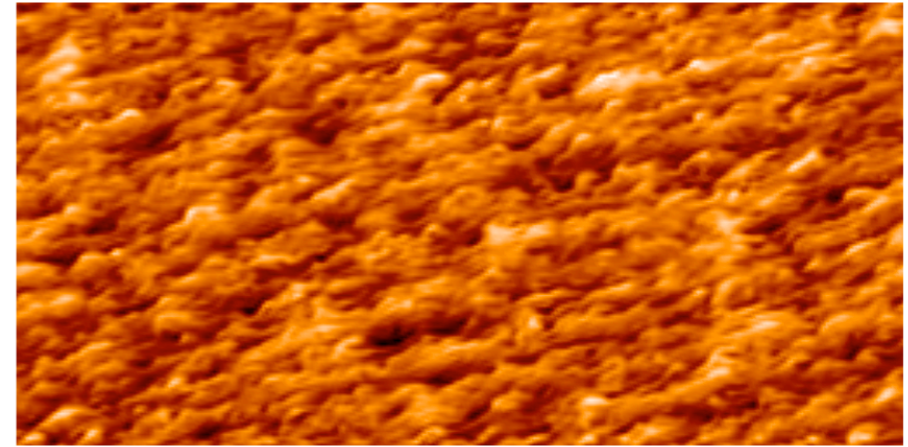
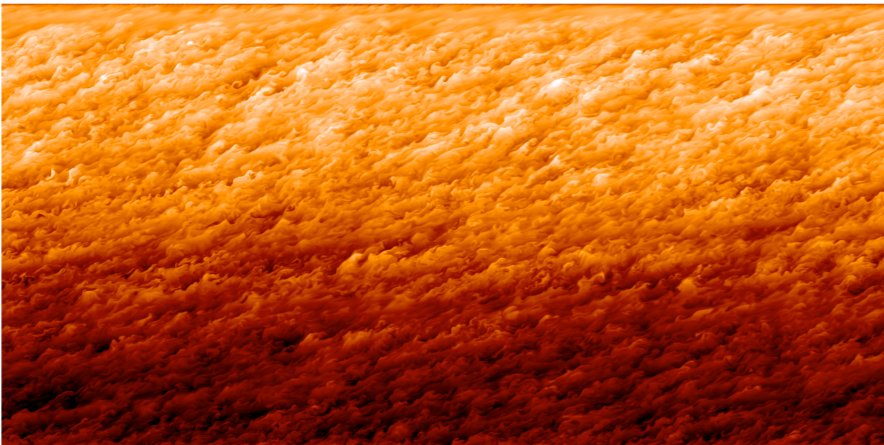
LES:  $256 \times 128^2$

$Rf = 0$



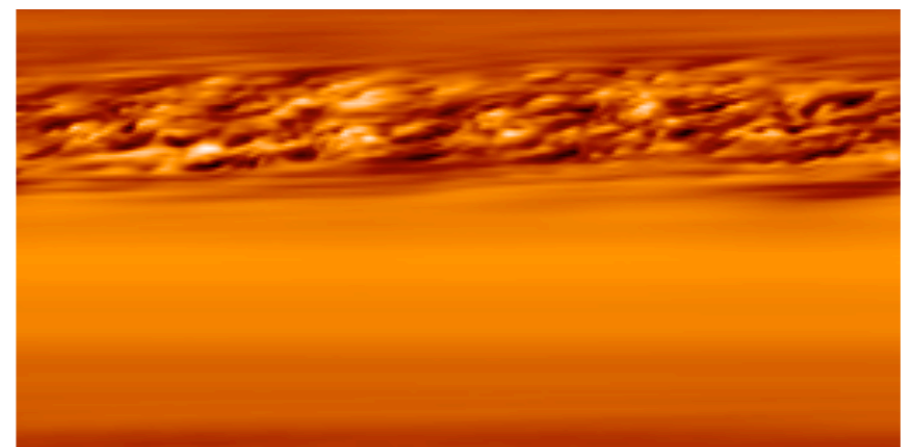
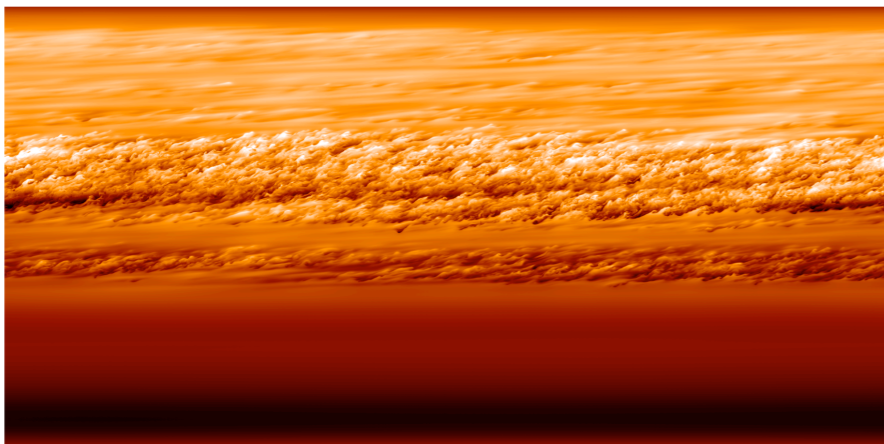
**0.10**

**0.12**



**0.16**

**0.19**



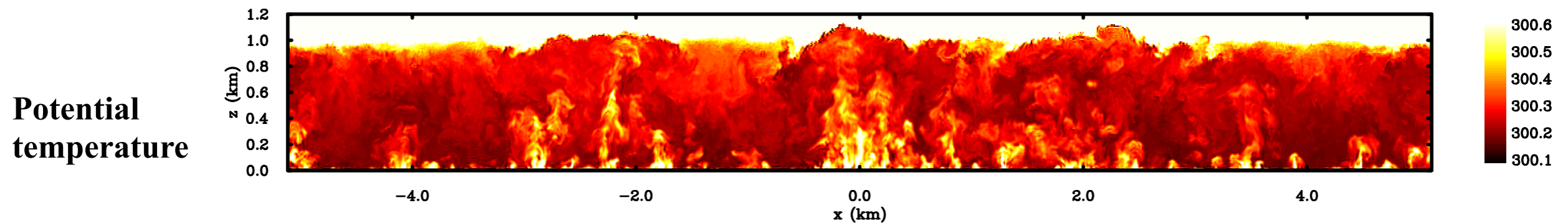
**0.18**

**0.20**

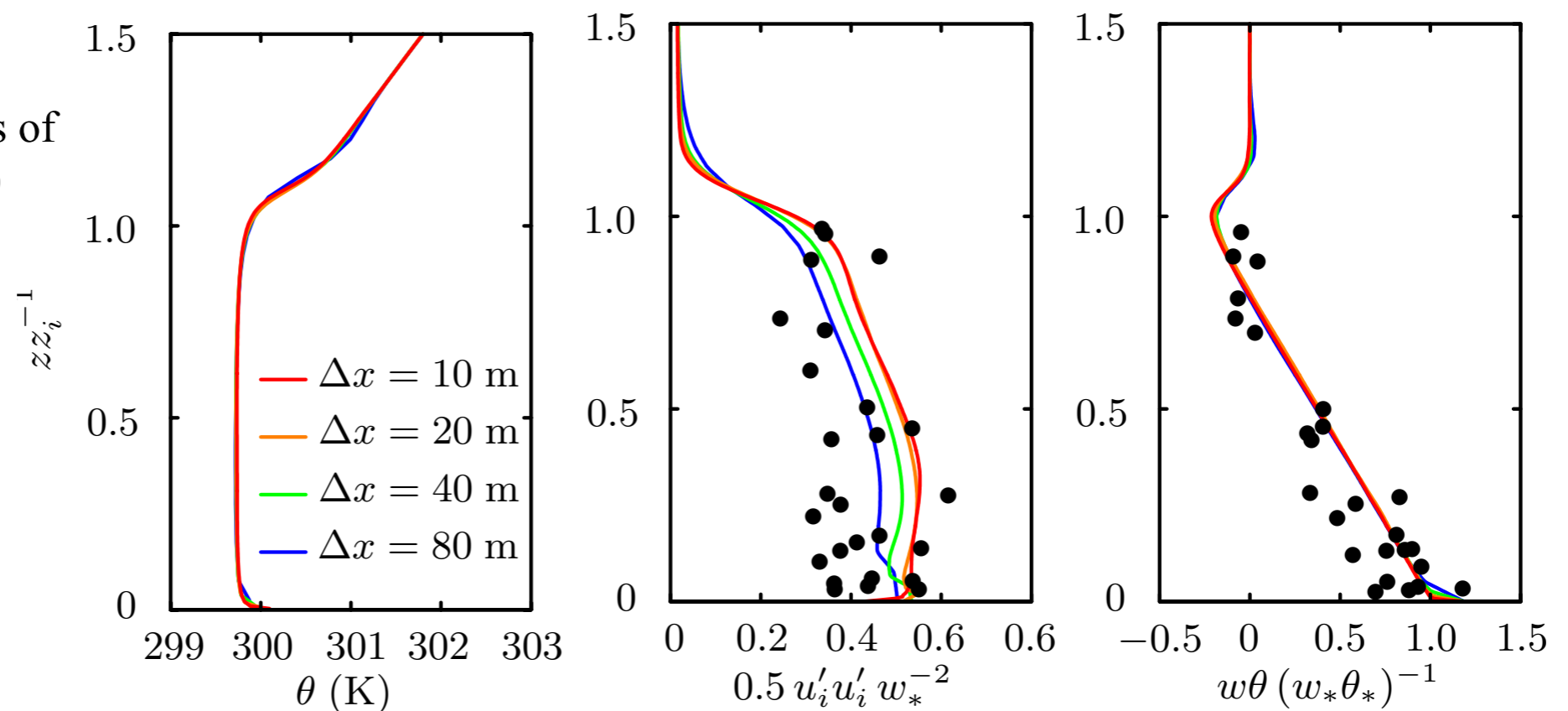


# Convective boundary layer

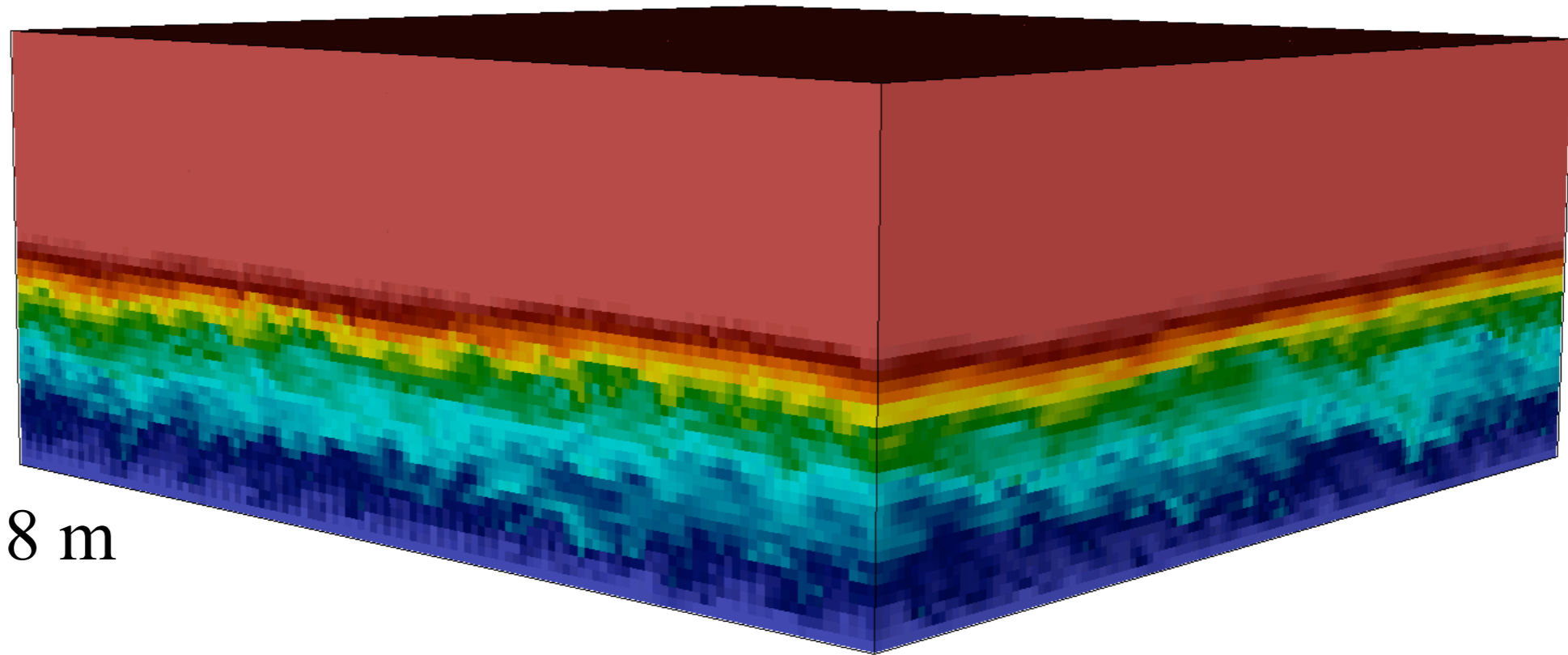
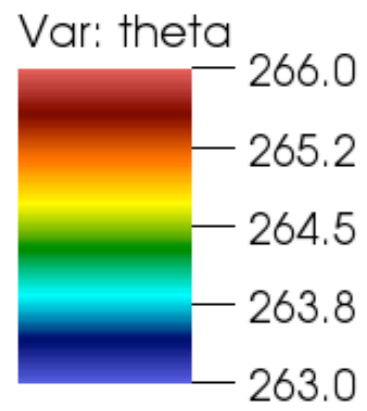
- Quality assessment of LES predictions: comparison to measurements and grid convergence (and theory – not shown here)
- Grid convergence is prerequisite for any predictive model



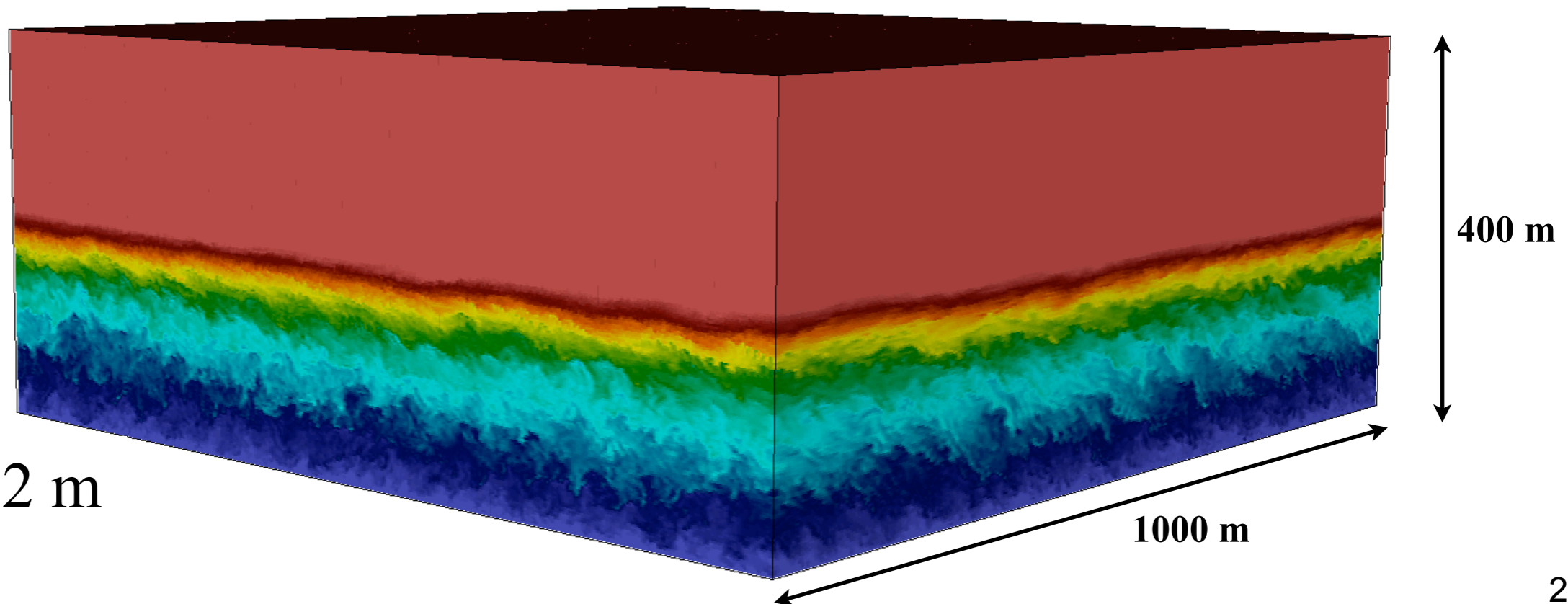
Circles: Measurements of Lenschow et al. (1980)



# Potential temperature field at two grid resolutions



$\Delta x = 2 \text{ m}$



# Monin-Obukhov local scaling

